ECONOMICS 207
SPRING 2008
PROBLEM SET 12

Problem 1. Consider the following matrix and vector.

$$P = \begin{bmatrix} 1 & 1 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 2 \end{bmatrix},$$

a. Use elementary row operations to find both the inverse of P and solve the equation Px=p in one set of operations.

Date: April 16, 2008.

b. Find the determinant of the matrix P.

$$P = \begin{bmatrix} 1 & 1 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 2 \end{bmatrix},$$

c. Find the inverse of the matrix P using the cofactor/adjoint method.

d. Solve the equation Px=p using the inverse you found in part 1c

e. Solve the equation Px=p using Cramer's rule.

$$P = \begin{bmatrix} 1 & 1 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 2 \end{bmatrix},$$

Problem 2. Consider the following matrix and vector.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -5 & 16 & -11 \\ 3 & -6 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$$

a. Use elementary row operations to find both the inverse of A and solve the equation Ax=b in one set of operations.

b. Find the determinant of the matrix A.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -5 & 16 & -11 \\ 3 & -6 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$$

c. Find the inverse of the matrix A using the cofactor/adjoint method.

d. Solve the equation Ax=b using the inverse you found in part 2c

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -5 & 16 & -11 \\ 3 & -6 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$$

e. Solve the equation Ax=b using Cramer's rule.

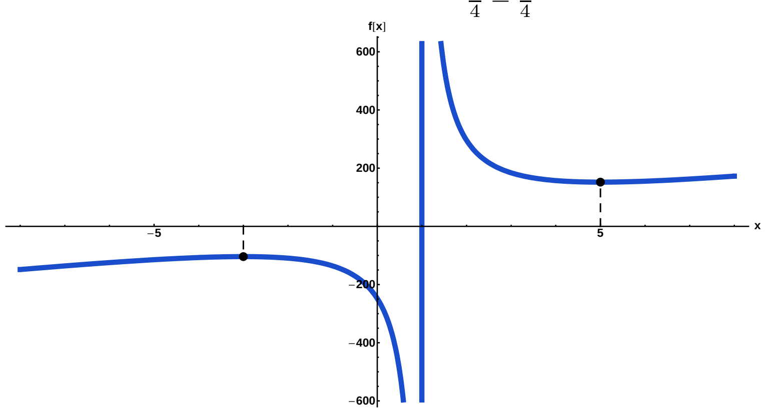
Problem 3. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection **at points other than** critical points.

a. $f(x) = \dfrac{\left(4x^2 - 2x + 62\right)}{\frac{x}{4} - \frac{1}{4}}$.

You need not find the points of inflection for this problem. Hint: The simplified version of the first derivative is $\dfrac{\left(x^2 - 2x - 15\right)}{(\frac{x}{4} - \frac{1}{4})^2} = \dfrac{16\left(x^2 - 2x - 15\right)}{(x-1)^2}$

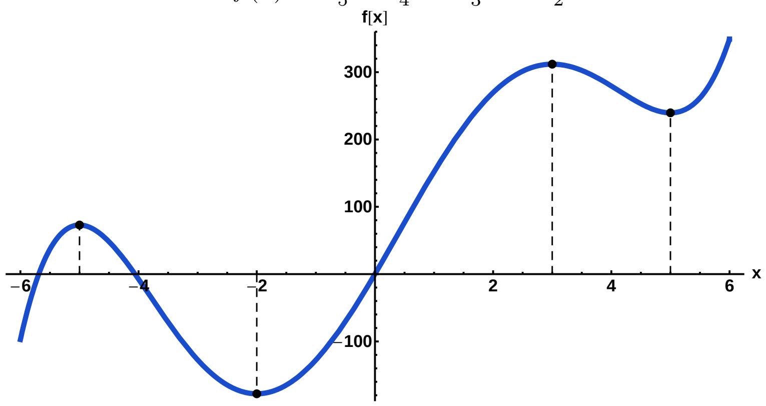
You should find the second derivative of f(x) but here is the answer: $\dfrac{512}{(-1+x)^3}$.

FIGURE 1. $f(x) = \dfrac{\left(4x^2 - 2x + 62\right)}{\frac{x}{4} - \frac{1}{4}}$



b. $f(x) = \frac{x^5}{5} - \frac{x^4}{4} - \frac{31x^3}{3} + \frac{25x^2}{2} + 150x$
You need not find the points of inflection for this problem as they are all imaginary numbers.
Hint: You will need to use long division twice.

Figure 2. $f(x) = \frac{x^5}{5} - \frac{x^4}{4} - \frac{31x^3}{3} + \frac{25x^2}{2} + 150x$



Problem 4. Solve the following system of equations.

$$1764x_1^{-2/3}x_2^{1/4} - 108 = 0$$

$$1323x_1^{1/3}x_2^{-3/4} - 343 = 0$$

Hint: $x_1 = 343$.

