ECONOMICS 207
SPRING 2008
LABORATORY EXERCISE 1
KEY

Problem 1. Consider the following six sets.

$$\begin{aligned} A &= \{1, 2, 4\} \\ B &= \{2, 3, 5\} \\ C &= \{1, 2, 3, 4\} \\ D &= \{2, 3, 4, 5, 6\} \\ E &= \{2, 3, 4, 5, 6\} \\ F &= \{0, 1, 2, 4, 5, 7\} \\ U &= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \end{aligned}$$

a. A is a subset of which other sets?

A is a subset of A, C, F, U.

b. What is A $\cap$ B?

$$A \cap B = \{2\}.$$

c. What is C $\cap$ F?

$$C \cap F = \{1, 2, 4\}.$$

Date: July 1, 2008.

d. What is A ∩ B ∩ C?

$$A \cap B = \{2\},$$
$$A \cap B \cap C = \{2\}.$$

e. What is A ∪ (B ∩ D)?

$$B \cap D = \{2, 3, 5\},$$
$$A \cup (B \cap D) = \{1, 2, 3, 4, 5\}.$$

f. What is (A ∪ B) ∩ D?

$$A \cup B = \{1, 2, 3, 4, 5\},$$
$$(A \cup B) \cap D = \{2, 3, 4, 5\}.$$

g. What is (A ∪ B) ∩ (A ∪ D)?

$$A \cup B = \{1, 2, 3, 4, 5\},$$
$$A \cup D = \{1, 2, 3, 4, 5, 6\},$$
$$(A \cup B) \cap (A \cup D) = \{1, 2, 3, 4, 5\}.$$

h. What is A ∩ (B ∪ D)?

$$B \cup D = \{2, 3, 4, 5, 6\},$$
$$A \cap (B \cup D) = \{2, 4\}.$$

i. What is (A ∩ B) ∪ (A ∩ D)?

$$A \cap B = \{2\},$$
$$A \cap D = \{2, 4\},$$
$$(A \cap B) \cup (A \cap D) = \{2, 4\}.$$

j. What is (A ∩ B) ∪ D?

$$A \cap B = \{2\},$$
$$(A \cap B) \cup D = \{2, 3, 4, 5, 6\}.$$

k. What is (A $\cup$ D) $\cap$ (B $\cup$ D)?

$$A \cup D = \{1, 2, 3, 4, 5, 6\},$$
$$B \cup D = \{2, 3, 4, 5, 6\},$$
$$(A \cup D) \cap (B \cup D) = \{2, 3, 4, 5, 6\}.$$

l. What is (A $\cap$ D) $\cup$ (B $\cap$ D)?

$$A \cap D = \{2, 4\},$$
$$B \cap D = \{2, 3, 5\},$$
$$(A \cap D) \cup (B \cap D) = \{2, 3, 4, 5\}.$$

m. Given U, what is A^C?

$$A^C = \{0, 3, 5, 6, 7, 8, 9, 10\}.$$

n. Given U, what is $(A \cup B)^C$?

$$A \cup B = \{1, 2, 3, 4, 5\},$$
$$(A \cup B)^C = \{0, 6, 7, 8, 9, 10\}.$$

o. Given U, what is $(A \cap B)^C$?

$$A \cap B = \{2\},$$
$$(A \cap B)^C = \{0, 1, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

p. What is D $\setminus$ E?

$$D \setminus E = \emptyset.$$

q. What is F $\setminus$ A?

$$F \setminus A = \{0, 5, 7\}.$$

Problem 2. Consider the following sets.

$$\begin{aligned}
A &= \left\{\frac{a}{b} : a \in \{0,1,2,3,4\}, -1 \leq b \leq 3 \text{ and } b \in \text{integers},\ b \neq 0\right\} \\
B &= \{\{x,y\} : x+y=5, x<3 \text{ and } x \in \text{natural numbers}, y \leq 7\} \\
C &= \{\{x,y\} : x+y=5, x<10 \text{ and } x \in \text{natural numbers},\ y \leq 7 \text{ and } y \in \text{integers}\} \\
D &= \{\{x,y\} : x+2y=12, x<10 \text{ and } x \in \text{natural numbers},\ y \leq 7 \text{ and } y \in \text{integers}\} \\
E &= \{\{x,y\} : x+2y=12, x<8 \text{ and } x \in \text{integers},\ y \leq 7\} \\
F &= \{\{x,y\} : 4x+y=-1, x<2 \text{ and } x \in \text{integers},\ y \leq 7\} \\
G &= \{\{x,y\} : x+y=5, x<2 \text{ and } x \in \text{integers},\ y \leq 7\} \\
X &= \{\{x,y\} : |x|<10,\ |y|<5\}
\end{aligned}$$

a. List or show the elements of each of the sets: A, B, C, D , E, F, G, and X.
Hints: For set A, first find acceptable numbers for b.
For sets B,C,D,E,F and G, the set will be composed of ordered pairs (x,y).

