ECONOMICS 207
SPRING 2008
LABORATORY EXERCISE 2
KEY

Problem 1. Carry out the following long division operations.

a.

$$\begin{array}{r} 13 \\ 11\,\overline{)\,143} \\ \underline{110} \\ 33 \\ \underline{33} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 16 \\ 13\,\overline{)\,208} \\ \underline{130} \\ 78 \\ \underline{78} \\ 0 \end{array}$$

c.

$$\begin{array}{r} 11 \\ 14\,\overline{)\,154} \\ \underline{140} \\ 14 \\ \underline{14} \\ 0 \end{array}$$

d.

$$\begin{array}{r} 40 \\ 35\,\overline{)\,1421} \\ \underline{1400} \\ 21 \end{array}$$

Date: February 4, 2008.

e.

$$
\begin{array}{r@{\,}r}
 & 94 \\ \cline{2-2}
73 \big) & 6872 \\
 & 6570 \\ \cline{2-2}
 & 302 \\
 & 292 \\ \cline{2-2}
 & 10
\end{array}
$$

f.

$$
\begin{array}{r@{\,}r}
 & 302 \\ \cline{2-2}
173 \big) & 52326 \\
 & 51900 \\ \cline{2-2}
 & 426 \\
 & 346 \\ \cline{2-2}
 & 80
\end{array}
$$

g.

$$
\begin{array}{r@{\,}r}
 & 119 \\ \cline{2-2}
141 \big) & 16779 \\
 & 14100 \\ \cline{2-2}
 & 2679 \\
 & 1410 \\ \cline{2-2}
 & 1269 \\
 & 1269 \\ \cline{2-2}
 & 0
\end{array}
$$

Problem 2. Complete the square in the following and then write as $(x+a)^2+b$.

a. x^2+4x

$(x+2)^2-4$

b. x^2-12x

1. Divide the coefficient of x by 2 to obtain -6.
2. Then square and add to the expression to obtain
3. $x^2-12x+36$
4. Now subtract 36 so that the expression has the same value.
5. $(x^2-12x+36)-36$
6. The first term is a perfect square so we can write
7. $(x-6)^2-36$

c. x^2-8x

1. Divide the coefficient of x by 2 to obtain -4.
2. Then square and add to the expression to obtain
3. $x^2-12x+16$
4. Now subtract 16 so that the expression has the same value.
5. $(x^2-8x+16)-16$
6. The first term is a perfect square so we can write
7. $(x-4)^2-16$

d. $2x^2-16x$

1. Factor out a two to obtain
2. $2(x^2-8x)$
3. Divide the coefficient of x by 2 to obtain -4.
4. Then square and add to the expression to obtain
5. $2(x^2-8x+16)$
6. Now subtract 32 so that the expression has the same value.
7. $2(x^2-8x+16)-32$
8. The first term is a perfect square so we can write
9. $2(x-4)^2-32$

e. $x^2+8x+20$

1. Divide the coefficient of x by 2 to obtain 4.
2. Then square and add to the expression to obtain
3. $x^2+8x+16+20$
4. Now subtract 16 so that the expression has the same value.
5. $(x^2+8x+16)+20-16$
6. The first term is a perfect square so we can write

7. $(x+4)^2+4$

f. $x^2 - \frac{2}{3}x + \frac{1}{9}$
1. Divide the coefficient of x by 2 to obtain $-\frac{1}{3}$.
2. Then square and add to the expression to obtain
3. $(x^2 - \frac{2}{3}x + \frac{1}{9}) + \frac{1}{9}$
4. Now subtract $\frac{1}{9}$ so that the expression has the same value.
5. $(x^2 - \frac{2}{3}x + \frac{1}{9}) + \frac{1}{9} - \frac{1}{9}$
6. The first term is a perfect square so we can write
7. $(x - \frac{1}{3})^2$

g. $x^2 - 4x\,(a+bt)$
1. Divide the coefficient of x by 2 to obtain $-2\,(a+bt)$.
2. Then square and add to the expression to obtain
3. $x^2 - 4x\,(a+bt) + 4\,(a+bt)^2$
4. Now subtract $4\,(a+bt)^2$ so that the expression has the same value.
5. $(x^2 - 4x\,(a+bt) + 4\,(a+bt)^2) - 4\,(a+bt)^2$
6. The first term is a perfect square so we can write
7. $(x - 2\,(a+bt))^2 - 4\,(a+bt)^2$

