ECONOMICS 207
SPRING 2008
LABORATORY EXERCISE 3
KEY

Problem 1. Solve the following equations for x.

a. $\frac{2x+5}{x+7} = \frac{19}{14}$

$$\begin{array}{rrcl} & \frac{2x+5}{x+7} & = & \frac{19}{14} \\ \Rightarrow & (2x+5)14 & = & (x+7)19 \\ \Rightarrow & 28x+70 & = & 19x+133 \\ \Rightarrow & 9x & = & 63 \\ \Rightarrow & x & = & 7. \end{array}$$

b. $\frac{2x-5}{11-3x} = \frac{-9}{17}$

$$\begin{array}{rrcl} & \frac{2x-5}{11-3x} & = & \frac{-9}{17} \\ \Rightarrow & (2x-5)17 & = & -(11-3x)9 \\ \Rightarrow & 34x-85 & = & -99+27x \\ \Rightarrow & 7x & = & -14 \\ \Rightarrow & x & = & -2. \end{array}$$

c. $\frac{3x+4}{-13} = \frac{-8x+5}{19}$

$$\begin{array}{rrcl} & \frac{3x+4}{-13} & = & \frac{-8x+5}{19} \\ \Rightarrow & (3x+4)19 & = & -13(-8x+5) \\ \Rightarrow & 57x+76 & = & 104x-65 \\ \Rightarrow & -47x & = & -141 \\ \Rightarrow & x & = & 3. \end{array}$$

d. $\frac{\frac{7x-12}{2x+6}}{4x-9} = \frac{1}{4}$

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\begin{array}{rrcl}
 & \frac{\frac{7x-12}{2x+6}}{4x-9} & = & \frac{1}{4} \\
\Rightarrow & \frac{7x-12}{2x+6} & = & \frac{4x-9}{4} \\
\Rightarrow & 28x-48 & = & (2x+6)(4x-9) \\
\Rightarrow & 28x-48 & = & 8x^2+6x-54 \\
\Rightarrow & 8x^2-22x-6 & = & 0 \\
\Rightarrow & 4x^2-11x-3 & = & 0 \\
\Rightarrow & (4x+1)(x-3) & = & 0 \\
\Rightarrow & x=-\frac{1}{4} & \text{or} & x=3
\end{array}
$$

Problem 2. Solve the following equations for x.

a. $3x^2 - 20x + 12 = 0$

$$\begin{aligned} && 3x^2 - 20x + 12 &= 0 \\ \Rightarrow && (x-6)(3x-2) &= 0 \\ \Rightarrow && x = 6 \quad &\text{or} \quad x = \tfrac{2}{3} \end{aligned}$$

b. $4x^2 - 27x + 18 \ = \ 0$

$$\begin{aligned} && 4x^2 - 27x + 18 &= 0 \\ \Rightarrow && (x-6)(4x-3) &= 0 \\ \Rightarrow && x = 6 \quad &\text{or} \quad x = \tfrac{3}{4} \end{aligned}$$

c. $36x^2 - 35x + 6 \ = \ 0$

$$\begin{aligned} && 36x^2 - 35x + 6 &= 0 \\ \Rightarrow && (4x-3)(9x-2) &= 0 \\ \Rightarrow && x = \tfrac{3}{4} \quad &\text{or} \quad x = \tfrac{2}{9} \end{aligned}$$

d. $x^2 - \frac{13}{2}x + 10 \ = \ 0$

$$\begin{aligned} && x^2 - \tfrac{13}{2}x + 10 &= 0 \\ \Rightarrow && \tfrac{1}{2}(x-4)(2x-5) &= 0 \\ \Rightarrow && x = \tfrac{2}{5} \quad &\text{or} \quad x = 4 \end{aligned}$$

Problem 3. Solve the following equations for x_1.

