ECONOMICS 207
SPRING 2008
LABORATORY EXERCISE 11

Problem 1. Consider the following matrix and vector.

$$P = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}, \quad p = \begin{bmatrix} 8 \\ 18 \end{bmatrix},$$

a. Use elementary row operations to find both the inverse of P and solve the equation Px=p in one set of operations.

Date: April 1, 2008.

b. Find the determinant of the matrix P.

$$P = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}, \quad p = \begin{bmatrix} 8 \\ 18 \end{bmatrix},$$

c. Find the inverse of the matrix P using the cofactor/adjoint method.

d. Solve the equation Px=p using the inverse you found in part 1c

e. Solve the equation Px=p using Cramer's rule.

$$P = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}, \quad p = \begin{bmatrix} 8 \\ 18 \end{bmatrix},$$

Problem 2. Consider the following matrix and vector.

$$Q = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, \quad q = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$

a. Use elementary row operations to find both the inverse of Q and solve the equation Qx=q in one set of operations.

b. Find the determinant of the matrix Q.

$$Q = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, \quad q = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$

c. Find the inverse of the matrix Q using the cofactor/adjoint method.

d. Solve the equation Qx=q using the inverse you found in part 2c

e. Solve the equation Qx=q using Cramer's rule.

$$Q \;=\; \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, \quad q \;=\; \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Problem 3. Consider the following matrix and vector.

$$F = \begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -6 \\ 2 & -3 & 3 \end{bmatrix}, \quad f = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

a. Use elementary row operations to find both the inverse of F and solve the equation Fx=f in one set of operations.

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b. Find the determinant of the matrix F.

$$F = \begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -6 \\ 2 & -3 & 3 \end{bmatrix}, \quad f = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

c. Find the inverse of the matrix F using the cofactor/adjoint method.

$$F = \begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -6 \\ 2 & -3 & 3 \end{bmatrix}, \quad f = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

d. Solve the equation Fx=f using the inverse you found in part 3c

$$F = \begin{bmatrix} 1 & -1 & 2 \\ -4 & 5 & -6 \\ 2 & -3 & 3 \end{bmatrix}, \quad f = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

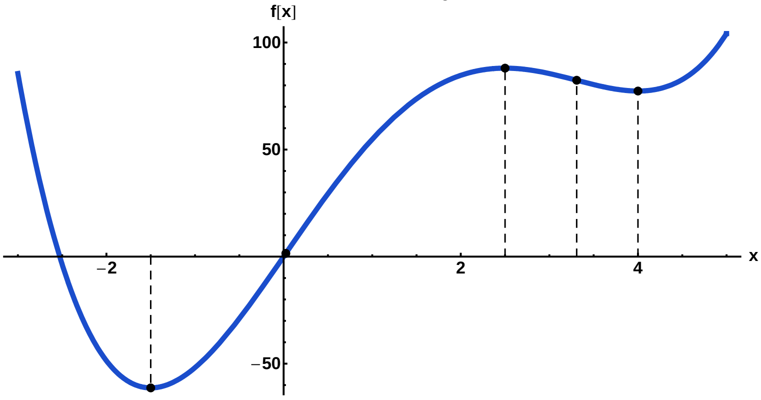
e. Solve the equation Fx=f using Cramer's rule.

Problem 4. This is a free problem.

Problem 5. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

a. $f(x) \ = x^4 - \frac{20}{3}x^3 + \frac{x^2}{2} + 60x$.

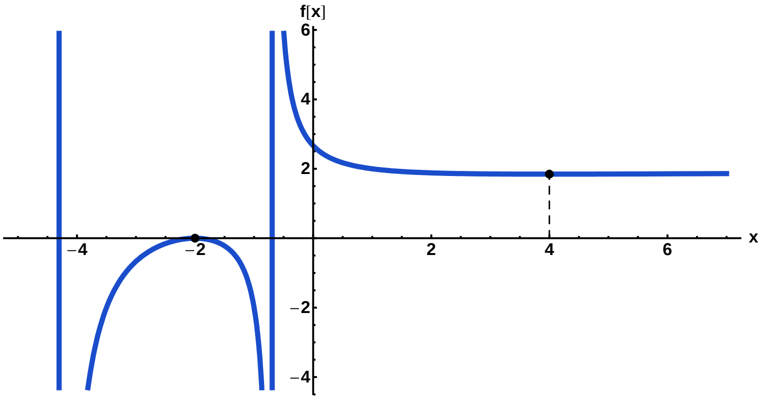
FIGURE 1. $f(x) \ = x^4 - \frac{20}{3}x^3 + \frac{x^2}{2} + 60x$



