ECONOMICS 207
SPRING 2008
EXAM 5

Some Hints:

(1) Allocate your time efficiently. Spend time on problems in proportion to points possible.
(2) $2^6 = 4^3 = 64 \Rightarrow 64^{1/3} = 4$
(3) $2^{12} = 2^6 \times 2^6 = 64^2 = 4096$
(4) $2^5 \times 2^6 = 2^{11} = 2048$
(5) $2^{11} \times 3 = 6144$
(6) $16^{9/4} = 2^9$

For each of the first three problems, write an equation that represents profit as a function of the two inputs x_1 and x_2. Find levels of of x_1 and x_2 that potentially maximize profit. Show that the levels of x_1 and x_2 you find actually maximize profit by checking second order conditions.

Date: April 23, 2008.

Problem 1 (25 Points)**.**

a. What is the profit equation for the following firm

$$
\begin{aligned}
f(x_1, x_2) &= 10x_1 + 30x_2 - x_1^2 + x_1x_2 - 2x_2^2 \\
p &= 3 \\
w_1 &= 9, \qquad w_2 \;=\; 6
\end{aligned}
$$

b. Fill in the following table of derivatives of the profit equation.

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2} = 84 + 3x_1 - 12x_2$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -6$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} =$ -8

c. Find potential profit maximizing levels of x_1 and x_2.

d. By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

Problem 2 (25 Points)**.**

a. What is the profit equation for the following firm

$$\begin{aligned} f(x_1, x_2) &= 100x_1 + 50x_2 - 2x_1^2 + x_1x_2 - 2x_2^2 \\ p &= 2 \\ w_1 &= 90, \qquad w_2 = 60 \\ x_1 &= 16 \end{aligned}$$

b. Fill in the following table of derivatives of the profit equation.

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 2$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

c. Find potential profit maximizing levels of x_1 and x_2.

d. By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

Problem 3 (35 Points).

a. What is the profit equation for the following firm

$$\begin{aligned} f(x_1, x_2) &= x_1^{1/4} x_2^{1/3} \\ p &= 24 \\ w_1 &= 3, \qquad w_2 = 1 \\ x_1 &= 16 \end{aligned}$$

b. Fill in the following table of derivatives of the profit equation.

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 2 x_1^{-3/4} x_2^{-2/3}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

c. Find potential profit maximizing levels of x_1 and x_2.

d. In this table fill in values of x_1 and x_2 given to obtain numerical answers for the Hessian matrix.

$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -\frac{1}{96}$

e. By evaluating the Hessian matrix of the profit equation at the critical values, verify the optimal levels of x_1 and x_2.

Problem 4 (15 Points)**.** Find the listed partial derivatives of each of the following function.

$$\mathcal{L}(x_1, x_2, \lambda) = \left(10x_1 + 30x_2 - x_1^2 + x_1x_2 - 2x_2^2\right) - \lambda\left(9x_1 + 6x_2 - 126\right)$$

$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1}$	$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2}$	$\dfrac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda}$
$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial x_2} =$	$-\frac{\partial^2 \mathcal{L}}{\partial x_1 \partial \lambda} = 9$
$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial x_2} =$	$-\frac{\partial^2 \mathcal{L}}{\partial x_2 \partial \lambda} =$
$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_1}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial x_2}$	$-\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda}$