Problem 1. Solve the following equations for $x$.

a. \[ \frac{2x+5}{x+7} = \frac{19}{14} \]

\[
\frac{2x+5}{x+7} = \frac{19}{14} \\
\Rightarrow (2x + 5)14 = (x + 7)19 \\
\Rightarrow 28x + 70 = 19x + 133 \\
\Rightarrow 9x = 63 \\
\Rightarrow x = 7.
\]

b. \[ \frac{2x-5}{11-3x} = -\frac{9}{17} \]

\[
\frac{2x-5}{11-3x} = -\frac{9}{17} \\
\Rightarrow (2x - 5)17 = -(11 - 3x)9 \\
\Rightarrow 34x - 85 = -99 + 27x \\
\Rightarrow 7x = -14 \\
\Rightarrow x = -2.
\]

c. \[ \frac{3x+4}{-13} = \frac{-8x+5}{19} \]

\[
\frac{3x+4}{-13} = \frac{-8x+5}{19} \\
\Rightarrow (3x + 4)19 = -13(-8x + 5) \\
\Rightarrow 57x + 76 = 104x - 65 \\
\Rightarrow -47x = -141 \\
\Rightarrow x = 3.
\]

d. \[ \frac{7x-12}{4x-9} = \frac{1}{4} \]

Date: July 2, 2008.
\[
\frac{7x - 12}{2x + 6} = \frac{1}{4}
\]
\Rightarrow
\[
\frac{7x - 12}{2x + 6} = \frac{4x - 9}{4}
\]
\Rightarrow
\[
28x - 48 = (2x + 6)(4x - 9)
\]
\Rightarrow
\[
28x - 48 = 8x^2 + 6x - 54
\]
\Rightarrow
\[
8x^2 - 22x - 6 = 0
\]
\Rightarrow
\[
4x^2 - 11x - 3 = 0
\]
\Rightarrow
\[
(4x + 1)(x - 3) = 0
\]
\Rightarrow
\[
x = -\frac{1}{4} \text{ or } x = 3
\]
Problem 2. Solve the following equations for x.

a. \( 3x^2 - 20x + 12 = 0 \)
   \[3x^2 - 20x + 12 = 0\]
   \[\Rightarrow (x - 6)(3x - 2) = 0\]
   \[\Rightarrow x = 6 \text{ or } x = \frac{2}{3}\]

b. \( 4x^2 - 27x + 18 = 0 \)
   \[4x^2 - 27x + 18 = 0\]
   \[\Rightarrow (x - 6)(4x - 3) = 0\]
   \[\Rightarrow x = 6 \text{ or } x = \frac{3}{4}\]

c. \( 36x^2 - 35x + 6 = 0 \)
   \[36x^2 - 35x + 6 = 0\]
   \[\Rightarrow (4x - 3)(9x - 2) = 0\]
   \[\Rightarrow x = \frac{3}{4} \text{ or } x = \frac{2}{3}\]

d. \( x^2 - \frac{13}{2}x + 10 = 0 \)
   \[x^2 - \frac{13}{2}x + 10 = 0\]
   \[\Rightarrow \frac{1}{2}(x - 4)(2x - 5) = 0\]
   \[\Rightarrow x = \frac{2}{5} \text{ or } x = 4\]
Problem 3. Solve the following equations for $x_1$.

a. $12x_1^{-1/2} - 6 = 0$

\[
12x_1^{-1/2} - 6 = 0 \\
\Rightarrow 12x_1^{-1/2} = 6 \\
\Rightarrow x_1^{-1/2} = 1/2 \\
\Rightarrow x_1^{1/2} = 2 \\
\Rightarrow x = 4
\]

b. $25x_1^{-2/3} - 16 = 0$

\[
25x_1^{-2/3} - 16 = 0 \\
\Rightarrow x_1^{-2/3} = 16/25 \\
\Rightarrow x_1^{2/3} = 25/16 \\
\Rightarrow x_1 = (25/16)^{3/2} \\
\Rightarrow x_1 = (5/4)^3 = 125/64
\]