$$\begin{aligned}
A &= \{-4,-3,-2,-1,0,\frac{1}{3},\frac{1}{2},\frac{2}{3},1,\frac{4}{3},\frac{3}{2},2,3,4\} \\
B &= \{\{1,4\},\{2,3\}\}, \\
C &= \{\{1,4\},\{2,3\},\{3,2\},\{4,1\},\{5,0\},\{6,-1\},\{7,-2\},\{8,-3\},\{9,-4\}\}, \\
D &= \{\{2,5\},\{4,4\},\{6,3\},\{8,2\}\}, \\
E &= \left\{\{-2,7\},\{-1,\frac{13}{2}\},\{0,6\},\{1,\frac{11}{2}\},\{2,5\},\{3,\frac{9}{2}\},\{4,4\},\{5,\frac{7}{2}\},\{6,3\},\{7,\frac{5}{2}\}\right\}, \\
F &= \{\{-2,7\},\{-1,3\},\{0,-1\},\{1,-5\}\}, \\
G &= \{\{-2,7\},\{-1,6\},\{0,5\},\{1,4\}\},
\end{aligned}$$

There are infinite elements in X, hence it can not be listed.

b. What is A ∩ B?

$$A \cap B = \emptyset.$$

c. What is B ∩ C?

$$B \cap C = \{\{1,4\},\{2,3\}\}.$$

d. What is B ∩ D?

$$B \cap D = \emptyset.$$

e. What is B ∩ E?

$$B \cap E = \emptyset.$$

f. What is E ∩ F?

$$E \cap F = \{\{-2,7\}\}.$$

g. What is E ∩ G?

$$E \cap G = \{\{-2,7\}\}.$$

h. What is E ∩ F ∩ G ?

$$E \cap F \cap G = \{\{-2,7\}\}.$$

Problem 3. Simplify the following fractions.

a. $\frac{16}{20}$

$$\frac{16}{20} = \frac{16/4}{20/4} = \frac{4}{5}.$$

b. $\frac{112}{77}$

$$\frac{112}{77} = \frac{112/7}{77/7} = \frac{16}{11}.$$

c. $\frac{441}{189}$

$$\frac{441}{189} = \frac{441/63}{189/63} = \frac{7}{3}.$$

d. $\frac{4158}{2160}$

$$\frac{4158}{2160} = \frac{4158/54}{2160/54} = \frac{77}{40}.$$

e. $\frac{426888}{27720}$

$$\frac{426888}{27720} = \frac{426888/5544}{27720/5544} = \frac{77}{5}.$$

f. $\frac{15015}{35343}$

$$\frac{15015}{35343} = \frac{15015/231}{35343/231} = \frac{65}{153}.$$

Problem 4. Complete the following operations.

a. $\frac{14}{16} + \frac{1}{4}$

$$\begin{aligned}\frac{14}{16} + \frac{1}{4} &= \frac{14}{16} + \frac{1 \times 4}{4 \times 4} \\ &= \frac{18}{16} \\ &= \frac{9}{8}.\end{aligned}$$

b. $\frac{\frac{112}{77}}{\frac{6}{7}}$

$$\begin{aligned}\frac{\frac{112}{77}}{\frac{6}{7}} &= \frac{\frac{112}{77} \times 77}{\frac{6}{7} \times 77} \\ &= \frac{112}{66} = \frac{112/2}{66/2} \\ &= \frac{56}{33}.\end{aligned}$$

c. $\frac{15}{28} + \frac{3}{7}$

$$\begin{aligned}\frac{15}{28} + \frac{3}{7} &= \frac{15}{28} + \frac{12}{28} \\ &= \frac{27}{28}.\end{aligned}$$

d. $\frac{17}{26} + \frac{7}{4}$

$$\begin{aligned}\frac{17}{26} + \frac{7}{4} &= \frac{17 \times 2}{26 \times 2} + \frac{7 \times 13}{4 \times 13} \\ &= \frac{34 + 91}{52} \\ &= \frac{125}{52}.\end{aligned}$$

e. $\frac{6}{7} + \frac{33}{84} + \frac{5}{6}$

$$\begin{aligned}\frac{6}{7}+\frac{33}{84}+\frac{5}{6} &= \frac{6\times 12}{7\times 12}+\frac{33}{84}+\frac{5\times 14}{6\times 14}\\ &= \frac{72+33+70}{84}\\ &= \frac{175}{84}=\frac{175/7}{84/7}\\ &= \frac{25}{12}.\end{aligned}$$

f. $\frac{7}{8}+\frac{11}{24}+\frac{1}{3}+\frac{7}{12}$

$$\begin{aligned}\frac{7}{8}+\frac{11}{24}+\frac{1}{3}+\frac{7}{12} &= \frac{7\times 3}{8\times 3}+\frac{11}{24}+\frac{1\times 8}{3\times 8}+\frac{7\times 2}{12\times 2}\\ &= \frac{21+11+8+14}{24}=\frac{54}{24}=\frac{54/6}{24/6}\\ &= \frac{9}{4}.\end{aligned}$$