ECONOMICS 207
SPRING 2008
PROBLEM SET 11

Problem 1. Consider the following matrix and vector.

$$P = \begin{bmatrix} 1 & 5 \\ 1 & 6 \end{bmatrix}, \quad p = \begin{bmatrix} 7 \\ 9 \end{bmatrix},$$

a. Use elementary row operations to find both the inverse of P and solve the equation Px=p in one set of operations.

Date: April 1, 2008.

b. Find the determinant of the matrix P.

$$P = \begin{bmatrix} 1 & 5 \\ 1 & 6 \end{bmatrix}, \quad p = \begin{bmatrix} 7 \\ 9 \end{bmatrix},$$

c. Find the inverse of the matrix P using the cofactor/adjoint method.

d. Solve the equation Px=p using the inverse you found in part 1c

e. Solve the equation Px=p using Cramer's rule.

$$P = \begin{bmatrix} 1 & 5 \\ 1 & 6 \end{bmatrix}, \quad p = \begin{bmatrix} 7 \\ 9 \end{bmatrix},$$

Problem 2. Consider the following matrix and vector.

$$Q = \begin{bmatrix} 2 & 2 \\ 5 & 4 \end{bmatrix}, \quad q = \begin{bmatrix} 4 \\ 12 \end{bmatrix},$$

a. Use elementary row operations to find both the inverse of Q and solve the equation Qx=q in one set of operations.

b. Find the determinant of the matrix Q.

$$Q = \begin{bmatrix} 2 & 2 \\ 5 & 4 \end{bmatrix}, \quad q = \begin{bmatrix} 4 \\ 12 \end{bmatrix},$$

c. Find the inverse of the matrix Q using the cofactor/adjoint method.

d. Solve the equation Qx=q using the inverse you found in part 2c

e. Solve the equation Qx=q using Cramer's rule.

$$Q = \begin{bmatrix} 2 & 2 \\ 5 & 4 \end{bmatrix}, \quad q = \begin{bmatrix} 4 \\ 12 \end{bmatrix},$$

Problem 3. Consider the following matrix and vector.

$$G = \begin{bmatrix} -3 & -3 & -4 \\ 0 & -1 & -2 \\ 2 & 1 & 1 \end{bmatrix}, \quad g = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

a. Use elementary row operations to find the inverse of G and solve the equation Gx=g in one set of operations.

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b. Find the determinant of the matrix G.

$$G = \begin{bmatrix} -3 & -3 & -4 \\ 0 & -1 & -2 \\ 2 & 1 & 1 \end{bmatrix}, \quad g = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

c. Find the inverse of the matrix G using the cofactor/adjoint method.

$$G = \begin{bmatrix} -3 & -3 & -4 \\ 0 & -1 & -2 \\ 2 & 1 & 1 \end{bmatrix}, \quad g = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

d. Solve the equation Gx=g using the inverse you found in part 3c

$$G = \begin{bmatrix} -3 & -3 & -4 \\ 0 & -1 & -2 \\ 2 & 1 & 1 \end{bmatrix}, \quad g = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

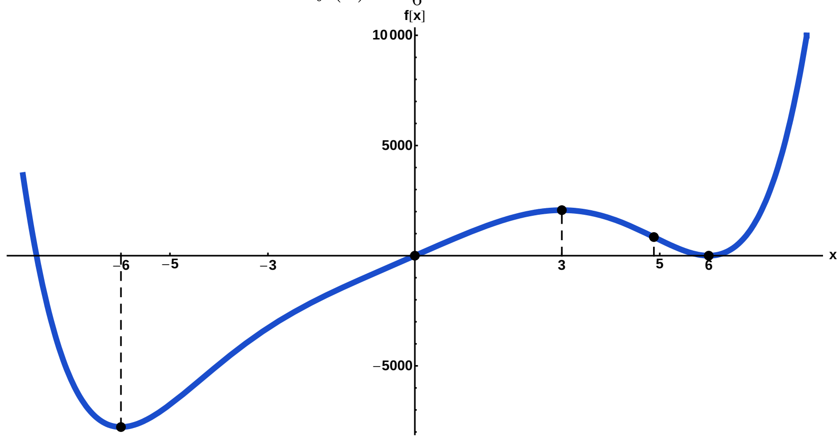
e. Solve the equation Gx=g using Cramer's rule.

