ECONOMICS 207
SPRING 2008
LABORATORY EXERCISE 12

Problem 1. Consider the following matrix and vector.

$$P = \begin{bmatrix} 2 & 4 \\ -2 & -6 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 6 \end{bmatrix},$$

a. Use elementary row operations to find both the inverse of P and solve the equation Px=p in one set of operations.

Date: April 9, 2008.

b. Find the determinant of the matrix P.

$$P = \begin{bmatrix} 2 & 4 \\ -2 & -6 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 6 \end{bmatrix},$$

c. Find the inverse of the matrix P using the cofactor/adjoint method.

d. Solve the equation Px=p using the inverse you found in part 1c

e. Solve the equation Px=p using Cramer's rule.

$$P = \begin{bmatrix} 2 & 4 \\ -2 & -6 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 6 \end{bmatrix},$$

Problem 2. Consider the following matrix and vector.

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 6 & -11 & 10 \\ 2 & -3 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -5 \\ -1 \end{bmatrix}$$

a. Use elementary row operations to find both the inverse of A and solve the equation Ax=b in one set of operations.

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$$A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ 2 & -1 & 2 \\ 4 & -1 & 1 \end{bmatrix}$$

b. Find the determinant of the matrix A.

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 6 & -11 & 10 \\ 2 & -3 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -5 \\ -1 \end{bmatrix}$$

c. Find the inverse of the matrix A using the cofactor/adjoint method.

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 6 & -11 & 10 \\ 2 & -3 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -5 \\ -1 \end{bmatrix}$$

d. Solve the equation Ax=b using the inverse you found in part 2c

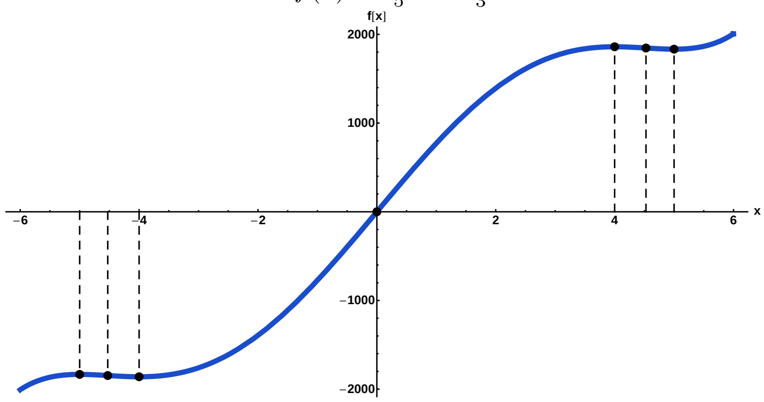
$$A = \begin{bmatrix} 1 & -2 & 2 \\ 6 & -11 & 10 \\ 2 & -3 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -5 \\ -1 \end{bmatrix}$$

e. Solve the equation Ax=b using Cramer's rule.

Problem 3. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

a. $f(x) \ = \dfrac{2x^5}{5} - \dfrac{82x^3}{3} + 800x.$

Figure 1. $f(x) \ = \frac{2}{5}x^5 - \frac{82}{3}x^3 + 800x$



The inflection points are $\pm\sqrt{\dfrac{41}{2}}$.

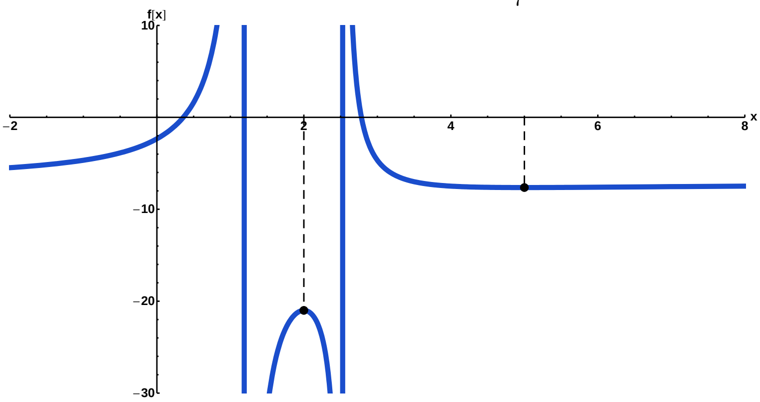
b. $f(x) = \dfrac{7x^2 - 22x + 7}{-x^2 + \frac{26x}{7} - 3}$

You need not find the points of inflection for this problem.

Hint: The simplified version of the first derivative is $\dfrac{4\left(10 - 7x + x^2\right)}{\left(-x^2 + \frac{26x}{7} - 3\right)^2} = \dfrac{196\left(10 - 7x + x^2\right)}{\left(21 - 26x + 7x^2\right)^2}$

You should find the second derivative of f(x) but here is the answer: $-\frac{196\left(-373+420x-147x^2+14x^3\right)}{\left(21-26x+7x^2\right)^3}$.