c. $4096x_1^{-3/4} - 64 = 0$

\[
4096x_1^{-3/4} - 64 = 0 \\
\Rightarrow x_1^{-3/4} = 64/4096 = 1/64 \\
\Rightarrow x_1^{3/4} = 64 \\
\Rightarrow x_1 = 64^{4/3} = 256
\]

d. $343x_1^{-3/2} - 1 = 0$

\[
343x_1^{-3/2} - 1 = 0 \\
\Rightarrow x_1^{-3/2} = 1/343 \\
\Rightarrow x_1 = 343^{2/3} = 49
\]
Problem 4. Solve the following equations for $x_1$.

a. $25x_1^{1/2} = x_1$

\[
25x_1^{1/2} = x_1 \\
\Rightarrow 25x_1^{1/2} - x_1 = 0 \\
\Rightarrow x_1^{1/2}(25 - x_1^{1/2}) = 0 \\
\Rightarrow x_1^{1/2} = 0 \text{ or } 25 \\
\Rightarrow x_1 = 0 \text{ or } x_1 = 625
\]

b. $12x_1^{1/2} = 6x_1$

\[
12x_1^{1/2} = 6x_1 \\
\Rightarrow 12x_1^{1/2} - 6x_1 = 0 \\
\Rightarrow x_1^{1/2}(12 - 6x_1^{1/2}) = 0 \\
\Rightarrow x_1^{1/2} = 0 \text{ or } 2 \\
\Rightarrow x_1 = 0 \text{ or } x_1 = 4
\]

c. $27x_1^{-1/3} = 3x_1^{1/6}$

\[
27x_1^{-1/3} = 3x_1^{1/6} \\
\Rightarrow 27x_1^{-1/3} - 3x_1^{1/6} = 0 \\
\Rightarrow x_1^{-1/3}(27 - 3x_1^{1/2}) = 0 \\
\Rightarrow (27 - 3x_1^{1/2}) = 0 \\
\Rightarrow x_1 = 81
\]

d. $256x_1^{1/6} = 8x_1^{7/6}$

\[
256x_1^{1/6} = 8x_1^{7/6} \\
\Rightarrow 256x_1^{1/6} - 8x_1^{7/6} = 0 \\
\Rightarrow 8x_1^{1/6}(32 - x_1) = 0 \\
\Rightarrow x_1 = 0 \text{ or } x_1 = 32
\]
Problem 5. Solve the following systems of equations for $x_1$ and $x_2$ using the method of substitution.

a.

\[
x_1 + 2x_2 = 7
\]
\[
7x_1 + 2x_2 = 13
\]

\[
x_1 + 2x_2 = 7
\]
\[
\Rightarrow x_1 = 7 - 2x_2
\]

Then substitute $x_1 = 7 - 2x_2$ into the second equation.

\[
7(7 - 2x_2) + 2x_2 = 13
\]
\[
49 - 14x_2 + 2x_2 - 13 = 0
\]
\[
36 - 12x_2 = 0
\]
\[
\Rightarrow x_2 = 3
\]

Then substitute $x_2 = 3$ into the first equation.

\[
x_1 = 7 - 2x_2
\]
\[
\Rightarrow x_1 = 7 - 6 = 1
\]

So, 

\[
x_1 = 1
\]
\[
x_2 = 3
\]

b.

\[
x_1 + 8x_2 = 4
\]
\[
3x_1 + 2x_2 = -10
\]

\[
x_1 + 8x_2 = 4
\]
\[
\Rightarrow x_1 = 4 - 8x_2
\]

Then substitute $x_1 = 4 - 8x_2$ into the second equation.

\[
3(4 - 8x_2) + 2x_2 = -10
\]
\[
12 - 24x_2 + 2x_2 = -10
\]
\[
-22x_2 = -22
\]
\[
\Rightarrow x_2 = 1
\]

Then substitute $x_2 = 1$ into the first equation.

\[
x_1 + 8 = 4
\]
\[
\Rightarrow x_1 = -4
\]

So, 

\[
x_1 = -4
\]
\[
x_2 = 1
\]
c.

