ECONOMICS 207
EXAM 4
SPRING 2008

Hints.

(1) Remember rules about dividing by 3 and 9.
(2) $\frac{171}{9} = 19$
(3) $19 \times 18 = 342$
(4) $729 \times 3 = 2187$
(5) $81 \times 50 = 4050$

Problem 1 (25 Points)**.** Consider the following matrix and vector.

$$P = \begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix}, \quad p = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

a. Use elementary row operations to find the inverse of P and solve the equation Px=p in one set of operations.

Date: April 7, 2008.

b. Find the determinant of the matrix P.

$$P = \begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix}, \quad p = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

c. Find the inverse of the matrix P using the cofactor/adjoint method.

d. Solve the equation Px=p using the inverse you found in part 1c

e. Solve the equation Px=p using Cramer's rule.

$$P = \begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix}, \quad p = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

Problem 2 (30 Points)**.**

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 7 & 3 \\ 4 & -6 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -8 \\ 7 \end{bmatrix}$$

a. Use elementary row operations to find the inverse of A and solve the equation Ax=b in one set of operations.

More space and partial answers on next page.

More space for problem 2.

$$A^{-1} = \begin{bmatrix} & & \\ & & \\ -10 & -2 & 1 \end{bmatrix}$$

b. Show that the determinant of the matrix A is equal to one, i.e., $|A| = 1$.

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 7 & 3 \\ 4 & -6 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -8 \\ 7 \end{bmatrix}$$

c. Find the inverse of the matrix A using the cofactor/adjoint method.

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 7 & 3 \\ 4 & -6 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -8 \\ 7 \end{bmatrix}$$

$$cof(A) = \begin{bmatrix} & & -10 \\ 0 & 1 & \\ & & 1 \end{bmatrix}$$

d. Solve the equation Ax=b using the inverse you found in part 2c

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 7 & 3 \\ 4 & -6 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -8 \\ 7 \end{bmatrix}$$

e. Solve the equation Ax=b using Cramer's rule.

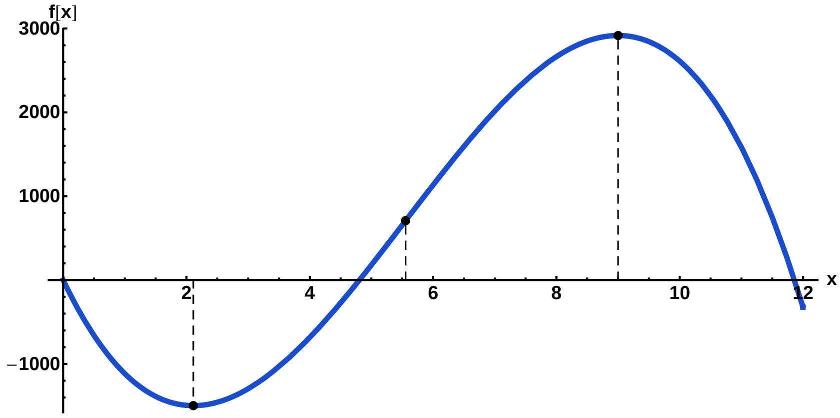
Hint: $\begin{vmatrix} 2 & -2 & -1 \\ -8 & 7 & 3 \\ 7 & -6 & -3 \end{vmatrix} = 1$

Problem 3. [15 Points]

For the following problem, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

$$f(x) \;=\; -27x^3 + 450x^2 - 1539x$$

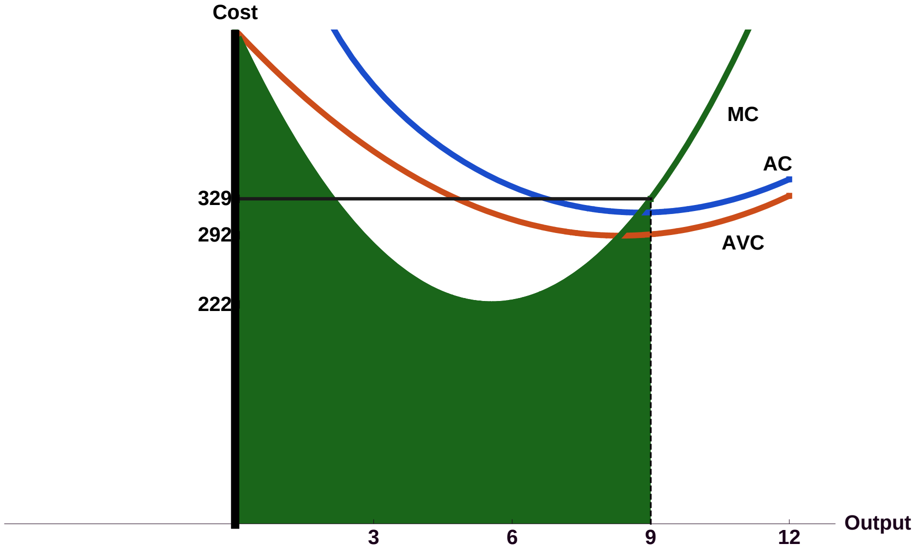
FIGURE 1. $f(x) \;=\; -27x^3 + 450x^2 - 1539x$



Problem 4. [20 Points]

The cost function for a firm is a rule or mapping that tells the minimum total cost of production of any output level produced by the firm for a fixed level of input prices. If the variable y represents the output of the firm, then the cost function is given by c(y,w) or c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output, i.e., Marginal Cost(MC) $= \frac{dc(y)}{dy}$. A graph of price and marginal cost is given in figure 2.

FIGURE 2. Profit Maximization Given Marginal Cost



a. Consider a competitive firm with the following technology (as represented by its cost function) and output price.

$$price = p = \$329$$

$$cost = c(y) = 200 + 500y - 50y^2 + 3y^3$$

Without writing down an equation for profit, find the levels of output which potentially maximize profit using what you have learned in general about profit maximization for a competitive firm.

b. Given that you have no second order conditions from profit maximization per se, make a coherent argument about which of the two potential output levels you found in part 4a maximizes profit.

c. Explain in words why setting price equal to marginal cost and solving for the optimal output y gives the same answers as taking the derivative of profit with respect to y, setting the result equal to zero and solving for the optimal y. Remember that

$$Profit = py - c(y)$$

$$Profit = 343y - \left[200 + 500y - 50\,y^2 + 3y^3\right]$$

d. Show that variable cost for this firm when it maximizes profit with a price of \$329 is \$2637.

e. Given that revenue for this profit maximizing firm is \$2961, what is producer surplus?

f. Cross-hatch this level of producer surplus in Figure 2.