The inflection points are $\frac{1}{6}\left(10 \pm \sqrt{97}\right)$.

b. $f(x) = \dfrac{2x^2+8x+8}{x^2+5x+3}$ You need not find the points of inflection for this problem. Hint: You should find the second derivative of f(x) but here is the answer: $-\frac{4\left(x^3-3x^2-24x-37\right)}{(x^2+5x+3)^3}$.

FIGURE 2. $f(x) = \dfrac{2x^2+8x+8}{x^2+5x+3}$



Problem 6. Solve the following system of equations.

$$360x_1^{-2/3}x_2^{2/5} - 160 = 0$$

$$432x_1^{1/3}x_2^{-3/5} - 162 = 0$$

$x_1 = 27.$

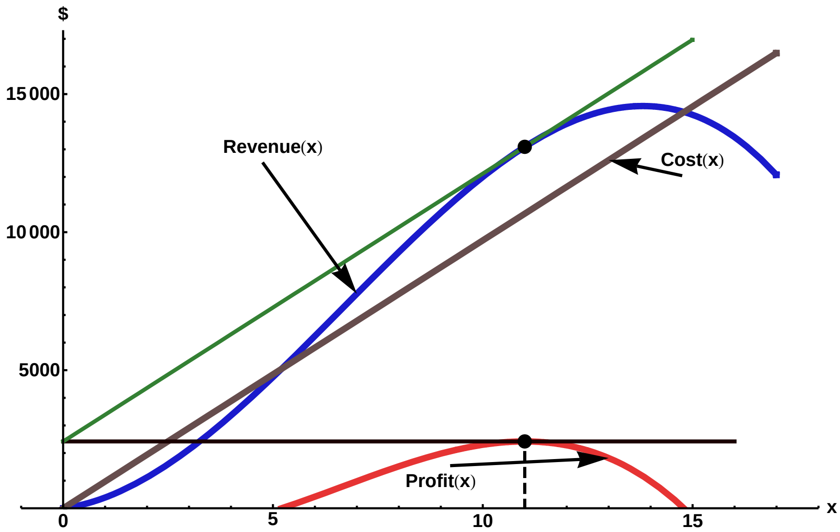
Problem 7. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input.

$$output\ price = p = 5$$

$$input\ price = w = 970$$

$$y = output = f(x) = 40x + 40x^2 - 2x^3$$

Figure 3. Revenue, Cost and Profit



a. Write down an equation that represents profit for the firm.

b. Maximize this function by taking its derivative with respect to the variable input x and setting the resulting equation equal to zero.

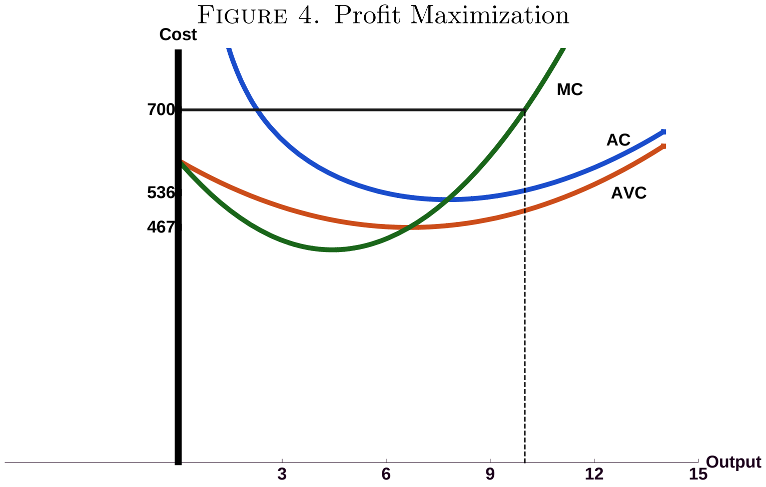
c. If you identify more than one critical value from setting the first derivative of profit equal to zero, show which ones, if any, maximize profit.