Problem 5. Find all first and second partial derivatives of each of the following

a. $f(x_1, x_2) \ = \ 60x_1 + 21x_2 - x_1^2 + x_1 x_2 - x_2^2$

$\frac{\partial f}{\partial x_1} =$	$\frac{\partial f}{\partial x_2} =$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 f}{\partial x_1 \partial x_2} =$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

b. $f(x_1, x_2) \ = \ f(x_1, x_2) \ = \ 80x_1 + 22x_2 - x_1^2 + 2x_1x_2 - 2x_2^2$

$\frac{\partial f}{\partial x_1} =$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

c. $f(x_1, x_2) \;=\; 10x_1 + 9x_2 - x_1^2 \;+\; x_1x_2 \;-\; 2x_2^2$

$\frac{\partial f}{\partial x_1} =$	$\frac{\partial f}{\partial x_2}$
$\frac{\partial^2 f}{\partial x_1 \partial x_1}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2}$
$\frac{\partial^2 f}{\partial x_2 \partial x_1}$	$\frac{\partial^2 f}{\partial x_2 \partial x_2}$

d. $f(x_1, x_2, x_3) \;=\; 100x_1^{1/2}x_2^{1/5}x_3^{1/4} \;-\; 6x_1 \;-\; 5x_2 \;-\; 2x_3$

$\frac{\partial f}{\partial x_1} = 50x_1^{-1/2}x_2^{1/5}x_3^{1/4} - 6$	$\frac{\partial f}{\partial x_2} =$	$\frac{\partial f}{\partial x_3} = 25x_1^{1/2}x_2^{1/5}x_3^{-3/4} - 2$
$\frac{\partial^2 f}{\partial x_1 \partial x_1} = -25x_1^{-3/2}x_2^{1/5}x_3^{1/4}$	$\frac{\partial^2 f}{\partial x_1 \partial x_2} =$	$\frac{\partial^2 f}{\partial x_1 \partial x_3} =$
$\frac{\partial^2 f}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 f}{\partial x_2 \partial x_2} =$	$\frac{\partial^2 f}{\partial x_2 \partial x_3} =$
$\frac{\partial^2 f}{\partial x_3 \partial x_1} =$	$\frac{\partial^2 f}{\partial x_3 \partial x_2} =$	$\frac{\partial^2 f}{\partial x_3 \partial x_3} =$

Problem 6. For each of the following problems, write an equation that represents profit as a function of the two inputs x_1 and x_2. Write it in the form $\pi = pf(x_1,x_2)-w_1x_1-w_2x_2$ and then simplify the expression. Then find all first and second partial derivatives of the function at the specified point.

a.

$$
\begin{aligned}
f(x_1,x_2) &= 60x_1 + 21x_2 - x_1^2 + x_1x_2 - x_2^2 \\
p &= 4 \\
w_1 &= 60, \qquad w_2 \;=\; 12 \\
x_1 &= 36, \qquad x_2 \;=\; 27
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

b.

$$
\begin{aligned}
f(x_1, x_2) &= 80x_1 + 22x_2 - x_1^2 + 2x_1x_2 - 2x_2^2 \\
p &= 4 \\
w_1 &= 24, \qquad w_2 \ = \ 16 \\
x_1 &= 83, \qquad x_2 \ = \ 46
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

c.

$$
\begin{aligned}
f(x_1, x_2) &= 10x_1 \ + \ 9x_2 \ - \ x_1^2 \ + \ x_1x_2 \ - \ 2x_2^2 \\
p &= 5 \\
w_1 &= 30, \qquad w_2 \ = \ 20 \\
x_1 &= 3, \qquad x_2 \ = \ 2
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1}$	$\frac{\partial \pi}{\partial x_2}$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2}$

d.

$$
\begin{aligned}
f(x_1, x_2) &= x_1^{1/3} x_2^{1/4} \\
p &= 5292 \\
w_1 &= 108, \qquad w_2 \;=\; 343 \\
x_1 &= 343, \qquad x_2 \;=\; 81
\end{aligned}
$$

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} =$

In this table fill in values of x_1 and x_2 given to obtain numerical answers.

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} =$

e.

$$\begin{aligned} f(x_1, x_2) &= x_1^{1/2} x_2^{1/4} \\ p &= 288 \\ w_1 &= 32, \qquad w_2 = 81 \\ x_1 &= 81, \qquad x_2 = 16 \end{aligned}$$

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2}$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1}$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} =$

In this table fill in values of x_1 and x_2 given to obtain numerical answers.

$\frac{\partial \pi}{\partial x_1} =$	$\frac{\partial \pi}{\partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_1 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_1 \partial x_2} =$
$\frac{\partial^2 \pi}{\partial x_2 \partial x_1} =$	$\frac{\partial^2 \pi}{\partial x_2 \partial x_2} =$