Problem 3. Simplify, add, subtract, multiply or divide the following fractions. Express all answers in reduced form.

a. $\frac{35}{36}$ + $\frac{3}{8}$

1. First find the prime factorization of each denominator.
   i) $36 = 9 \times 4 = 2^2 \times 3^2$
   ii) $8 = 2^3$
2. To obtain the least common denominator consider multiplying together all these prime numbers. But only multiply the highest power of each prime number that appears in the list. For this example we obtain $2^3 \times 3^2 = 8 \times 9 = 72$.
3. Multiply the numerator in each term by the prime factors in the overall product that do not appear in the denominator for that term. The only term missing for 36 is 2, so we multiply 36 by 2 to obtain 72 and 35 by 2 to obtain 70. So the first fraction is $\frac{70}{72}$. The term in the overall product missing for 8 is 3^2 so we multiply 8 by 9 to obtain 72 and 3 by 9 to obtain 27. So the second fraction is $\frac{27}{72}$
4. Adding the terms together we obtain
   $\frac{70}{72} + \frac{27}{72} = \frac{97}{72}$
   This fraction does not simplify given 97 is a prime number.

b. $\frac{7}{25}$ + $\frac{17}{24}$

1. First find the prime factorizations of each denominator.
   i) 255^2
   ii) $24 = 3 \times 2^3$
2. To obtain the least common denominator consider multiplying together all these prime numbers. But only multiply the highest power of each prime number that appears in the list. Unfortunately in this case, there are no primes that appear in more than one term so we have to multiply the entire list. This gives $5^2 \times 3 \times 2^3 = 25 \times 3 \times 8 = 75 \times 8 = 600$.
3. Multiply the numerator in each term by the prime factors in the overall product that do not appear in the denominator for that term. So we multiply 7 by 24 to obtain $\frac{168}{600}$ and 17 by 25 to obtain $\frac{425}{600}$.
4. Adding the terms together we obtain
   $\frac{168}{600} + \frac{425}{600} = \frac{593}{600}$
   This fraction does not simplify because 593 is a prime number. If you do not believe me that 593 is a prime number consider how easily we can show that it is a prime number. Given that $25^2 = 625$, we need only consider numbers less than 25 as possible factors of 593. All the even numbers are out. The sum of the digits is 17 so 3 and 9 are out. It does not end in 0 or 5 so 5 is out. If we drop the last digit and subtract twice it from what remains we obtain 59-6 = 53, and 53 is not divisible by 7, to 7 is out. Eleven is out because $5 + 3 = 8 \neq 9$. We are now left with just the primes 13, 17, 19, and 23. We need not worry about 15 given that 593 is not divisible by 3 or 5, similarly 593 is not divisible by 21 given it is not divisible by 3. So we need only perform four actual long division problems to demonstrate that 593 is prime.

c. $\left(\frac{75}{144}\right)\left(\frac{5}{\frac{1}{4}}\right)$

1. Rewrite the expression as $\left(\frac{75\times 20}{144}\right)$
2. Write the prime factorization of each number to obtain $\left(\frac{5^2\times 3\times 2^2\times 5}{2^4\times 3^2}\right)$
3. Cancel terms to obtain $\frac{125}{12}$

