Problem 1. Consider the following six sets.

\[ A = \{1, 2, 4\} \]
\[ B = \{2, 3, 5\} \]
\[ C = \{1, 2, 3, 4\} \]
\[ D = \{2, 3, 4, 5, 6\} \]
\[ E = \{2, 3, 4, 5, 6\} \]
\[ F = \{0, 1, 2, 4, 5, 7\} \]
\[ U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \]

a. A is a subset of which other sets?

\[ A \text{ is a subset of } A, C, F, U. \]

b. What is \( A \cap B \)?

\[ A \cap B = \{2\}. \]

c. What is \( C \cap F \)?

\[ C \cap F = \{1, 2, 4\}. \]
d. What is $A \cap B \cap C$?

\[
A \cap B = \{2\}, \\
A \cap B \cap C = \{2\}.
\]

e. What is $A \cup (B \cap D)$?

\[
B \cap D = \{2, 3, 5\}, \\
A \cup (B \cap D) = \{1, 2, 3, 4, 5\}.
\]

f. What is $(A \cup B) \cap D$?

\[
A \cup B = \{1, 2, 3, 4, 5\}, \\
(A \cup B) \cap D = \{2, 3, 4, 5\}.
\]

g. What is $(A \cup B) \cap (A \cup D)$?

\[
A \cup B = \{1, 2, 3, 4, 5\}, \\
A \cup D = \{1, 2, 3, 4, 5, 6\}, \\
(A \cup B) \cap (A \cup D) = \{1, 2, 3, 4, 5\}.
\]

h. What is $A \cap (B \cup D)$?

\[
B \cup D = \{2, 3, 4, 5, 6\}, \\
A \cap (B \cup D) = \{2, 4\}.
\]

i. What is $(A \cap B) \cup (A \cap D)$?

\[
A \cap B = \{2\}, \\
A \cap D = \{2, 4\}, \\
(A \cap B) \cup (A \cap D) = \{2\}.
\]

j. What is $(A \cap B) \cup D$?

\[
A \cap B = \{2\}, \\
(A \cap B \cup D) = \{2, 3, 4, 5, 6\}.
\]
k. What is \((A \cup D) \cap (B \cup D)\)?

\[
A \cup D = \{1, 2, 3, 4, 5, 6\},
\]

\[
B \cup D = \{2, 3, 4, 5, 6\},
\]

\[
(A \cup D) \cap (B \cup D) = \{2, 3, 4, 5, 6\}.
\]

l. What is \((A \cap D) \cup (B \cap D)\)?

\[
A \cap D = \{2, 4\},
\]

\[
B \cap D = \{2, 3, 5\},
\]

\[
(A \cap D) \cup (B \cap D) = \{2, 3, 4, 5\}.
\]

m. Given U, what is \(A^C\)?

\[
A^C = \{0, 3, 5, 6, 7, 8, 9, 10\}.
\]

n. Given U, what is \((A \cup B)^C\)?

\[
A \cup B = \{1, 2, 3, 4, 5\},
\]

\[
(A \cup B)^C = \{0, 6, 7, 8, 9, 10\}.
\]

o. Given U, what is \((A \cap B)^C\)?

\[
A \cap B = \{2\},
\]

\[
(A \cap B)^C = \{0, 1, 3, 4, 5, 6, 7, 8, 9, 10\}.
\]

p. What is \(D \setminus E\)?

\[
D \setminus E = \emptyset.
\]

q. What is \(F \setminus A\)?

\[
F \setminus A = \{0, 5, 7\}.
\]
Problem 2. Consider the following sets.

\[ A = \left\{ \frac{a}{b} : a \in \{0,1,2,3,4\}, -1 \leq b \leq 3 \text{ and } b \in \text{integers, } b \neq 0 \right\} \]

\[ B = \{ \{x, y\} : x + y = 5, x < 3 \text{ and } x \in \text{natural numbers, } y \leq 7 \} \]

\[ C = \{ \{x, y\} : x + y = 5, x < 10 \text{ and } x \in \text{natural numbers, } y \leq 7 \text{ and } y \in \text{integers} \} \]

\[ D = \{ \{x, y\} : x + 2y = 12, x < 10 \text{ and } x \in \text{natural numbers, } y \leq 7 \text{ and } y \in \text{integers} \} \]

\[ E = \{ \{x, y\} : x + 2y = 12, x < 8 \text{ and } x \in \text{integers, } y \leq 7 \} \]

\[ F = \{ \{x, y\} : 4x + y = -1, x < 2 \text{ and } x \in \text{integers, } y \leq 7 \} \]

\[ G = \{ \{x, y\} : x + y = 5, x < 2 \text{ and } x \in \text{integers, } y \leq 7 \} \]

\[ X = \{ \{x, y\} : |x| < 10, |y| < 5 \} \]

a. List or show the elements of each of the sets: A, B, C, D, E, F, G, and X.

