Problem 1. Solve the following equations for x.

a. \( \frac{2x + 5}{x + 4} = \frac{19}{14} \)

b. \( \frac{2x - 5}{11 - 3x} = \frac{-9}{17} \)

c. \( \frac{3x + 4}{13} = \frac{-8x + 5}{19} \)

d. \( \frac{7x - 12}{4x - 9} = \frac{1}{4} \)

Date: February 1, 2008.
Problem 2. Solve the following equations for x.
   a. \( 3x^2 - 20x + 12 = 0 \)
   b. \( 4x^2 - 27x + 18 = 0 \)
   c. \( 36x^2 - 35x + 6 = 0 \)
   d. \( x^2 - \frac{13}{2}x + 10 = 0 \)
Problem 3. Solve the following equations for $x_1$.

a. $12x_1^{-1/2} - 6 = 0$

b. $25x_1^{-2/3} - 16 = 0$

c. $4096x_1^{-3/4} - 64 = 0$

d. $343x_1^{-3/2} - 1 = 0$
Problem 4. Solve the following equations for $x_1$.

a. $25x_1^{1/2} = x_1$

b. $12x_1^{1/2} = 6x_1$

c. $27x_1^{-1/3} = 3x_1^{1/6}$

d. $256x_1^{1/6} = 8x_1^{7/6}$
Problem 5. Solve the following systems of equations for $x_1$ and $x_2$ using the method of substitution

a.  

\begin{align*}
  x_1 + 2x_2 &= 7 \\
  7x_1 + 2x_2 &= 13
\end{align*}

b.  

\begin{align*}
  x_1 + 8x_2 &= 4 \\
  3x_1 + 2x_2 &= -10
\end{align*}
c.

\[ 2x_1 + 3x_2 = 14 \]
\[ 6x_1 - 2x_2 = 20 \]

d.

\[ x_1 + 3x_2 = 7 \]
\[ 2x_1 + 6x_2 = 12 \]
e.

\[ 2x_1 - 3x_2 = 7 \]
\[ 3x_1 - 5x_2 = 12 \]
Problem 6. Solve the following systems of equations for $x_1$, $x_2$, and $x_3$ using the method of substitution.

a. 
\[
\begin{align*}
\{ & x_1 = 1, \ x_2 = 3, \ x_3 = 2 \\
-2x_1 + \frac{1}{2}x_2 + 2x_3 = \frac{7}{2} \\
6x_1 - x_2 - 5x_3 = -7 \\
2x_1 - 2x_2 - 4x_3 = -12 
\end{align*}
\]

Solve the first equation for $x_1$ as a function of $x_2$ and $x_3$ as follows
\[
-2x_1 = \frac{7}{2} - \frac{1}{2}x_2 - 2x_3 \\
\Rightarrow x_1 = -\frac{7}{4} + \frac{1}{4}x_2 + x_3 \ \ \ \text{First formula for } x_1
\]

Now substitute the formula for $x_1$ into the second equation and get a formula for $x_2$ in terms of $x_3$.
\[
6x_1 - x_2 - 5x_3 = -7 \\
\Rightarrow 6 \left( -\frac{7}{4} + \frac{1}{4}x_2 + x_3 \right) - x_2 - 5x_3 = -7 \\
\Rightarrow -\frac{21}{2} + \frac{3}{2}x_2 + 6x_3 - x_2 - 5x_3 = -7 \\
\Rightarrow \frac{1}{2}x_2 = -7 + \frac{21}{2} - x_3 \\
\Rightarrow x_2 = -14 + 21 - 2x_3 \\
= 7 - 2x_3 \ \ \ \text{Formula for } x_2
\]

Now substitute the formulas for $x_1$ and $x_2$ into the third equation.
\[
x_1 = -\frac{7}{4} + \frac{1}{4}x_2 + x_3 \ \ \ \text{First formula for } x_1
\]
\[
= -\frac{7}{4} + \frac{1}{4}(7 - 2x_3) + x_3 \\
= \frac{1}{2}x_3 \ \ \ \text{Second formula for } x_1
\]

Now substitute the formulas for $x_1$ and $x_2$ into the third equation.
b.

\[ \{ x_1 = 2, \ x_2 = -1, \ x_3 = 2 \} \]

\[ x_1 + 2x_2 + 4x_3 = 8 \]

\[ 3x_1 + 7x_2 + 10x_3 = 19 \]

\[ 2x_1 + 3x_2 + 11x_3 = 23 \]
c.

\[
\begin{align*}
    &x_1 = 2, \ x_2 = 2, \ x_3 = -1 \\
    &x_1 - 2x_2 + 4x_3 = -6 \\
    &2x_1 - 5x_2 + 9x_3 = -15 \\
    &3x_1 - 2x_2 + 7x_3 = -5
\end{align*}
\]
Problem 7. Solve the following systems of equations for $x_1$ and $x_2$ using the method of substitution.

a.

\[
\begin{align*}
9x_1^{-1/2}x_2^{1/3} - 9 &= 0 \\
6x_1^{1/2}x_2^{-2/3} - 2 &= 0
\end{align*}
\]
b. 
\[ 48x_1^{-1/2}x_2^{1/4} - 32 = 0 \]
\[ 24x_1^{1/2}x_2^{-3/4} - 9 = 0 \]