d. $\frac{5}{12}+\frac{1}{3}$ + $\frac{5}{36}$ - $\frac{7}{16}$

1. First find the prime factorizations of each denominator.
   i) $12 = 3\times 4 = 3\times 2^2$
   ii) $3 = 3$
   iii) $36 = 4\times 9 = 2^2\times 3^2$
   iv) $16 = 2^4$
2. To obtain the least common denominator consider multiplying together all these prime numbers. But only multiply the highest power of each prime number that appears in the list. For this example we obtain $2^4\times 3^2 = 16\times 9 = 144$.
3. Multiply the numerator in each term by the prime factors in the overall product that do not appear in the denominator for that term. The terms missing for 12 are $2^2\times 3$, so we multiply 12 by 12 to obtain 144 and 5 by 12 to obtain 60. So the first fraction is $\frac{60}{144}$. The term in the overall product missing for 3 is $2^4\times 3 = 48$ so we multiply 3 by 48 to obtain 144 and 1 by 48 to obtain 48. So the second fraction is $\frac{48}{144}$. For 36, we are missing $2^2 = 4$ so we multiply 36 by 4 to obtain 144 and 5 by 4 to obtain 20. So the third fraction is $\frac{20}{144}$. The part of the overall product missing for 16 is $3^2 = 9$ so we obtain $9\times 16 = 144$ and $9\times 7 = 63$. The final fraction is $\frac{63}{144}$
4. Combining the terms together we obtain
   $\frac{60}{144}+\frac{48}{144}+\frac{20}{144}-\frac{63}{144}=\frac{65}{144}$
   This fraction does not simplify as can be seen by writing the prime factorization of the numerator and the denominator.
   $\frac{5\times 13}{2^2\times 3^2}$

e. $\frac{1}{5}+\frac{12}{49}$ + $\frac{5}{84}$

1. First find the prime factorizations of each denominator.
   i) $5 = 5$
   ii) $49 = 7\times 7$
   iii) $84 = 7\times 12 = 7\times 2^2\times 3$
2. To obtain the least common denominator consider multiplying together all these prime numbers. But only multiply the highest power of each prime number that appears in the list. For this example we obtain $2^2\times 3\times 5\times 7^2 = 60\times 49 = 2940$. Unfortunately this is a big number for use without a calculator. You win some, you lose some.
3. Multiply the numerator in each term by the prime factors in the overall product that do not appear in the denominator for that term. The terms missing for 5 are $2^2\times 3\times 7^2 = 588$, so we multiply 5 by 588 to obtain 2940 and 1 by 588 to obtain 588. So the first fraction is $\frac{588}{2940}$. The terms in the overall product missing for 49 are $2^2\times 3\times 5 = 60$ so we multiply 49 by 60 to

obtain 2940 and 12 by 60 to obtain 720. So the second fraction is $\frac{720}{2940}$ The terms in the overall product missing for 84 are $5 \times 7 = 35$ so we multiply 84 by 35 to obtain 2940 and 5 by 35 to obtain 175. So the third fraction is $\frac{175}{2940}$

4. Adding the terms together we obtain
$\frac{588}{2940} + \frac{720}{2940} + \frac{175}{2940} = \frac{1483}{2940}$
This fraction does not simplify as can be seen by writing the prime factorization of the numerator and denominator.
$\frac{1483}{2940} = \frac{1483 \times 1}{2^2 \times 3 \times 5 \times 7^2}$

f. $\frac{7}{65} + \frac{3}{77} + \frac{6}{91}$

1. First find the prime factorizations of each denominator.
   i) $65 = 5 \times 13$
   ii) $77 = 7 \times 11$
   iii) $91 = 7 \times 13$
2. To obtain the least common denominator consider multiplying together all these prime numbers. But only multiply the highest power of each prime number that appears in the list. This gives $5 \times 7 \times 11 \times 13 = 35 \times 11 \times 13 = 385 \times 13 = 5005$.
3. Multiply the numerator in each term by the prime factors in the overall product that do not appear in the denominator for that term. The terms missing for 65 are $7 \times 11 = 77$, so we multiply 65 by 77 to obtain 5005 and 7 by 77 to obtain 539. So the first fraction is $\frac{539}{505}$. The terms in the overall product missing for 77 are $5 \times 13 = 65$ so we multiply 77 by 65 to obtain 5005 and 3 by 66 to obtain 195. So the second fraction is $\frac{195}{5005}$ The terms in the overall product missing for 91 are $5 \times 11 = 55$ so we multiply 91 by 55 to obtain 5005 and 6 by 55 to obtain 330. So the third fraction is $\frac{330}{5005}$
4. Adding the terms together we obtain
$\frac{539}{5005} + \frac{195}{5005} + \frac{330}{5005} = \frac{1064}{5005}$
We can look for common factors and see that 2,3, 4, 5, and 6 are quickly eliminated. If we use the seven rule on 5005 where we drop the last digit and subtract twice if from what is remaining we obtain

$$\begin{array}{r} 500 \\ -10 \\ \hline 490 \end{array}$$

Because 490 is divisible by 7, we know that 5005 is divisible by 7.

