

ECONOMICS 207
SPRING 2008
LABORATORY EXERCISE 4
KEY

Problem 1. Solve the following equations for x .

a. $26x^2 - 43x + 6 = 0$

$$\begin{aligned} & 26x^2 - 43x + 6 = 0 \\ \Rightarrow & (2x - 3)(13x - 2) = 0 \\ \Rightarrow & \quad \quad \quad x = \frac{2}{3} \text{ or } x = \frac{2}{13} \end{aligned}$$

b. $288x^2 - 300x + 72 = 0$

$$\begin{aligned} & 288x^2 - 300x + 72 = 0 \\ \Rightarrow & \quad 24x^2 - 25x + 6 = 0 \\ \Rightarrow & \quad (8x - 3)(3x - 2) = 0 \\ \Rightarrow & \quad \quad \quad x = \frac{2}{3} \text{ or } x = \frac{3}{8}. \end{aligned}$$

Problem 2. Solve the following equations for x_1 .

a. $128x_1^{-2/3} - 2 = 0$

$$\begin{aligned} 128x_1^{-2/3} - 2 &= 0 \\ \Rightarrow 128x_1^{-2/3} &= 2 \\ \Rightarrow x_1^{-2/3} &= 2/128 = 1/64 \\ \Rightarrow x_1^{2/3} &= 2/128 = 64 \\ \Rightarrow x_1 &= 64^{3/2} \\ \Rightarrow x_1 &= 512 \end{aligned}$$

b. $343x_1^{-3/4} - 1 = 0$

$$\begin{aligned} 343x_1^{-3/4} - 1 &= 0 \\ \Rightarrow 343x_1^{-3/4} &= 1 \\ \Rightarrow x_1^{-3/4} &= 1/343 \\ \Rightarrow x_1^{3/4} &= 343 \\ \Rightarrow x_1 &= 343^{4/3} \\ \Rightarrow x_1 &= 2401 \end{aligned}$$

Problem 3. Solve the following equations for x_1 .

a. $2x_1^{2/3} = x_1^{5/6}$

$$\begin{aligned} & 2x_1^{2/3} = x_1^{5/6} \\ \Rightarrow & 2x_1^{2/3} - x_1^{5/6} = 0 \\ \Rightarrow & x_1^{2/3}(2 - x_1^{1/6}) = 0 \\ \Rightarrow & x_1 = 0 \text{ or } (2 - x_1^{1/6}) = 0 \\ \Rightarrow & x_1 = 0 \text{ or } x_1^{1/6} = 2 \\ \Rightarrow & x_1 = 0 \text{ or } x = 64 \end{aligned}$$

b. $2x_1^{3/4} = x_1^{5/6}$

$$\begin{aligned} & 2x_1^{3/4} = x_1^{5/6} \\ \Rightarrow & 2x_1^{3/4} - x_1^{5/6} = 0 \\ \Rightarrow & x_1^{3/4}(2 - x_1^{1/12}) = 0 \\ \Rightarrow & x_1 = 0 \text{ or } (2 - x_1^{1/12}) = 0 \\ \Rightarrow & x_1 = 0 \text{ or } x_1 = 2^{12} \\ \Rightarrow & x_1 = 0 \text{ or } x_1 = 4096 \end{aligned}$$

Problem 4. Solve the following systems of equations for x_1 and x_2 using the method of elimination.

a.

$$\begin{aligned}x_1 + 2x_2 &= 4 \\2x_1 + 5x_2 &= 9\end{aligned}$$

Multiply the first equation by -2 .

$$\Rightarrow \begin{cases} -2x_1 - 4x_2 = -8 & (1) \\ 2x_1 + 5x_2 = 9 & (2) \end{cases}$$

Add above equation (1) to equation (2).

$$\begin{aligned}\Rightarrow -2x_1 - 4x_2 + 2x_1 + 5x_2 &= -8 + 9 \\ \Rightarrow x_2 &= 1\end{aligned}$$

Substitute $x_2 = 1$ into the second equation.

$$\begin{aligned}\Rightarrow 2x_1 + 5 \times 1 &= 9 \\ \Rightarrow x_1 &= 2\end{aligned}$$

So the solution is

$$\begin{aligned}x_1 &= 2 \\ x_2 &= 1\end{aligned}$$

b.