Problem 4. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

a. $f(x) \ = \frac{x^6}{6} - 9x^4 - 9x^3 + 972x$.

Hint: You might consider dividing the polynomial in the first order condition by appropriate polynomials one after another or maybe try dividing by $(x^3 - 27)$. The only real inflection point is zero. Show that this is true.

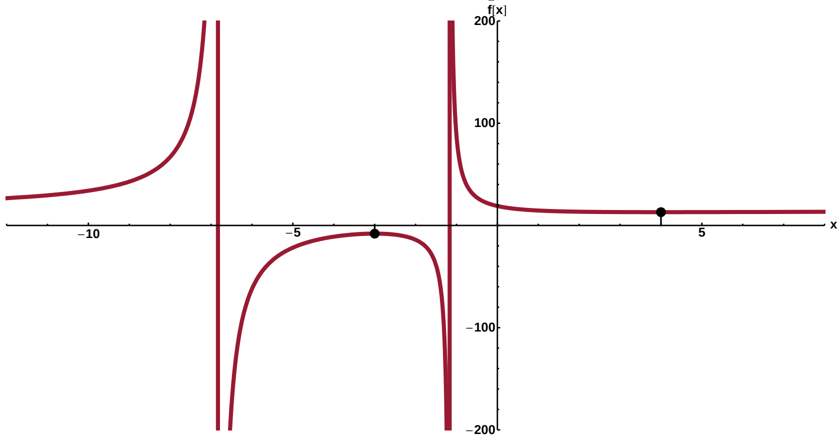
FIGURE 1. $f(x) = \frac{x^6}{6} - 9x^4 - 9x^3 + 972x$



b. $f(x) = \dfrac{4x^2 + 20x + 38}{\frac{x^2}{4} + 2x + 2}$

You need not find the points of inflection for this problem. Hint: The first derivative simplifies to $\frac{48(x^2-x-12)}{(x^2+8x+8)^2}$. You should find the second derivative of f(x) but here is the answer: $-\frac{48(2x^3-3x^2-7x-184)}{(x^2+8x+8)^3}$.

Figure 2. $f(x) = \dfrac{4x^2 + 20x + 38}{\frac{x^2}{4} + 2x + 2}$



Problem 5. Solve the following system of equations.

$$1944x_1^{-5/6}x_2^{1/3} - 32 = 0$$

$$3888x_1^{1/6}x_2^{-2/3} - 729 = 0$$

Hint: $x_1 = 729$.

Problem 6. Find all first partial derivatives of each of the following

a. $f(x) \;=\; 5x_1^3 - 3x_2^2 + 30x_1$

b. $y \;=\; 2x_1^3 x_2^{1/2}$

c. $f(x) \;=\; 11664 x_1^{1/6} x_2^{1/3} - 32x_1 - 729x_2$

d. $f(x) \;=\; 30x_1^{2/3}x_2^{5/6} + 6x_3^{5/6}$

e. $f(x) \;=\; 30x_1^{1/3}x_2^{3/5}x_3^{1/2} - 4x_1 - 5x_2 - x_3$

f. $f(x) = 4{x_1}^{1/2} + 4{x_2}^{1/2} + 6{x_3}^{1/2} + 3x_1 + 4{x_1}^{1/2}{x_2}^{1/2} + 6{x_1}^{1/2}{x_3}^{1/2} + 2x_2 - 2{x_2}^{1/2}{x_3}^{1/2} + 7x_3$