Now use the seven rule on 1064 where we drop the last digit and subtract twice if from what is remaining we obtain

$$\begin{array}{r} 106 \\ -\ 8 \\ \hline 98 \end{array}$$

If we don't rember what $98 = 7 \times 14$ we can apply the seven rule again.

$$\begin{array}{r} 9 \\ -16 \\ \hline -7 \end{array}$$

Because -7 is divisible by 7, we know that 1064 is divisible by 7.

Carring out the two divisions we obtain.

$$\begin{array}{r} 152 \\ 7\overline{)1064} \\ \underline{700} \\ 364 \\ \underline{350} \\ 14 \\ \underline{14} \\ 0 \end{array}$$

and

$$\begin{array}{r} 715 \\ 7\overline{)5005} \\ \underline{4900} \\ 105 \\ \underline{70} \\ 35 \\ \underline{35} \\ 0 \end{array}$$

So we obtain

$$\frac{1064}{5005} = \frac{152}{715}$$

The prime factorizaion of 152 is $2^3 \times 19$ while the prime factorization of 715 is $5 \times 11 \times 13$ so $\frac{152}{715}$ is in reduced form.

Problem 4. Factor the following.

a. $x^2 + 4x + 4$

$(x + 2)^2$

b. $x^2 - 8x + 16$

$(x - 4)^2$

c. $4x^2 - 32x + 64$

$$\begin{aligned} 4x^2 - 32x + 64 =& 4(x^2 - 8x + 16) \\ =& 4(x - 4)^2 \end{aligned}$$

d. $2x^2 - 8x - 42$

$$\begin{aligned} 2x^2 - 8x - 42 =& 2(x^2 - 4x - 21) \\ =& 2(x - 7)(x + 3) \end{aligned}$$

e. $6x^2 - x - 12$

$(3x + 4)(2x - 3)$

f. $12x^2 - 23x - 24$

$(3x + 4)(2x - 3)$

Problem 5. Solve the following equations for x.

a. $4x + 5 = 29$

$$\begin{aligned} 4x + 5 &= 29 \\ \Rightarrow 4x &= 24 && \text{Subtract 5 from both sides of the equation.} \\ \Rightarrow x &= 6 && \text{Divide both sides of the equation by 4.} \end{aligned}$$

b. $4x + 3 = 24 - 3x$

$$\begin{aligned} 4x + 3 &= 24 - 3x \\ \Rightarrow 7x + 3 &= 24 && \text{Add 3x to both sides of the equation.} \\ \Rightarrow 7x &= 21 && \text{Subtract 3 from both sides of the equation.} \\ \Rightarrow x &= 3 && \text{Divide both sides of the equation by 3.} \end{aligned}$$

c. $\frac{3x+5}{4x-5} = 2$

$$\begin{aligned} \frac{3x+5}{4x-5} &= 2 \\ \Rightarrow 8x - 10 &= 3x + 5 && \text{Cross multiply.} \\ \Rightarrow 5x - 10 &= 5 && \text{Subtract 3x from both sides of the equation.} \\ \Rightarrow 5x &= 15 && \text{Add 10 to both sides of the equation.} \\ \Rightarrow x &= 3 && \text{Divide both sides of the equation by 5.} \end{aligned}$$

d. $\frac{x+5}{4x+3} = \frac{10}{23}$

$$\begin{aligned} \frac{x+5}{4x+3} &= \frac{10}{23} \\ \Rightarrow 40x + 30 &= 23x + 115 && \text{Cross multiply.} \\ \Rightarrow 17x + 30 &= 115 && \text{Subtract 23x from both sides of the equation.} \\ \Rightarrow 17x &= 85 && \text{Subtract 30 from both sides of the equation.} \\ \Rightarrow x &= 5 && \text{Divide both sides of the equation by 17.} \end{aligned}$$