$$\begin{aligned}x_1 + 5x_2 &= 14 \\2x_1 + 9x_2 &= 25\end{aligned}$$

Multiply the first equation by -2 .

$$\Rightarrow \begin{cases} -2x_1 - 10x_2 = -28 & (1) \\ 2x_1 + 9x_2 = 25 & (2) \end{cases}$$

Add above equation (1) to equation (2).

$$\begin{aligned}\Rightarrow -2x_1 - 10x_2 + 2x_1 + 9x_2 &= -28 + 25 \\ \Rightarrow x_2 &= 3\end{aligned}$$

Substitute $x_2 = 3$ into the second equation.

$$\begin{aligned}\Rightarrow 2x_1 + 9 \times 3 &= 25 \\ \Rightarrow x_1 &= -1\end{aligned}$$

So the solution is

$$\begin{aligned}x_1 &= -1 \\ x_2 &= 3\end{aligned}$$

c.

$$\begin{aligned} -x_1 + x_2 &= 4 \\ 3x_1 - 5x_2 &= -9 \end{aligned}$$

Multiply the first equation by 3.

$$\Rightarrow \begin{cases} -3x_1 + 3x_2 = 12 & (1) \\ 3x_1 - 5x_2 = -9 & (2) \end{cases}$$

Add above equation (1) to equation (2).

$$\begin{aligned} \Rightarrow -3x_1 + 3x_2 + 3x_1 - 5x_2 &= 12 - 9 \\ \Rightarrow x_2 &= -3/2 \end{aligned}$$

Substitute $x_2 = -3/2$ into the second equation.

$$\begin{aligned} \Rightarrow 3x_1 - 5 \times (-3/2) &= -9 \\ \Rightarrow x_1 &= -11/2 \end{aligned}$$

So the solution is

$$\begin{aligned} x_1 &= -11/2 \\ x_2 &= -3/2 \end{aligned}$$

d.

$$\begin{aligned} 5x_1 + 2x_2 &= 3 \\ 2x_1 + x_2 &= 2 \end{aligned}$$

Multiply the first equation by -2 and multiply the second equation by 5 .

$$\Rightarrow \begin{cases} -10x_1 - 4x_2 = -6 & (1) \\ 10x_1 + 5x_2 = 10 & (2) \end{cases}$$

Add above equation (1) to equation (2).

$$\begin{aligned} \Rightarrow -10x_1 - 4x_2 + 10x_1 + 5x_2 &= -6 + 10 \\ \Rightarrow x_2 &= 4 \end{aligned}$$

Substitute $x_2 = 4$ into the second equation.

$$\begin{aligned} \Rightarrow 2x_1 + 4 &= 2 \\ \Rightarrow x_1 &= -1 \end{aligned}$$

So the solution is

$$\begin{aligned} x_1 &= -1 \\ x_2 &= 4 \end{aligned}$$

e.

$$\begin{aligned}x_1 + 2x_2 &= 7 \\7x_1 + 2x_2 &= 13\end{aligned}$$

Multiply the first equation by -7 .

$$\Rightarrow \begin{cases} -7x_1 - 14x_2 = -49 & (1) \\ 7x_1 + 2x_2 = 13 & (2) \end{cases}$$

Add above equation (1) to equation (2).

$$\begin{aligned}\Rightarrow -7x_1 - 14x_2 + 7x_1 + 2x_2 &= -49 + 13 \\ \Rightarrow x_2 &= 3\end{aligned}$$

Substitute $x_2 = 3$ into the second equation.

$$\begin{aligned}\Rightarrow 7x_1 + 2 \times 3 &= 13 \\ \Rightarrow x_1 &= 1\end{aligned}$$

So the solution is

$$\begin{aligned}x_1 &= 1 \\ x_2 &= 3\end{aligned}$$

Problem 5. Solve the following systems of equations for x_1 , x_2 , and x_3 using the method of elimination.

a.

$$\{x_1 = 1, x_2 = 2, x_3 = -1\}$$

$$x_1 + 2x_2 + 4x_3 = 1$$

$$3x_1 + 7x_2 + 10x_3 = 7$$

$$2x_1 + 3x_2 + 11x_3 = -3$$

Multiply the first equation by -3 and this gives.

$$-3x_1 - 6x_2 - 12x_3 = -3. \quad (1)$$

Add (1) to the second equation and this gives.

