ECONOMICS 207
SPRING 2008
LABORATORY EXERCISE 7

Problem 1. Find the second derivative of each of the following functions with respect to x

a. $f(x) \;=\; -400 + 200x + 20x^2 - 2x^3$

b. $f(x) \;=\; 180x^{1/4}z^{2/5} \;-\; 15x \;-\; 8z$

c. $f(x) \;=\; (2x^3 \;-\; 4x^2)^3$

Date: 27 February 2008.

d. $f(x) = 4x^3e^{2x^3-4x^2}$

Problem 2. Find the definite integral of each of the following functions.

a. $\int_2^4 (6x+3)\ dx$

b. $\int_{16}^{81} \left(45x^{-3/4}z^{2/5} - 15\right) dx, \quad z = 243.$

c. $\int_0^{10} \left(6x^2 - 40x + 200\right)\, dx$

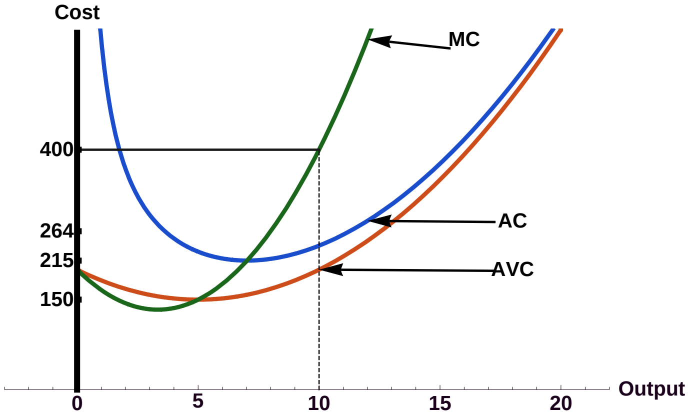
d. $\int_0^{12} \left(-27x^2 + 360x - 432\right)\, dx$

Problem 3. Solve the following systems of equations.

$$45x_1^{-3/4}x_2^{2/5} - 15 = 0$$
$$72x_1^{1/4}x_2^{-3/5} - 8 = 0$$

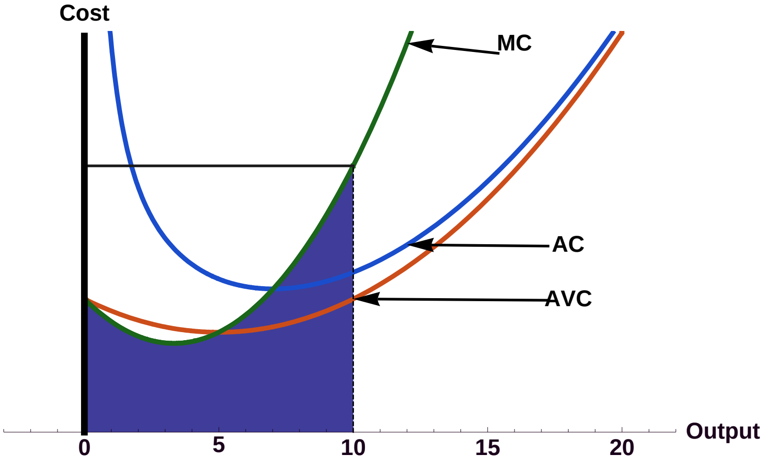
Problem 4. The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output,i.e., Marginal Cost(MC) = $\frac{dc(y)}{dy}$. A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price as in the figure 1.

FIGURE 1. Profit Maximization



The area below the cost curve is a measure of variable cost and can be found by integrating the marginal cost curve from 0 to any given output level y. The shaded area in figure 2 represents the variable cost of production for the cost function $c(y) = 400 + 200y - 20\,y^2 + 2y^3$.

FIGURE 2. Variable Cost of Production and Producer Surplus



Producer surplus is the area below a given price and above the marginal cost curve. Producer surplus is the unshaded area below the horizontal line at 400 in figure 2. Producer surplus can be computed by subtracting the shaded area from total revenue.

a. Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$price \;=\; p \;= \$400$$

$$cost \;=\; c(y) \;= 400 \;+\; 200y \;-\; 20y^2 \;+\; 2y^3$$

b. What is revenue minus variable cost for this firm when price is \$400?

c. Find producer surplus for this firm assuming you are only given the following marginal cost function: $MC(y) = 200 - 40y + 6y^2$ and a price of \$400.

Problem 5. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input.