a. $12x_1^{-1/2} - 6 = 0$

$$\begin{aligned} & 12x_1^{-1/2} - 6 &=& \; 0 \\ \Rightarrow \quad & 12x_1^{-1/2} &=& \; 6 \\ \Rightarrow \quad & x_1^{-1/2} &=& \; 1/2 \\ \Rightarrow \quad & x_1^{1/2} &=& \; 2 \\ \Rightarrow \quad & x &=& \; 4 \end{aligned}$$

b. $25x_1^{-2/3} - 16 = 0$

$$\begin{aligned} & 25x_1^{-2/3} - 16 &=& \; 0 \\ \Rightarrow \quad & x_1^{-2/3} &=& \; 16/25 \\ \Rightarrow \quad & x_1^{2/3} &=& \; 25/16 \\ \Rightarrow \quad & x_1 &=& \; (25/16)^{3/2} \\ \Rightarrow \quad & x_1 &=& \; (5/4)^3 = \tfrac{125}{64} \end{aligned}$$

c. $4096x_1^{-3/4} - 64 = 0$

$$\begin{aligned} & 4096x_1^{-3/4} - 64 &=& \; 0 \\ \Rightarrow \quad & x_1^{-3/4} &=& \; \tfrac{64}{4096} = \tfrac{1}{64} \\ \Rightarrow \quad & x_1^{3/4} &=& \; 64 \\ \Rightarrow \quad & x_1 &=& \; 64^{4/3} = 256 \end{aligned}$$

d. $343x_1^{-3/2} - 1 = 0$

$$\begin{aligned} & 343x_1^{-3/2} - 1 &=& \; 0 \\ \Rightarrow \quad & x_1^{-3/2} &=& \; 1/343 \\ \Rightarrow \quad & x_1 &=& \; 343^{2/3} = 49 \end{aligned}$$

Problem 4. Solve the following equations for x_1.

a. $25x_1^{1/2} \;=\; x_1$

$$\begin{aligned} & 25x_1^{1/2} &=& \; x_1 \\ \Rightarrow \quad & 25x_1^{1/2} - x_1 &=& \; 0 \\ \Rightarrow \quad & x_1^{1/2}(25 - x_1^{1/2}) &=& \; 0 \\ \Rightarrow \quad & x_1^{1/2} &=& \; 0 \text{ or } 25 \\ \Rightarrow \quad & x_1 = 0 & \text{or} & \; x_1 = 625 \end{aligned}$$

b. $12x_1^{1/2} \;=\; 6x_1$

$$\begin{aligned} & 12x_1^{1/2} &=& \; 6x_1 \\ \Rightarrow \quad & 12x_1^{1/2} - 6x_1 &=& \; 0 \\ \Rightarrow \quad & x_1^{1/2}(12 - 6x_1^{1/2}) &=& \; 0 \\ \Rightarrow \quad & x_1^{1/2} &=& \; 0 \text{ or } 2 \\ \Rightarrow \quad & x_1 = 0 & \text{or} & \; x_1 = 4 \end{aligned}$$

c. $27x_1^{-1/3} \;=\; 3x_1^{1/6}$

$$\begin{aligned} & 27x_1^{-1/3} &=& \; 3x_1^{1/6} \\ \Rightarrow \quad & 27x_1^{-1/3} - 3x_1^{1/6} &=& \; 0 \\ \Rightarrow \quad & x_1^{-1/3}(27 - 3x_1^{1/2}) &=& \; 0 \\ \Rightarrow \quad & (27 - 3x_1^{1/2}) &=& \; 0 \\ \Rightarrow \quad & x_1 &=& \; 81 \end{aligned}$$

d. $256x_1^{1/6} \;=\; 8x_1^{7/6}$

$$\begin{aligned} & 256x_1^{1/6} &=& \; 8x_1^{7/6} \\ \Rightarrow \quad & 256x_1^{1/6} - 8x_1^{7/6} &=& \; 0 \\ \Rightarrow \quad & 8x_1^{1/6}(32 - x_1) &=& \; 0 \\ \Rightarrow \quad & x_1 = 0 & \text{or} & \; x_1 = 32 \end{aligned}$$