\begin{align*}
2x_1 + 3x_2 &= 14 \\
6x_1 - 2x_2 &= 20
\end{align*}

\begin{align*}
2x_1 + 3x_2 &= 14 \\
\Rightarrow \quad 2x_1 &= 14 - 3x_2
\end{align*}

Then substitute $2x_1 = 14 - 3x_2$ into the second equation.

\begin{align*}
\Rightarrow \quad 3(14 - 3x_2) - 2x_2 &= 20 \\
\Rightarrow \quad 42 - 11x_2 &= 20 \\
\Rightarrow \quad x_2 &= 2
\end{align*}

Then substitute $x_2 = 2$ into the first equation.

\begin{align*}
\Rightarrow \quad 2x_1 + 3 \times 2 &= 14 \\
\Rightarrow \quad x_1 &= 4
\end{align*}

So,

\begin{align*}
x_1 &= 4 \\
x_2 &= 2
\end{align*}

d.

\begin{align*}
x_1 + 3x_2 &= 7 \\
2x_1 + 6x_2 &= 12
\end{align*}

\begin{align*}
x_1 + 3x_2 &= 7 \\
\Rightarrow \quad x_1 &= 7 - 3x_2
\end{align*}

Then substitute $x_1 = 7 - 3x_2$ into the second equation.

\begin{align*}
\Rightarrow \quad 2(7 - 3x_2) + 6x_2 &= 12 \\
\Rightarrow \quad 14 - 6x_2 + 6x_2 &= 12 \\
\Rightarrow \quad 14 &= 12
\end{align*}

So there is no solution since $14 \neq 12$. 
e.

\[ \begin{align*}
2x_1 - 3x_2 &= 7 \\
3x_1 - 5x_2 &= 12
\end{align*} \]

\[ x_1 = \frac{(7 + 3x_2)}{2} \]

Substitute \( x_1 = \frac{(7 + 3x_2)}{2} \) into the second equation.

\[ \begin{align*}
3 \left( \frac{7 + 3x_2}{2} \right) - 5x_2 &= 12 \\
21 + 9x_2 - 10x_2 &= 24 \\
x_2 &= -3
\end{align*} \]

Substitute \( x_2 = -3 \) into the first equation.

\[ \begin{align*}
2x_1 + 9 &= 7 \\
x_1 &= -1
\end{align*} \]

So,

\[ \begin{align*}
x_1 &= -1 \\
x_2 &= -3
\end{align*} \]
Problem 6. Solve the following systems of equations for $x_1$, $x_2$, and $x_3$ using the method of substitution.

a.

\[
\begin{align*}
\{x_1 &= 1, \ x_2 = 3, \ x_3 = 2\} \\
-2x_1 + \frac{1}{2}x_2 + 2x_3 &= \frac{7}{2} \\
6x_1 - x_2 - 5x_3 &= -7 \\
2x_1 - 2x_2 - 4x_3 &= -12
\end{align*}
\]

Solve the first equation for $x_1$ as a function of $x_2$ and $x_3$ as follows

\[
-2x_1 = \frac{7}{2} - \frac{1}{2}x_2 - 2x_3
\]

\[
\Rightarrow x_1 = -\frac{7}{4} + \frac{1}{4}x_2 + x_3 \quad \text{First formula for } x_1
\]

Now substitute the formula for $x_1$ into the second equation and get a formula for $x_2$ in terms of $x_3$.

\[
6x_1 - x_2 - 5x_3 = -7
\]

\[
\Rightarrow 6\left( -\frac{7}{4} + \frac{1}{4}x_2 + x_3 \right) - x_2 - 5x_3 = -7
\]

\[
\Rightarrow -\frac{21}{2} + \frac{3}{2}x_2 + 6x_3 - x_2 - 5x_3 = -7
\]

\[
\Rightarrow \frac{1}{2}x_2 = -7 + \frac{21}{2} - x_3
\]

\[
\Rightarrow x_2 = -14 + 21 - 2x_3
\]

\[
= 7 - 2x_3 \quad \text{Formula for } x_2
\]