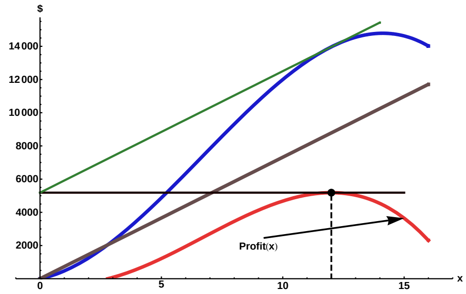
a. Write down an equation that represents profit for the firm.
b. Maximize this function by taking its derivative with respect to the variable input x and setting the derivative equal to zero.
c. Solve the equation in part 5b for the potentially profit maximizing level of x.
d. Determine using the second order conditions which of the roots represents maximum profit.

$$output\ price \; = \; p \; = 3$$

$$input\ price \; = \; w \; = 732$$

$$y \; = \; output \; = \; f(x) \; = \; 100x \; + \; 60x^2 \; - 3x^3$$

FIGURE 3. Revenue, Cost and Profit



You can use the next page if necessary.

Space for Problem 5

Problem 6. Solve the following system of equations for x_1, x_2, and x_3.

$$\{x_1 = 3,\ x_2 = 2,\ x_3 = 1\}$$

$$x_1 + 2x_2 - 3x_3 = 4$$

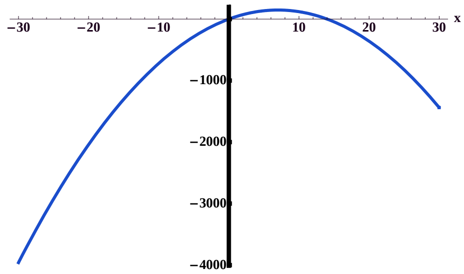
$$-3x_1 - 5x_2 + 8x_3 = -11$$

$$2x_1 + 5x_2 - 6x_3 = 10$$

Problem 7. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise.

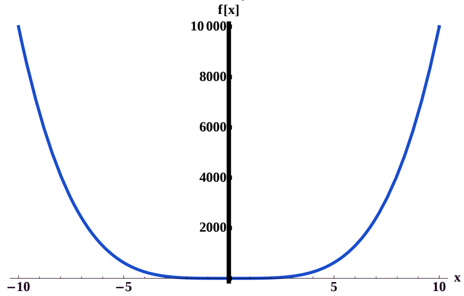
a. $y = 42x - 3x^2$

FIGURE 4. $f(x) = 42x - 3x^2$



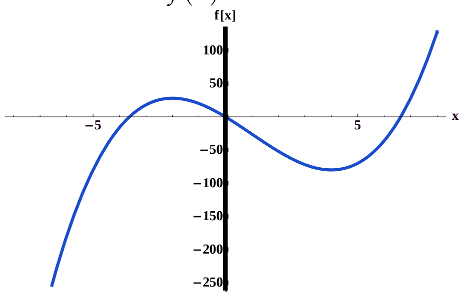
b. $y = x^4$

FIGURE 5. $f(x) = x^4$



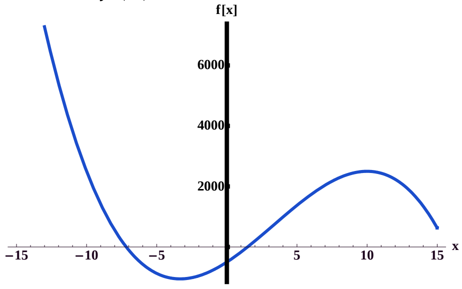
c. $f(x) = x^3 - 3x^2 - 24x$

FIGURE 6. $f(x) = x^3 - 3x^2 - 24x$



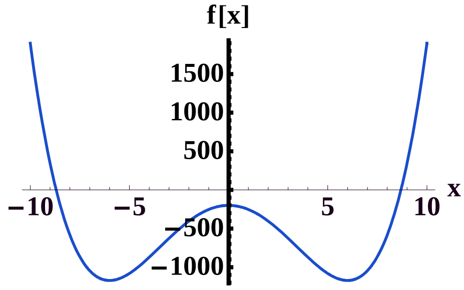
d. $f(x) = -3x^3 + 30x^2 + 300x - 500$

FIGURE 7. $f(x) = -3x^3 + 30x^2 + 300x - 500$



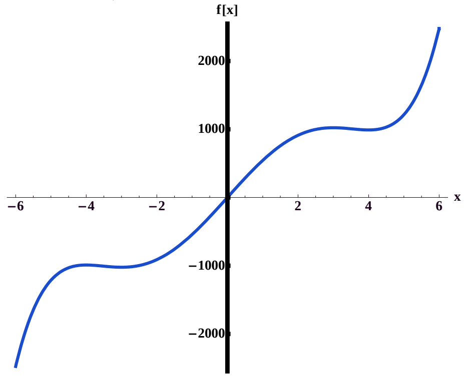
e. $f(x) = 3/4x^4 - 54x^2 - 200$

FIGURE 8. $f(x) = 3/4x^4 - 54x^2 - 200$



f. $f(x) = 4/5x^5 - 100/3x^3 + 576x$

FIGURE 9. $f(x) = 4/5x^5 - 100/3x^3 + 576x$



g. $f(x) = 30x^{3/5} - 2x$;

FIGURE 10. $f(x) = 30x^{3/5} - 2x$