Problem 1. Consider the following matrix and vector.

\[
P = \begin{bmatrix} 1 & 1 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 2 \end{bmatrix},
\]

a. Use elementary row operations to find both the inverse of \( P \) and solve the equation \( Px = p \) in one set of operations.

Augment matrix \( P \) with a \( 2 \times 2 \) identity matrix and matrix \( p \). That is, let

\[
\tilde{A} = \begin{pmatrix} P & I_{2 \times 2} & p \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 & -2 \\ 3 & 7 & 0 & 1 & 2 \end{pmatrix}
\]

Then use elementary row operations on \( \tilde{A} \) to create an identity matrix on the left side. To begin with, based on matrix \( \tilde{A} \), subtract the first row multiplied by 3 from the second row.

\[
\begin{pmatrix} 1 & 1 & 1 & 0 \\ 3 - 1 \times 3 & 7 - 1 \times 3 & 0 - 1 \times 3 & 1 - 2 \times 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 & -2 \\ 0 & 4 & -3 & 1 & 8 \end{pmatrix} \quad (1)
\]

Based on the matrix on the right side of equation (1), divide the second row by 4.

\[
\begin{pmatrix} 1 & 1 & 1 & 0 & -2 \\ 0 & 4/4 & -3/4 & 1/4 & 8/4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 & -2 \\ 0 & 1 & -3/4 & 1/4 & 2 \end{pmatrix} \quad (2)
\]

Based on the matrix on the right side of equation (2), subtract the second row by the first row.

\[
\begin{pmatrix} 1 & 1 & 1 & 0 & -2 \\ 0 - 1 - 1 \times (-3/4) & 0 - 1/4 & 0 - 2 - 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 & -2 \\ 0 & 1 & -3/4 & 1/4 & 2 \end{pmatrix} \quad (3)
\]

So \( P^{-1} = \begin{pmatrix} 7/4 & -1/4 \\ -3/4 & 1/4 \end{pmatrix} \) and the solutions for \( Px = p \) is \( x = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \).

Date: August 25, 2008.
b. Find the determinant of the matrix P.

\[ P = \begin{bmatrix} 1 & 1 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \]

\[ \det[P] = 1 \times 7 - 1 \times 3 = 4 \]

c. Find the inverse of the matrix P using the cofactor/adjoint method.

The adjoint matrix of \( P \) is given by

\[ \text{adj}(P) = \begin{pmatrix} 7 & -3 \\ -1 & 1 \end{pmatrix}^T = \begin{pmatrix} 7 & -1 \\ -3 & 1 \end{pmatrix} \]

Then the inverse of \( P \) is given by

\[ P^{-1} = \frac{\text{adj}(P)}{\det[P]} = \frac{1}{4} \begin{pmatrix} 7 & -1 \\ -3 & 1 \end{pmatrix} \]

\[ = \begin{pmatrix} 7/4 & -1/4 \\ -3/4 & 1/4 \end{pmatrix} \]

d. Solve the equation \( Px = p \) using the inverse you found in part 1c

Using the inverse of \( P \), the solution is given by

\[ x = P^{-1}p = \begin{pmatrix} 7/4 & -1/4 \\ -3/4 & 1/4 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} \]

\[ = \begin{pmatrix} (7/4) \times (-2) + (-1/4) \times 2 \\ (-3/4) \times (-2) + (1/4) \times 2 \end{pmatrix} \]

\[ = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \]
e. Solve the equation $Px=p$ using Cramer’s rule.

$$P = \begin{bmatrix} 1 & 1 \\ 3 & 7 \end{bmatrix}, \quad p = \begin{bmatrix} -2 \\ 2 \end{bmatrix},$$

By Cramer’s rule, the solutions is given by

$$x = \frac{\begin{vmatrix} -2 & 1 \\ 2 & 7 \end{vmatrix}}{\text{det}[P]} = \frac{1}{4} \begin{bmatrix} -16 \\ 8 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}.$$
Problem 2. Consider the following matrix and vector.

\[
A = \begin{bmatrix}
1 & -3 & 2 \\
-5 & 16 & -11 \\
3 & -6 & 3
\end{bmatrix}, \quad b = \begin{bmatrix}
1 \\
-5 \\
7
\end{bmatrix}
\]

a. Use elementary row operations to find both the inverse of A and solve the equation \(Ax = b\) in one set of operations.