Problem 5. Solve the following systems of equations for x_1 and x_2 using the method of substitution

a.

$$\begin{aligned} x_1 + 2x_2 &= 7 \\ 7x_1 + 2x_2 &= 13 \end{aligned}$$

$$\begin{aligned} & x_1 + 2x_2 &= \; & 7 \\ \Rightarrow \quad & x_1 &= \; & 7 - 2x_2 \end{aligned}$$

Then substitute $x_1 = 7 - 2x_2$ into the second equation.

$$\begin{aligned} \Rightarrow \quad & 7(7 - 2x_2) + 2x_2 &= \; & 13 \\ \Rightarrow \quad & 49 - 14x_2 + 2x_2 - 13 &= \; & 0 \\ \Rightarrow \quad & 36 - 12x_2 &= \; & 0 \\ \Rightarrow \quad & x_2 &= \; & 3 \end{aligned}$$

Then substitute $x_2 = 3$ into the first equation.

$$\begin{aligned} \Rightarrow \quad & x_1 &= \; & 7 - 2x_2 \\ \Rightarrow \quad & x_1 &= \; & 7 - 6 = 1 \end{aligned}$$

So,

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 3 \end{aligned}$$

b.

$$\begin{aligned} x_1 + 8x_2 &= 4 \\ 3x_1 + 2x_2 &= -10 \end{aligned}$$

$$\begin{aligned} & x_1 + 8x_2 &= \; & 4 \\ \Rightarrow \quad & x_1 &= \; & 4 - 8x_2 \end{aligned}$$

Then substitue $x_1 = 4 - 8x_2$ into the second equation.

$$\begin{aligned} \Rightarrow \quad & 3(4 - 8x_2) + 2x_2 &= \; & -10 \\ \Rightarrow \quad & 12 - 24x_2 + 2x_2 &= \; & -10 \\ \Rightarrow \quad & -22x_2 &= \; & -22 \\ \Rightarrow \quad & x_2 &= \; & 1 \end{aligned}$$

Then substitue $x_2 = 1$ into the first equation.

$$\begin{aligned} \Rightarrow \quad & x_1 + 8 &= \; & 4 \\ \Rightarrow \quad & x_1 &= \; & -4 \end{aligned}$$

So,

$$\begin{aligned} x_1 &= -4 \\ x_2 &= 1 \end{aligned}$$

c.

$$\begin{aligned} 2x_1 + 3x_2 &= 14 \\ 6x_1 - 2x_2 &= 20 \end{aligned}$$

$$\begin{aligned} 2x_1 + 3x_2 &= 14 \\ \Rightarrow \quad 2x_1 &= 14 - 3x_2 \end{aligned}$$

Then subsitute $2x_1 = 14 - 3x_2$ into the second equation.

$$\begin{aligned} \Rightarrow \quad 3(14 - 3x_2) - 2x_2 &= 20 \\ \Rightarrow \quad 42 - 11x_2 &= 20 \\ \Rightarrow \quad x_2 &= 2 \end{aligned}$$

Then subsitue $x_2 = 2$ into the first equation.

$$\begin{aligned} \Rightarrow \quad 2x_1 + 3 \times 2 &= 14 \\ \Rightarrow \quad x_1 &= 4 \end{aligned}$$

So,

$$\begin{aligned} x_1 &= 4 \\ x_2 &= 2 \end{aligned}$$

d.

$$\begin{aligned} x_1 + 3x_2 &= 7 \\ 2x_1 + 6x_2 &= 12 \end{aligned}$$

$$\begin{aligned} x_1 + 3x_2 &= 7 \\ \Rightarrow \quad x_1 &= 7 - 3x_2 \end{aligned}$$

Then substitude $x_1 = 7 - 3x_2$ into the second equation.

$$\begin{aligned} \Rightarrow \quad 2(7 - 3x_2) + 6x_2 &= 12 \\ \Rightarrow \quad 14 - 6x_2 + 6x_2 &= 12 \\ \Rightarrow \quad 14 &= 12 \end{aligned}$$

