ECONOMICS 207
SPRING 2008
LABORATORY EXERCISE 10

For this laboratory exercise, consider the following matrices and vectors. You may want to tear this page off so it is easy to view.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & -1 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ -2 & 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -5 & 2 & 0 \\ 1 & -1 & -1 \\ -4 & 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -2 \\ 4 & -6 & 3 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & -4 \\ -4 & -5 & 3 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 1 & 1 \\ 7 & 11 & 10 \\ -8 & -13 & -12 \end{bmatrix}$$

$$a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Date: March 25, 2008.

Problem 1.

a. Use elementary row operations to solve the following system of equations.

$$\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$$

b. Use elementary row operations to solve the following system of equations.

$$\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 20 \\ 10 \end{pmatrix}$$

Problem 2.

a. Use elementary row operations to solve the following system of equations. The answers are x_1 = -9, x_2 = -4, x_3 = 3.

$$Dx = b$$

$$\begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -2 \\ 4 & -6 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$

b. Use elementary row operations to solve the following system of equations. Use elementary row operations to solve the following system of equations. The answers are $x_1 = 7$, $x_2 = -5$, $x_3 = 0$.

$$Ex = b$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & -4 \\ -4 & -5 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$

Problem 3.

a. Use elementary row operations to find the inverse of the following matrix.

$$\begin{pmatrix} -2 & 3 \\ 3 & 4 \end{pmatrix}$$

The answer is $\begin{pmatrix} \frac{-4}{17} & \frac{3}{17} \\ \frac{3}{17} & \frac{2}{17} \end{pmatrix}$.

b. Use elementary row operations to find the inverse of the following matrix.

$$\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$$

The answer is $\begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$.

Problem 4.

a. Use elementary row operations to find the inverse of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & -1 & 6 \end{bmatrix}$$

The answer is

$$A^{-1} = \begin{bmatrix} 14 & -5 & -1 \\ -24 & 9 & 2 \\ -11 & 4 & 1 \end{bmatrix}$$

b. Use elementary row operations to find the inverse of the following matrix.

$$F = \begin{bmatrix} 1 & 1 & 1 \\ 7 & 11 & 10 \\ -8 & -13 & -12 \end{bmatrix}$$

The answer is

$$F^{-1} = \begin{bmatrix} 2 & 1 & 1 \\ -4 & 4 & 3 \\ 3 & -5 & -4 \end{bmatrix}$$

Problem 5.

a. Use the inverse you found in problem 3a to solve the following equation.

$$\begin{pmatrix} -2 & 3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \end{pmatrix}$$

The answer is

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

b. Use the inverse you found in problem 3b to solve the following equation.

$$\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Problem 6. Use the inverse you found in problem 4a to solve the following equation.

$$Ax = a$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & -1 & 6 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Problem 7. Compute the determinants of the following matrices.

a.
$$\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$$

b.
$$\begin{pmatrix} -2 & 3 \\ 3 & 4 \end{pmatrix}$$

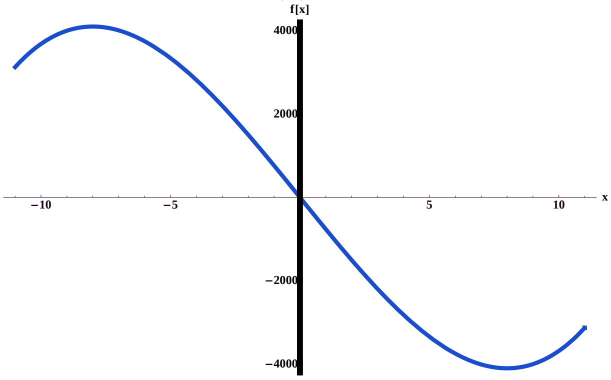
c.
$$\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix}$$

d.
$$\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$$

Problem 8. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

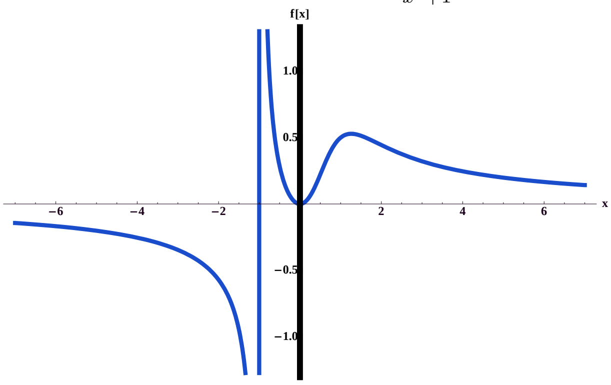
a. $f(x) = 4x^3 - 768x$.

FIGURE 1. $f(x) = 4x^3 - 768x$



b. $f(x) = \frac{x^2}{x^3+1}$

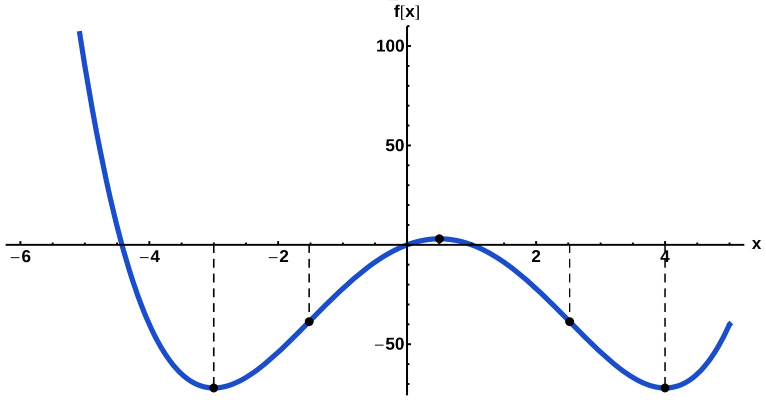
FIGURE 2. $f(x) = \frac{x^2}{x^3+1}$



Hints: The second derivative of the function is $\frac{2(x^6-7x^3+1)}{(x^3+1)^3}$. To solve the following equation $x^6 - 7x^3 + 1 = 0$, use the substitution z = x^3. The real points of inflection (you still need to find them) are $\left(\frac{7\pm3\sqrt{5}}{2}\right)^{1/3}$.

c. $f(x) \; = \; \dfrac{x^4}{2} - x^3 - \frac{23}{2}x^2 + 12x$

FIGURE 3. $f(x) \; = \; \dfrac{x^4}{2} - x^3 - \frac{23}{2}x^2 + 12x$



The inflection points are $\frac{3 \pm 7\sqrt{3}}{6}$.