Figure 2. $f(x) = \dfrac{7x^2 - 22x + 7}{-x^2 + \frac{26x}{7} - 3}$



Problem 4. Solve the following system of equations.

$$72x_1^{-3/5}x_2^{1/4} - 8 = 0$$

$$45x_1^{2/5}x_2^{-3/4} - 15 = 0$$

This problem is for extra credit only

Problem 5. Find all first and second partial derivatives of each of the following

a. $f(x_1, x_2) = -2x_1^2 - x_2^2 + 100x_1 + 50x_2$

$\frac{\partial f}{\partial x_1} = -4x_1 + 100$	$\frac{\partial f}{\partial x_2} = -2x_2 + 50$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} = -4$	$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

b. $f(x_1, x_2) = -4x_1^2 + 3x_1x_2 - 2x_2^2 + 80x_1 + 40x_2$

$\frac{\partial f}{\partial x_1} = -8x_1 + 3x_2 + 80$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} = -8$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

c. $f(x_1, x_2) = -2x_1^2 + x_1 x_2 + 3x_2^2 + 40x_1 - 60x_2$

$\frac{\partial f}{\partial x_1} = -4x_1 + x_2 + 40$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

d. $f(x_1, x_2, x_3) = 100x_1^{1/2}x_2^{1/5}x_3^{1/3} - 4x_1 - 3x_2 - 5x_3$

$\frac{\partial f}{\partial x_1} = 50x_1^{-1/2}x_2^{1/5}x_3^{1/3} - 4$	$\frac{\partial f}{\partial x_2} =$	$\frac{\partial f}{\partial x_3} = \frac{100}{3}x_1^{1/2}x_2^{1/5}x_3^{-2/3} - 5$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} = -25x_1^{-3/2}x_2^{1/5}x_3^{1/3}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2} =$	$\frac{\partial^2 f}{\partial x_1 \partial x_3} =$
$\frac{\partial^2 f}{\partial x_2 \partial x_1} = 10x_1^{-1/2}x_2^{-4/5}x_3^{1/3}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$	$\frac{\partial^2 f}{\partial x_2 \partial x_3}$
$\frac{\partial^2 f}{\partial x_3 \partial x_1}$	$\frac{\partial^2 f}{\partial x_3 \partial x_2}$	$\frac{\partial^2 f}{\partial x_3 \partial x_3} = -\frac{200}{9}x_1^{1/2}x_2^{1/5}x_3^{-5/3}$

Problem 6. For each of the following problems, write an equation that represents profit as a function of the two inputs x_1 and x_2. Write it in the form π = $pf(x_1,x_2)-w_1x_1-w_2x_2$ and then simplify the expression. Then find all first and second partial derivatives of the function at the specified point.

a.

$$f(x_1, x_2) = 60x_1 + 42x_2 - x_1^2 + x_1x_2 - x_2^2$$

$$p = 4$$

$$w_1 = 60, \qquad w_2 = 12$$

$$x_1 = 43, \qquad x_2 = 41$$

$$\begin{aligned}\pi =& 4(60x_1 + 42x_2 - x_1^2 + x_1x_2 - x_2^2) - 60x_1 - 12x_2 \\ =& 240x_1 + 168x_2 - 4x_1^2 + 4x_1x_2 - 4x_2^2 - 60x_1 - 12x_2 \\ =& 180x_1 + 156x_2 - 4x_1^2 + 4x_1x_2 - 4x_2^2\end{aligned}$$

$\frac{\partial \pi}{\partial x_1} = 180 - 8x_1 + 4x_2$	$\frac{\partial \pi}{\partial x_2} = 156 + 4x_1 - 8x_2$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -8$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 4$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

b.

$$
\begin{aligned}
f(x_1, x_2) &= 30x_1 \ + \ 15x_2 \ - \ 2x_1^2 \ + \ x_1x_2 \ - \ x_2^2 \\
p &= 4 \\
w_1 &= 60, \qquad w_2 \ = \ 12 \\
x_1 &= 6, \qquad x_2 \ = \ 9
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

c.

$$
\begin{aligned}
f(x_1, x_2) &= 40x_1 \ + \ 10x_2 \ - \ 2x_1^2 \ + \ 2x_1x_2 \ - \ x_2^2 \\
p &= 1 \\
w_1 &= 34, \qquad w_2 \ = \ 2 \\
x_1 &= 7, \qquad x_2 \ = \ 11
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

d.

$$
\begin{aligned}
f(x_1, x_2) &= x_1^{3/5} x_2^{1/3} \\
p &= 240 \\
w_1 &= 64, \qquad w_2 \;=\; 135 \\
x_1 &= 243, \qquad x_2 \;=\; 64
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -\frac{288}{5} x_2^{1/3} x_1^{-7/5}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 48 x_1^{-2/5} x_2^{-2/3}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} =$

In this table fill in values of x_1 and x_2 given to obtain numerical answers.

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -\frac{45}{32}$

e.

$$\begin{aligned} f(x_1, x_2) &= x_1^{1/4} x_2^{3/7} \\ p &= 224 \\ w_1 &= 7, \qquad w_2 \;=\; 24 \\ x_1 &= 256, \qquad x_2 \;=\; 128 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -\frac{384}{7} x_1^{1/4} x_2^{-11/7}$

In this table fill in values of x_1 and x_2 given to obtain numerical answers.

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -\frac{21}{1024}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -\frac{3}{28}$