e. $\frac{x+3}{2x+7} = \frac{7}{15}$

$\frac{x+3}{2x+7} = \frac{7}{15}$	
$\Rightarrow 14x + 49 = 15x + 45$	Cross multiply.
$\Rightarrow -x + 49 = 45$	Subtract 15x from both sides of the equation.
$\Rightarrow -x = -4$	Subtract 49 from both sides of the equation.
$\Rightarrow x = 4$	Divide both sides of the equation by -1.

f. $x^2 + 4x + 4 = 0$

$x^2 + 4x + 4 = 0$	
$\Rightarrow (x+2)(x+2) = 0$	Factor.
$\Rightarrow x + 2 = 0$	If a product is equal to zero, one of the terms must be equal to zero.
$\Rightarrow x = -2$	Subtract 2 from both sides of the equation.

g. $x^2 - 7x - 18 = 0$

$x^2 - 7x - 18 = 0$	
$\Rightarrow (x-9)(x+2) = 0$	Factor.
$\Rightarrow x - 9 = 0$	or $(x+2) = 0$
$\Rightarrow x = 9$	Add 9 to both sides of the equation. And
$\Rightarrow x = -2$	Subtract 2 from both sides of the equation.

h. $6x^2 - x - 12 = 0$

$6x^2 - x - 12 = 0$	
$\Rightarrow (3x+4)(2x-3) = 0$	Factor.
$\Rightarrow 3x + 4 = 0$	or $(2x-3) = 0$
$\Rightarrow x = -\frac{4}{3}$	Subtract 4 from both sides of the equation and divide by 3.
$\Rightarrow x = \frac{3}{2}$	Add 3 to both sides of the equation and divide by 2.

Problem 6. Solve the following equations for x_1.

a. $16x_1^{-1/2} - 2 = 0$

$$\begin{aligned} 16x_1^{-1/2} - 2 &= 0 \\ \Rightarrow 16x_1^{-1/2} &= 2 && \text{Add 2 to both sides of the equation.} \\ \Rightarrow x_1^{-1/2} &= \frac{2}{16} = \frac{1}{8} && \text{Divide both sides of the equation by 16 and simplify.} \\ \Rightarrow \left(x_1^{-1/2}\right)^{-2} &= \left(\frac{1}{8}\right)^{-2} && \text{Raise both sides of the equation to the (-2) power.} \\ \Rightarrow x_1 &= 8^2 && \text{Simplify both sides of equation.} \\ \Rightarrow x_1 &= 2^6 = 64 && \text{Further simplify.} \end{aligned}$$

b. $12x_1^{-1/2} - 4 = 0$

$$\begin{aligned} 12x_1^{-1/2} - 4 &= 0 \\ \Rightarrow 12x_1^{-1/2} &= 4 && \text{Add 4 to both sides of the equation.} \\ \Rightarrow x_1^{-1/2} &= \frac{4}{12} = \frac{1}{3} && \text{Divide both sides of the equation by 12 and simplify.} \\ \Rightarrow \left(x_1^{-1/2}\right)^{-2} &= \left(\frac{1}{3}\right)^{-2} && \text{Raise both sides of the equation to the (-2) power.} \\ \Rightarrow x_1 &= 3^2 && \text{Simplify both sides of equation.} \\ \Rightarrow x_1 &= 9 && \text{Further simplify.} \end{aligned}$$

c. $18x_1^{-1/3} - 6 = 0$

$$\begin{aligned} 18x_1^{-1/3} - 6 &= 0 \\ \Rightarrow 18x_1^{-1/3} &= 6 && \text{Add 6 to both sides of the equation.} \\ \Rightarrow x_1^{-1/3} &= \frac{6}{18} = \frac{1}{3} && \text{Divide both sides of the equation by 18 and simplify.} \\ \Rightarrow \left(x_1^{-1/3}\right)^{-3} &= \left(\frac{1}{3}\right)^{-3} && \text{Raise both sides of the equation to the (-3) power.} \\ \Rightarrow x_1 &= 3^3 && \text{Simplify both sides of equation.} \\ \Rightarrow x_1 &= 27 && \text{Further simplify.} \end{aligned}$$

d. $20x_1^{-2/3} - 5 = 0$

$$20x_1^{-2/3} - 5 = 0$$

$$\Rightarrow 20x_1^{-2/3} = 5$$ Add 5 to both sides of the equation.