Now substitute the formula for $x_2$ into the formula for $x_1$ to get a formula for $x_1$ that only depends on $x_3$.

\[
x_1 = -\frac{7}{4} + \frac{1}{4}(7 - 2x_3) + x_3 \quad \text{First formula for } x_1
\]

\[
= -\frac{7}{4} + \frac{1}{4}(7 - 2x_3) + x_3
\]

\[
= \frac{1}{2}x_3 \quad \text{Second formula for } x_1
\]

Now substitute the formulas for $x_1$ and $x_2$ into the third equation.
b. 

\[ \{ x_1 = 2, \ x_2 = -1, \ x_3 = 2 \} \]

\[ x_1 + 2x_2 + 4x_3 = 8 \]
\[ 3x_1 + 7x_2 + 10x_3 = 19 \]
\[ 2x_1 + 3x_2 + 11x_3 = 23 \]

Solve the first equation for \( x_1 \) as a function of \( x_2 \) and \( x_3 \) as follows.

\[ x_1 + 2x_2 + 4x_3 = 8 \]
\[ \Rightarrow \quad x_1 = 8 - 2x_2 - 4x_3 \quad \text{First formula for } x_1. \]

Now substitute the formula for \( x_1 \) into the second and get a formula of \( x_2 \) in terms of \( x_3 \).

\[ 3x_1 + 7x_2 + 10x_3 = 19 \]
\[ \Rightarrow \quad 3(8 - 2x_2 - 4x_3) + 7x_2 + 10x_3 = 19 \]
\[ \Rightarrow \quad 24 - 6x_2 - 12x_3 + 7x_2 + 10x_3 = 19 \]
\[ \Rightarrow \quad x_2 = 2x_3 - 5 \quad \text{Formula for } x_2. \]

Substitute the formula for \( x_2 \) into the formula for \( x_1 \) to get a formula for \( x_1 \) depending only on \( x_3 \).

\[ x_1 = 8 - 2x_2 - 4x_3 \]
\[ = 8 - 2(2x_3 - 5) - 4x_3 \]
\[ = 18 - 8x_3 \quad \text{Second formula for } x_1. \]

Now substitute the formula for \( x_1 \) and \( x_2 \) into the third equation.

\[ 2x_1 + 3x_2 + 11x_3 = 23 \]
\[ \Rightarrow \quad 2(18 - 8x_3) + 3(2x_3 - 5) + 11x_3 = 23 \]
\[ \Rightarrow \quad x_3 = 2 \]

Then,

\[ x_2 = 2x_3 - 5 \]
\[ \Rightarrow \quad x_2 = -1 \]
\[ x_1 = 18 - 8x_3 \]
\[ \Rightarrow \quad x_1 = 2 \]

So the solution is

\[ x_1 = 2, \ x_2 = -1, \ x_3 = 2 \]
c. 

\[ \begin{align*} 
\{ &x_1 = 2, \ x_2 = 2, \ x_3 = -1 \} \\
&x_1 - 2x_2 + 4x_3 = -6 \\
&2x_1 - 5x_2 + 9x_3 = -15 \\
&3x_1 - 2x_2 + 7x_3 = -5 
\end{align*} \]

Solve the first equation for \( x_1 \) as a function of \( x_2 \) and \( x_3 \) as follows.

\[ x_1 - 2x_2 + 4x_3 = -6 \]
\[ \Rightarrow \quad x_1 = -6 + 2x_2 - 4x_3 \quad \text{First formula for} \ x_1. \]

Now substitute the formula for \( x_1 \) into the second and get a formula of \( x_2 \) in terms of \( x_3 \).

\[ 2x_1 - 5x_2 + 9x_3 = -15 \]
\[ \Rightarrow \quad 2(-6 + 2x_2 - 4x_3) - 5x_2 + 9x_3 = -15 \]
\[ \Rightarrow \quad -12 + 4x_2 - 8x_3 - 5x_2 + 9x_3 = -15 \]
\[ \Rightarrow \quad x_2 = x_3 + 3 \quad \text{Formula for} \ x_2. \]