$$-3x_1 - 6x_2 - 12x_3 + 3x_1 + 7x_2 + 10x_3 = -3 + 7.$$

That is,

$$x_2 - 2x_3 = 4. \quad (2)$$

Multiply the first equation by -2 and this gives.

$$-2x_1 - 4x_2 - 8x_3 = -2. \quad (3)$$

Add (3) to the third equation and this gives.

$$-2x_1 - 4x_2 - 8x_3 + 2x_1 + 3x_2 + 11x_3 = -2 - 3.$$

That is,

$$-x_2 + 3x_3 = -5. \quad (4)$$

Add equation (2) to (4) and this gives.

$$x_2 - 2x_3 - x_2 + 3x_3 = 4 - 5 \Rightarrow x_3 = -1.$$

Add equation (2) multiplied by $3/2$ to (4) and this gives.

$$\frac{3}{2}x_2 - 3x_3 - x_2 + 3x_3 = 4 \times \frac{3}{2} - 5 \Rightarrow x_2 = 2.$$

Substitute $x_2 = 2$, $x_3 = -1$ into the first equation.

$$x_1 + 2 \times 2 + 4 \times (-1) = 1 \Rightarrow x_1 = 1.$$

So the solution is

$$x_1 = 1, x_2 = 2, x_3 = -1.$$

b.

$$\{x_1 = 1, x_2 = -3, x_3 = 2\}$$

$$x_1 - 2x_2 + 4x_3 = 15$$

$$2x_1 - 5x_2 + 9x_3 = 35$$

$$3x_1 - 2x_2 + 7x_3 = 23$$

Multiply the first equation by -2 and this gives.

$$-2x_1 + 4x_2 - 8x_3 = -30. \quad (5)$$

Add (5) to the second equation and this gives.

$$-2x_1 + 4x_2 - 8x_3 + 2x_1 - 5x_2 + 9x_3 = -30 + 35.$$

That is,

$$-x_2 + x_3 = 5. \quad (6)$$

Multiply the first equation by -3 and this gives.

$$-3x_1 + 6x_2 - 12x_3 = -45. \quad (7)$$

Add (7) to the third equation and this gives.

$$-3x_1 + 6x_2 - 12x_3 + 3x_1 - 2x_2 + 7x_3 = -45 + 23.$$

That is,

$$4x_2 - 5x_3 = -22. \quad (8)$$

Add equation (6) multiplied by 4 to equation (8).

$$-4x_2 + 4x_3 + 4x_2 - 5x_3 = 20 - 22 \Rightarrow x_3 = 2.$$

Add equation (6) multiplied by 5 to equation (8).

$$-5x_2 + 5x_3 + 4x_2 - 5x_3 = 25 - 22 \Rightarrow x_2 = -3.$$

Substitute $x_2 = -3$, $x_3 = 2$ into the first equation.

$$x_1 - 2 \times (-3) + 4 \times 2 = 15 \Rightarrow x_1 = 1.$$

So the solution is

$$x_1 = 1, x_2 = -3, x_3 = 2.$$

c.

$$\begin{aligned} &\{x_1 = 1, x_2 = 2, x_3 = 1\} \\ &-2x_1 + \frac{1}{2}x_2 + 2x_3 = 1 \\ &6x_1 - x_2 - 5x_3 = -1 \\ &2x_1 - 2x_2 - 4x_3 = -6 \end{aligned}$$

Add the first equation to the third equation.

$$-2x_1 + \frac{1}{2}x_2 + 2x_3 + 2x_1 - 2x_2 - 4x_3 = 1 - 6.$$

That is,

$$-\frac{3}{2}x_2 - 2x_3 = -5. \tag{9}$$

Multiply the first equation by 3 and this gives.

$$-6x_1 + \frac{3}{2}x_2 + 6x_3 = 3. \tag{10}$$

Add (10) to the second equation and this gives.

$$-6x_1 + \frac{3}{2}x_2 + 6x_3 + 6x_1 - x_2 - 5x_3 = 3 - 1.$$

That is,

$$\frac{1}{2}x_2 + x_3 = 2. \tag{11}$$

Add (11) multiplied by 3 to (9).

$$3 \times \frac{1}{2}x_2 + 3x_3 - \frac{3}{2}x_2 - 2x_3 = 6 - 5 \Rightarrow x_3 = 1.$$

Add (11) multiplied by 2 to (9).