So there is no solution since $14 \neq 12$.

e.

$$\begin{aligned} 2x_1 - 3x_2 &= 7 \\ 3x_1 - 5x_2 &= 12 \end{aligned}$$

$$\begin{aligned} 2x_1 - 3x_2 &= 7 \\ \Rightarrow \quad x_1 &= (7 + 3x_2)/2 \end{aligned}$$

Substitute $x_1 = (7 + 3x_2)/2$ into the second equation.

$$\begin{aligned} \Rightarrow \quad 3(7 + 3x_2)/2 - 5x_2 &= 12 \\ \Rightarrow \quad 3(7 + 3x_2) - 10x_2 &= 24 \\ \Rightarrow \quad 21 + 9x_2 - 10x_2 &= 24 \\ \Rightarrow \quad x_2 &= -3 \end{aligned}$$

Substitue $x_2 = -3$ into the first equation.

$$\begin{aligned} \Rightarrow \quad 2x_1 + 9 &= 7 \\ \Rightarrow \quad x_1 &= -1 \end{aligned}$$

So,

$$\begin{aligned} x_1 &= -1 \\ x_2 &= -3 \end{aligned}$$

Problem 6. Solve the following systems of equations for x_1, x_2, and x_3 using the method of substitution.

a.

$$\{x_1 = 1,\ x_2 = 3,\ x_3 = 2\}$$

$$\begin{aligned} -2x_1 \ + \ \frac{1}{2}x_2 \ + \ 2x_3 \ &= \frac{7}{2} \\ 6x_1 \ - \ x_2 \ - \ 5x_3 \ &= -7 \\ 2x_1 \ - \ 2x_2 \ - \ 4x_3 \ &= -12 \end{aligned}$$

Solve the first equation for x_1 as a function of x_2 and x_3 as follows

$$\begin{aligned} -2x_1 \ &= \frac{7}{2} - \frac{1}{2}x_2 - 2x_3 \\ \Rightarrow \ x_1 \ &= \frac{-7}{4} + \frac{1}{4}x_2 + x_3 \quad \textbf{First formula for } \mathbf{x_1} \end{aligned}$$

Now substitute the formula for x_1 into the second equation and get a formula for x_2 in terms of x_3.

$$\begin{aligned} 6x_1 \ - \ x_2 \ - \ 5x_3 \ &= \ -7 \\ \Rightarrow 6\left(\frac{-7}{4} + \frac{1}{4}x_2 + x_3\right) - x_2 - 5x_3 \ &= \ -7 \\ \Rightarrow \frac{-21}{2} + \frac{3}{2}x_2 + 6x_3 - x_2 - 5x_3 \ &= \ -7 \\ \Rightarrow \frac{1}{2}x_2 \ &= \ -7 + \frac{21}{2} - x_3 \\ \Rightarrow x_2 \ &= \ -14 + 21 - 2x_3 \\ &= 7 - 2x_3 \quad \textbf{Formula for } \mathbf{x_2} \end{aligned}$$

Now substitute the formula for x_2 into the formula for x_1 to get a formula for x_1 that only depends on x_3.

$$\begin{aligned} x_1 \ &= \frac{-7}{4} + \frac{1}{4}(x_2) + x_3 \quad \textbf{First formula for } \mathbf{x_1} \\ &= \frac{-7}{4} + \frac{1}{4}(7 - 2x_3) + x_3 \\ &= \frac{1}{2}x_3 \quad \textbf{Second formula for } \mathbf{x_1} \end{aligned}$$

Now substitute the formulas for x_1 and x_2 into the third equation.

b.

$$\{x_1 = 2,\ x_2 = -1,\ x_3 = 2\}$$

$$\begin{aligned} x_1 \ + \ 2x_2 \ + \ 4x_3 \ &= 8 \\ 3x_1 \ + \ 7x_2 \ + \ 10x_3 \ &= 19 \\ 2x_1 \ + \ 3x_2 \ + \ 11x_3 \ &= 23 \end{aligned}$$