Hint: For set A, first find acceptable numbers for b.

For sets B, C, D, E, F, and G, the set will be composed of ordered pairs (x, y).

\[ A = \{-4, -3, -2, -1, 0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1, \frac{4}{3}, \frac{3}{2}, 2, 3, 4\} \]

\[ B = \{\{1, 4\}, \{2, 3\}\} \]

\[ C = \{\{1, 4\}, \{2, 3\}, \{3, 2\}, \{4, 1\}, \{5, 0\}, \{6, -1\}, \{7, -2\}, \{8, -3\}, \{9, -4\}\} \]

\[ D = \{\{2, 5\}, \{4, 4\}, \{6, 3\}, \{8, 2\}\} \]

\[ E = \left\{ \{-2, 7\}, \{-1, \frac{13}{2}\}, \{0, 6\}, \{1, \frac{11}{2}\}, \{2, 5\}, \{3, \frac{9}{2}\}, \{4, 4\}, \{5, \frac{7}{2}\}, \{6, 3\}, \{7, \frac{5}{2}\} \right\} \]

\[ F = \{\{-2, 7\}, \{-1, 3\}, \{0, -1\}, \{1, -5\}\} \]

\[ G = \{\{-2, 7\}, \{-1, 6\}, \{0, 5\}, \{1, 4\}\} \]

There are infinite elements in X, hence it can not be listed.
b. What is $A \cap B$?

$$A \cap B = \emptyset.$$  


c. What is $B \cap C$?

$$B \cap C = \{\{1, 4\}, \{2, 3\}\}.$$  

d. What is $B \cap D$?

$$B \cap D = \emptyset.$$  

e. What is $B \cap E$?

$$B \cap E = \emptyset.$$  

f. What is $E \cap F$?

$$E \cap F = \{-2, 7\}.$$  

g. What is $E \cap G$?

$$E \cap G = \{-2, 7\}.$$  

h. What is $E \cap F \cap G$?

$$E \cap F \cap G = \{-2, 7\}.$$
**Problem 3.** Simplify the following fractions.

a. \(\frac{16}{20}\)

\[
\frac{16}{20} = \frac{16/4}{20/4} = \frac{4}{5}.
\]

b. \(\frac{112}{77}\)

\[
\frac{112}{77} = \frac{112/7}{77/7} = \frac{16}{11}.
\]

c. \(\frac{441}{189}\)

\[
\frac{441}{189} = \frac{441/63}{189/63} = \frac{7}{3}.
\]

d. \(\frac{4158}{2160}\)

\[
\frac{4158}{2160} = \frac{4158/54}{2160/54} = \frac{77}{40}.
\]

e. \(\frac{426888}{27720}\)

\[
\frac{426888}{27720} = \frac{426888/5544}{27720/5544} = \frac{77}{5}.
\]

f. \(\frac{15015}{35343}\)

\[
\frac{15015}{35343} = \frac{15015/231}{35343/231} = \frac{65}{153}.
\]
Problem 4. Complete the following operations.

a. $\frac{14}{16} + \frac{1}{4}$

\[
\frac{14}{16} + \frac{1}{4} = \frac{14 + 1 \times 4}{4 \times 4} = \frac{18}{16} = \frac{9}{8}.
\]

b. $\frac{112}{7}$

\[
\frac{112}{7} = \frac{112 \times 77}{7 \times 77} = \frac{112}{66} = \frac{112/2}{66/2} = \frac{56}{33}.
\]

c. $\frac{15}{28} + \frac{3}{7}$

\[
\frac{15}{28} + \frac{3}{7} = \frac{15 + 12}{28} = \frac{27}{28}.
\]

d. $\frac{17}{26} + \frac{7}{4}$

\[
\frac{17}{26} + \frac{7}{4} = \frac{17 \times 2 + 7 \times 13}{26 \times 2 + 4 \times 13} = \frac{34 + 91}{52} = \frac{125}{52}.
\]

e. $\frac{6}{7} + \frac{33}{84} + \frac{5}{6}$
\[
\begin{align*}
\frac{6}{7} + \frac{33}{84} + \frac{5}{6} &= \frac{6 \times 12}{7 \times 12} + \frac{33}{84} + \frac{5 \times 14}{6 \times 14} \\
&= \frac{72 + 33 + 70}{84} \\
&= \frac{175}{84} = \frac{175}{84}/7 \\
&= \frac{25}{12}.
\end{align*}
\]

\[
\begin{align*}
f. \frac{7}{8} + \frac{11}{24} + \frac{1}{3} + \frac{7}{12} &= \frac{7 \times 3}{8 \times 3} + \frac{11}{24} + \frac{1 \times 8}{3 \times 8} + \frac{7 \times 2}{12 \times 2} \\
&= \frac{21 + 11 + 8 + 14}{24} = \frac{54}{24} = \frac{54}{6} \\
&= \frac{9}{4}.
\end{align*}
\]