d. Explain in words why the value of the marginal product for this firm is equal to the price of the single variable input at the profit maximizing level of input use. You can use the following information in explaining this phenomenon. Say something about the benefits of using an input not being less than the cost of the input.

$$\begin{aligned}
Output =& y = f(x) \\
MP =& \text{Marginal Product} = \frac{d\,f(x)}{dx} = f'(x) = \frac{\Delta y}{\Delta x} \\
Revenue =& pf(x) \\
Cost =& wx \\
Profit = \pi =& Revenue - Cost = pf(x) - wx \\
\frac{d\pi}{dx} =&
\end{aligned}$$

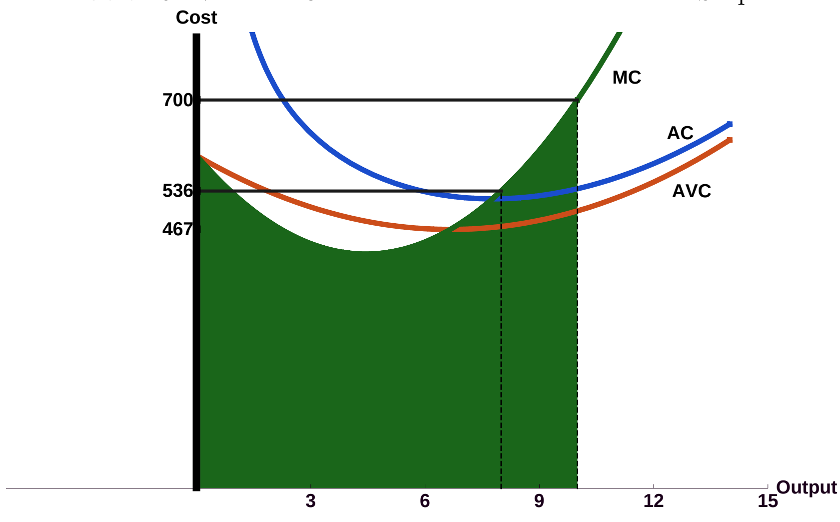
Problem 8. The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output,i.e., Marginal Cost(MC) $= \frac{dc(y)}{dy}$. A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price as in the figure 4.

FIGURE 4. Profit Maximization



The area below the cost curve is a measure of variable cost and can be found by integrating the marginal cost curve from 0 to any given output level y. The shaded area in figure 5 represents the variable cost of production for the cost function $c(y) = 400 + 600y - 40\,y^2 + 3y^3$.

FIGURE 5. Variable Cost of Production and Producer Surplus



Producer surplus is the area below a given price and above the marginal cost curve. Producer surplus is the unshaded area below the horizontal line at 700 in figure 5. Producer surplus can be computed by subtracting the shaded area from total revenue.

a. Without writing down an equation for profit, find the profit maximizing level of output for the following firm.

$$price = p = \$700$$

$$cost = c(y) = 400 + 600y - 40y^2 + 3y^3$$

Given that you have no second order conditions from profit maximization per se, how do you know which level of output to choose? You might check the derivative of marginal cost and whether MC is upward or downward sloping at each point.

b. Find the profit maximizing level of output for the same firm when the price is \$536.

c. Explain in words why setting price equal to marginal cost and solving for the optimal output y gives the same answers as taking the derivative of profit with respect to y, setting the result equal to zero and solving for the optimal y. Remember that

$$Profit = py - c(y)$$

$$Profit = 700y - \left[400 + 600y - 40\,y^2 + 3y^3\right]$$

d. Show that variable cost for this firm when it maximizes profit with a price of \$700 is \$5000.

e. What is producer surplus for this profit maximizing firm when the price is \$700?

f. Show that variable cost for this firm when it maximizes profit with a price of \$536 is \$3776.

g. What is producer surplus for this profit maximizing firm when the price is \$536?

h. How much is the firm worse off when price falls from \$700 to \$536?

i. Cross-hatch the change in producer surplus in Figure 5.