Solve the first equation for x_1 as a function of x_2 and x_3 as follows.

$$\begin{aligned} & x_1 + 2x_2 + 4x_3 &=& \ 8 \\ \Rightarrow & \qquad\qquad x_1 &=& \ 8 - 2x_2 - 4x_3 \qquad \textbf{First formula for } x_1. \end{aligned}$$

Now substitute the formula for x_1 into the second and get a formula of x_2 in terms of x_3.

$$\begin{aligned} & 3x_1 + 7x_2 + 10x_3 &=& \ 19 \\ \Rightarrow & \ 3(8 - 2x_2 - 4x_3) + 7x_2 + 10x_3 &=& \ 19 \\ \Rightarrow & \ 24 - 6x_2 - 12x_3 + 7x_2 + 10x_3 &=& \ 19 \\ \Rightarrow & \qquad\qquad x_2 &=& \ 2x_3 - 5 \qquad \textbf{Formula for } x_2. \end{aligned}$$

Substitute the formula for x_2 into the formula for x_1 to get a formula for x_1 depending only on x_3.

$$\begin{aligned} x_1 &= 8 - 2x_2 - 4x_3 \\ &= 8 - 2(2x_3 - 5) - 4x_3 \\ &= 18 - 8x_3 \qquad \textbf{Second formula for } x_1. \end{aligned}$$

Now substitute the formula for x_1 and x_2 into the third equation.

$$\begin{aligned} & 2x_1 + 3x_2 + 11x_3 &=& \ 23 \\ \Rightarrow & \ 2(18 - 8x_3) + 3(2x_3 - 5) + 11x_3 &=& \ 23 \\ \Rightarrow & \qquad\qquad x_3 &=& \ 2 \end{aligned}$$

Then,

$$\begin{aligned} & x_2 &=& \ 2x_3 - 5 \\ \Rightarrow & x_2 &=& \ -1 \\ & x_1 &=& \ 18 - 8x_3 \\ \Rightarrow & x_1 &=& \ 2 \end{aligned}$$

So the solution is

$x_1 = 2,\ x_2 = -1,\ x_3 = 2$

c.

$$\{x_1 \ = \ 2, \ x_2 \ = \ 2, \ x_3 \ = \ -1\}$$

$$\begin{aligned} x_1 \ - \ 2x_2 \ + \ 4x_3 \ &= -6 \\ 2x_1 \ - \ 5x_2 \ + \ 9x_3 \ &= -15 \\ 3x_1 \ - \ 2x_2 \ + \ 7x_3 \ &= -5 \end{aligned}$$

Solve the first equation for x_1 as a function of x_2 and x_3 as follows.

$$\begin{aligned} & x_1 - 2x_2 + 4x_3 &=& -6 \\ \Rightarrow \quad & x_1 &=& -6 + 2x_2 - 4x_3 \qquad \textbf{First formula for } x_1. \end{aligned}$$

Now substitute the formula for x_1 into the second and get a formula of x_2 in terms of x_3.

$$\begin{aligned} & 2x_1 - 5x_2 + 9x_3 &=& -15 \\ \Rightarrow \quad & 2(-6 + 2x_2 - 4x_3) - 5x_2 + 9x_3 &=& -15 \\ \Rightarrow \quad & -12 + 4x_2 - 8x_3 - 5x_2 + 9x_3 &=& -15 \\ \Rightarrow \quad & x_2 &=& x_3 + 3 \qquad \textbf{Formula for } x_2. \end{aligned}$$

Substitute the formula for x_2 into the formula for x_1 to get a formula for x_1 depending only on x_3.

$$\begin{aligned} x_1 &= -6 + 2x_2 - 4x_3 \\ &= -6 + 2(x_3 + 3) - 4x_3 \\ &= -2x_3 \end{aligned}$$