$$2 \times \frac{1}{2}x_2 + 2x_3 - \frac{3}{2}x_2 - 2x_3 = 4 - 5 \Rightarrow x_2 = 2.$$

Substitute $x_2 = 2$, $x_3 = 1$ into the third equation.

$$2x_1 - 2 \times 2 - 4 \times 1 = -6 \Rightarrow x_1 = 1.$$

So the solution is

$$x_1 = 1, x_2 = 2, x_3 = 1.$$

Problem 6. Solve the following systems of equations for x_1 and x_2 using the method of substitution.

a.

$$\begin{aligned} 72x_1^{-3/5}x_2^{1/4} - 8 &= 0 \\ 45x_1^{2/5}x_2^{-3/4} - 15 &= 0 \end{aligned}$$

From first equation,

$$\begin{aligned} 72x_1^{-3/5}x_2^{1/4} - 8 &= 0 \\ \Rightarrow x_1^{-3/5}x_2^{1/4} &= \frac{1}{9} \\ \Rightarrow x_2^{1/4} &= \frac{1}{9}x_1^{3/5} \quad \text{and} \quad x_1 \neq 0 \\ \Rightarrow x_2 &= \left(\frac{1}{9}\right)^4 x_1^{12/5} \quad \text{and} \quad x_1 \neq 0 \end{aligned}$$

Substitute $x_2 = \left(\frac{1}{9}\right)^4 x_1^{12/5}$ ($x_1 \neq 0$) into the second equation.

$$\begin{aligned} 45x_1^{2/5} \left(\left(\frac{1}{9}\right)^4 x_1^{12/5}\right)^{-3/4} - 15 &= 0 \\ \Rightarrow 45x_1^{2/5} \cdot 9^3 x_1^{-9/5} &= 15 \\ \Rightarrow x_1^{-7/5} &= \frac{15}{45} \times 9^{-3} = 3^{-7} \\ \Rightarrow x_1 &= 3^5 = 243. \end{aligned}$$

Substitute $x_1 = 243$ into $x_2 = \left(\frac{1}{9}\right)^4 x_1^{12/5}$

$$\begin{aligned} x_2 &= \left(\frac{1}{9}\right)^4 243^{12/5} = 3^{-8} \times 3^{12} = 3^4 \\ &= 81. \end{aligned}$$

So the solution is

$$\begin{aligned} x_1 &= 243 \\ x_2 &= 81. \end{aligned}$$

b.

$$\begin{aligned} 144x_1^{-4/5}x_2^{2/3} - 81 &= 0 \\ 480x_1^{1/5}x_2^{-1/3} - 320 &= 0 \end{aligned}$$

From the second equation,

$$\begin{aligned} 480x_1^{1/5}x_2^{-1/3} - 320 &= 0 \\ \Rightarrow x_1^{1/5}x_2^{-1/3} &= 320/480 \\ \Rightarrow x_2^{-1/3} &= \frac{2}{3}x_1^{-1/5} \\ \Rightarrow x_2^{2/3} &= \left(\frac{3}{2}\right)^2 x_1^{2/5} \quad \text{and} \quad x_1 \neq 0 \end{aligned}$$

Substitute $x_2^{2/3} = \left(\frac{3}{2}\right)^2 x_1^{2/5}$ ($x_1 \neq 0$) into the first equation.

$$\begin{aligned} 44x_1^{-4/5} \left(\frac{3}{2}\right)^2 x_1^{2/5} - 81 &= 0 \\ \Rightarrow x_1^{-4/5} \cdot \frac{9}{4}x_1^{2/5} &= \frac{81}{144} \\ \Rightarrow x_1^{-2/5} &= \frac{81}{144} \times \frac{4}{9} = \frac{1}{4} \\ \Rightarrow x_1^{2/5} &= 4 \\ \Rightarrow x_1 &= 32 \end{aligned}$$

Substitute $x_1 = 32$ into $x_2^{2/3} = \left(\frac{3}{2}\right)^2 x_1^{2/5}$.

$$\begin{aligned} x_2^{2/3} &= \left(\frac{3}{2}\right)^2 x_1^{2/5} \\ \Rightarrow x_2^{2/3} &= \frac{9}{4}(32)^{2/5} = 9 \\ \Rightarrow x_2 &= 27 \end{aligned}$$

So the solution is

$$\begin{aligned} x_1 &= 32 \\ x_2 &= 27. \end{aligned}$$