$$\Rightarrow x_1^{-2/3} = \frac{5}{20} = \frac{1}{4}$$ Divide both sides of the equation by 20 and simplify.

$$\Rightarrow \left(x_1^{-2/3}\right)^{-3/2} = \left(\frac{1}{4}\right)^{-3/2}$$ Raise both sides of the equation to the $\left(-\frac{3}{2}\right)$ power.

$$\Rightarrow x_1 = 4^{3/2}$$ Simplify both sides of equation.

$$\Rightarrow x_1 = \left(4^{1/2}\right)^3$$ Further simplify.

$$\Rightarrow x_1 = 2^3 = 8$$ And simplify some more.

e. $27x_1^{-3/4} - 8 = 0$

$$27x_1^{-3/4} - 8 = 0$$

$$\Rightarrow 27x_1^{-3/4} = 8$$ Add 8 to both sides of the equation.

$$\Rightarrow x_1^{-3/4} = \frac{8}{27}$$ Divide both sides of the equation by 27.

$$\Rightarrow \left(x_1^{-3/4}\right)^{-4/3} = \left(\frac{8}{27}\right)^{-4/3}$$ Raise both sides of the equation to the $\left(-\frac{4}{3}\right)$ power.

$$\Rightarrow x_1 = \left(\frac{27}{8}\right)^{4/3}$$ Simplify both sides of equation.

$$\Rightarrow x_1 = \left(\left(\frac{27}{8}\right)^{1/3}\right)^4$$ Further simplify.

$$\Rightarrow x_1 = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$$ And simplify some more.

f. $18x_1^{-2/5} - 2 = 0$

$$18x_1^{-2/5} - 2 = 0$$

$$\Rightarrow 18x_1^{-2/5} = 2 \qquad \text{Add 2 to both sides of the equation.}$$

$$\Rightarrow x_1^{-2/5} = \frac{2}{18} = \frac{1}{9} \qquad \text{Divide both sides of the equation by 18 and simplify.}$$

$$\Rightarrow \left(x_1^{-2/5}\right)^{-5/2} = \left(\frac{1}{9}\right)^{-5/2} \qquad \text{Raise both sides of the equation to the } \left(-\frac{5}{2}\right) \text{ power.}$$

$$\Rightarrow x_1 = 9^{5/2} \qquad \text{Simplify both sides of equation.}$$

$$\Rightarrow x_1 = \left(9^{1/2}\right)^5 \qquad \text{Further simplify.}$$

$$\Rightarrow x_1 = 3^5 = 243 \qquad \text{And simplify some more.}$$

g. $48x_1^{-4/5} - 3 = 0$

$$48x_1^{-4/5} - 3 = 0$$

$$\Rightarrow 48x_1^{-4/5} = 3 \qquad \text{Add 3 to both sides of the equation.}$$

$$\Rightarrow x_1^{-4/5} = \frac{3}{48} = \frac{1}{16} \qquad \text{Divide both sides of the equation by 48 and simplify.}$$

$$\Rightarrow \left(x_1^{-4/5}\right)^{-5/4} = \left(\frac{1}{16}\right)^{-5/4} \qquad \text{Raise both sides of the equation to the } \left(-\frac{5}{4}\right) \text{ power.}$$

$$\Rightarrow x_1 = 16^{5/4} \qquad \text{Simplify both sides of equation.}$$

$$\Rightarrow x_1 = \left(16^{1/4}\right)^5 \qquad \text{Further simplify.}$$

$$\Rightarrow x_1 = 2^5 = 32 \qquad \text{And simplify some more.}$$