Substitute the formula for \( x_2 \) into the formula for \( x_1 \) to get a formula for \( x_1 \) depending only on \( x_3 \).

\[ x_1 = -6 + 2x_2 - 4x_3 \]
\[ = -6 + 2(x_3 + 3) - 4x_3 \]
\[ = -2x_3 \]

Now substitute the formula for \( x_1 \) and \( x_2 \) into the third equation.

\[ 3x_1 - 2x_2 + 7x_3 = -5 \]
\[ \Rightarrow \quad -6x_3 - 2(x_3 + 3) + 7x_3 = -5 \]
\[ \Rightarrow \quad x_3 = -1 \]

Then,

\[ x_2 = x_3 + 3 = 2 \]
\[ x_1 = -2x_3 = 2 \]

So the solution is

\[ x_1 = 2, \ x_2 = 2, \ x_3 = -1 \]
Problem 7. Solve the following systems of equations for $x_1$ and $x_2$ using the method of substitution.

**a.**

\[
\begin{align*}
9x_1^{-1/2}x_2^{1/3} - 9 &= 0 \\
6x_1^{1/2}x_2^{-2/3} - 2 &= 0
\end{align*}
\]

\[
\begin{align*}
9x_1^{-1/2}x_2^{1/3} - 9 &= 0 \\
\Rightarrow & \quad x_1^{-1/2}x_2^{1/3} = 1 \\
\Rightarrow & \quad x_1^{1/2} = x_2^{1/3}
\end{align*}
\]

Substitute $x_1^{1/2} = x_2^{1/3}$ into the second equation.

\[
\begin{align*}
6x_1^{1/2}x_2^{-2/3} - 2 &= 0 \\
\Rightarrow & \quad 6x_2^{1/3}x_2^{-2/3} - 2 = 0 \\
\Rightarrow & \quad 6x_2^{-1/3} - 2 = 0 \\
\Rightarrow & \quad x_2^{-1/3} = 1/3 \\
\Rightarrow & \quad x_2 = 27
\end{align*}
\]

Substitute $x_2 = 27$ into $x_1^{1/2} = x_2^{1/3}$.

\[
\begin{align*}
x_1^{1/2} &= x_2^{1/3} \\
\Rightarrow & \quad x_1^{1/2} = 3 \\
\Rightarrow & \quad x_1 = 9
\end{align*}
\]

So,

\[
\begin{align*}
x_1 &= 9 \\
x_2 &= 27
\end{align*}
\]
b. 

\[ 48x_1^{-1/2}x_2^{1/4} - 32 = 0 \]
\[ 24x_1^{1/2}x_2^{-3/4} - 9 = 0 \]

\[ 48x_1^{-1/2}x_2^{1/4} - 32 = 0 \]
\[ \Rightarrow 48x_1^{-1/2}x_2^{1/4} = 32 \]
\[ \Rightarrow 3x_1^{-1/2}x_2^{1/4} = 2 \]
\[ \Rightarrow 3x_2^{1/4} = 2x_1^{1/2} \]

Substitute \( 2x_1^{1/2} = 3x_2^{1/4} \) into the second equation.

\[ 24x_1^{1/2}x_2^{-3/4} - 9 = 0 \]
\[ \Rightarrow 12(3x_2^{1/4})x_2^{-3/4} - 9 = 0 \]
\[ \Rightarrow 36x_2^{-1/2} = 9 \]
\[ \Rightarrow x_2^{-1/2} = 1/4 \]
\[ \Rightarrow x_2^{1/2} = 4 \]
\[ \Rightarrow x_2 = 16 \]

Substitute \( x_2 = 16 \) into \( 2x_1^{1/2} = 3x_2^{1/4} \).

\[ 2x_1^{1/2} = 3x_2^{1/4} \]
\[ \Rightarrow 2x_1^{1/2} = 3 \times 2 \]
\[ \Rightarrow x_1^{1/2} = 3 \]
\[ \Rightarrow x_1 = 9 \]

So,

\[ x_1 = 9 \]
\[ x_2 = 16 \]