Now substitute the formula for x_1 and x_2 into the third equation.

$$\begin{aligned} & 3x_1 - 2x_2 + 7x_3 &=& -5 \\ \Rightarrow \quad & -6x_3 - 2(x_3 + 3) + 7x_3 &=& -5 \\ \Rightarrow \quad & x_3 &=& -1 \end{aligned}$$

Then,

$$\begin{aligned} x_2 &= x_3 + 3 = 2 \\ x_1 &= -2x_3 = 2 \end{aligned}$$

So the solution is

$x_1 = 2, \ x_2 = 2, \ x_3 = -1$

Problem 7. Solve the following systems of equations for x_1 and x_2 using the method of substitution.

a.

$$\begin{aligned} 9x_1^{-1/2}x_2^{1/3} - 9 &= 0 \\ 6x_1^{1/2}x_2^{-2/3} - 2 &= 0 \end{aligned}$$

$$\begin{aligned} 9x_1^{-1/2}x_2^{1/3} - 9 &= 0 \\ \Rightarrow \quad x_1^{-1/2}x_2^{1/3} &= 1 \\ \Rightarrow \quad x_1^{1/2} &= x_2^{1/3} \end{aligned}$$

Substitute $x_1^{1/2} = x_2^{1/3}$ into the second equation.

$$\begin{aligned} 6x_1^{1/2}x_2^{-2/3} - 2 &= 0 \\ \Rightarrow \quad 6x_2^{1/3}x_2^{-2/3} - 2 &= 0 \\ \Rightarrow \quad 6x_2^{-1/3} - 2 &= 0 \\ \Rightarrow \quad x_2^{-1/3} &= 1/3 \\ \Rightarrow \quad x_2 &= 27 \end{aligned}$$

Substitute $x_2 = 27$ into $x_1^{1/2} = x_2^{1/3}$.

$$\begin{aligned} x_1^{1/2} &= x_2^{1/3} \\ \Rightarrow \quad x_1^{1/2} &= 3 \\ \Rightarrow \quad x_1 &= 9 \end{aligned}$$

So,

$$\begin{aligned} x_1 &= 9 \\ x_2 &= 27 \end{aligned}$$

b.

$$\begin{aligned} 48x_1^{-1/2}x_2^{1/4} - 32 &= 0 \\ 24x_1^{1/2}x_2^{-3/4} - 9 &= 0 \end{aligned}$$

$$\begin{aligned} 48x_1^{-1/2}x_2^{1/4} - 32 &= 0 \\ \Rightarrow \quad 48x_1^{-1/2}x_2^{1/4} &= 32 \\ \Rightarrow \quad 3x_1^{-1/2}x_2^{1/4} &= 2 \\ \Rightarrow \quad 3x_2^{1/4} &= 2x_1^{1/2} \end{aligned}$$

Substitute $2x_1^{1/2} = 3x_2^{1/4}$ into the second equation.

$$\begin{aligned} 24x_1^{1/2}x_2^{-3/4} - 9 &= 0 \\ \Rightarrow \quad 12(3x_2^{1/4})x_2^{-3/4} - 9 &= 0 \\ \Rightarrow \quad 36x_2^{-1/2} &= 9 \\ \Rightarrow \quad x_2^{-1/2} &= 1/4 \\ \Rightarrow \quad x_2^{1/2} &= 4 \\ \Rightarrow \quad x_2 &= 16 \end{aligned}$$

Substitute $x_2 = 16$ into $2x_1^{1/2} = 3x_2^{1/4}$.

$$\begin{aligned} 2x_1^{1/2} &= 3x_2^{1/4} \\ \Rightarrow \quad 2x_1^{1/2} &= 3 \times 2 \\ \Rightarrow \quad x_1^{1/2} &= 3 \\ \Rightarrow \quad x_1 &= 9 \end{aligned}$$

So,

$$\begin{aligned} x_1 &= 9 \\ x_2 &= 16 \end{aligned}$$