Augment matrix \(A\) with a \(3 \times 3\) identity matrix and matrix \(b\). That is, let

\[
\tilde{A} = [A \ I_{3 \times 3} \ b] = \begin{bmatrix}
1 & -3 & 2 & 1 & 0 & 0 & 1 \\
-5 & 16 & -11 & 0 & 1 & 0 & -5 \\
3 & -6 & 3 & 0 & 0 & 1 & 7
\end{bmatrix}
\]

Then use elementary row operations on matrix \(\tilde{A}\) to create an identity matrix on the left side. To begin with, based on matrix \(\tilde{A}\), add the first row multiplied by 5 to the second row; and add the first row multiplied by \(-3\) to the third row.

\[
\begin{bmatrix}
1 & -3 & 2 & 1 & 0 & 0 & 1 \\
0 & 1 & -1 & 5 & 1 & 0 & 0 \\
0 & 3 & -3 & -3 & 0 & 1 & 4
\end{bmatrix}
\]

Based on the matrix on the right side of equation (5), subtract the second row multiplied by \(3\) from the third row.

\[
\begin{bmatrix}
1 & -3 & 2 & 1 & 0 & 0 & 1 \\
0 & 1 & -1 & 5 & 1 & 0 & 0 \\
0 & 3 & -3 & -3 & 0 & 1 & 4
\end{bmatrix}
\]

Since the first 3 elements of the third row of matrix on the right side of equation (7) are all zeros, to create an identity matrix is not possible. Thus, the inverse of \(A\) does not exist.

Further, the last element of the third row of matrix on the right side of equation (7) is not zero. As a result, there is no solution for the equation.
b. Find the determinant of the matrix $A$.

\[
A = \begin{bmatrix} 1 & -3 & 2 \\ -5 & 16 & -11 \\ 3 & -6 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}
\]

\[
det[A] = \begin{vmatrix} 16 & -11 \\ -6 & 3 \end{vmatrix} - (-3) \begin{vmatrix} -5 & -11 \\ 3 & 3 \end{vmatrix} + 2 \begin{vmatrix} -5 & 16 \\ 3 & -6 \end{vmatrix}
\]

\[
= (48 - 66) + 3(-15 + 33) + 2(30 - 48)
\]

\[
= 0
\]

c. Find the inverse of the matrix $A$ using the cofactor/adjoint method.

If the inverse of the matrix $A$ does exist, it is given by

\[
A^{-1} = \frac{\text{adj}(A)}{\det[A]}
\]

However, $\det[A] = 0$. As a result, the inverse of the matrix $A$ does not exist.
d. Solve the equation $Ax=b$ using the inverse you found in part 2c

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -5 & 16 & -11 \\ 3 & -6 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$$

Since the determinant of $A$ is zero, there is no solution for the equation.

e. Solve the equation $Ax=b$ using Cramer’s rule.

Since the determinant of $A$ is zero, there is no solution for the equation.
Problem 3. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Check to see if there are potential points of inflection at points other than critical points.

a. \( f(x) = \frac{(4x^2 - 2x + 62)}{\frac{x}{4} - \frac{1}{4}} \).

You need not find the points of inflection for this problem. Hint: The simplified version of the first derivative is \( \frac{(x^2 - 2x - 15)}{(\frac{x}{4} - \frac{1}{4})^2} = \frac{16(x^2 - 2x - 15)}{(x - 1)^2} \).

You should find the second derivative of \( f(x) \) but here is the answer: \( \frac{512}{(-1 + x)^3} \).

First, simplify \( f(x) = \frac{(4x^2 - 2x + 62)}{\frac{x}{4} - \frac{1}{4}} \).

\[
\begin{align*}
16x + 8 & \quad \frac{1}{4}x - \frac{1}{4} \\
4x^2 - 2x + 62 & \quad 4x^2 + 4x \\
-4x^2 + 4x & \quad 2x + 62 \\
-2x + 2 & \quad 64
\end{align*}
\]

That is,

\[
f(x) = 16x + 8 + 64 \left( \frac{x}{4} - \frac{1}{4} \right)^{-1} = 16x + 8 + 256(x - 1)^{-1}
\]

So the derivatives of \( f(x) \) are given by

\[
\begin{align*}
f'(x) & = 16 - 256(x - 1)^{-2} \quad (8) \\
f''(x) & = 512(x - 1)^{-3} \quad (9)
\end{align*}
\]
Set the first derivative, equation (8), to zero.

\[ f'(x) = 16 - 256(x - 1)^{-2} = 0 \]

\[ \Rightarrow \quad (x - 1)^{-2} = \frac{16}{256} = \frac{1}{16} \]

\[ \Rightarrow \quad (x - 1)^2 = 16 \]

\[ \Rightarrow \quad (x - 1) = \pm 4 \]

\[ \Rightarrow \quad x = 5 \quad \text{or} \quad x = -3 \]

Check the second derivative, equation (9), for \( x = 5 \) and \( x = -3 \) respectively.

\[ f''(5) = 512(5 - 1)^{-3} > 0 \]

\[ f''(-3) = 512(-3 - 1)^{-3} < 0 \]

So \( x = -3 \) is a local maximum point; \( x = 5 \) is a local minimum point.
b. \( f(x) = \frac{x^5}{5} - \frac{x^4}{4} - \frac{31x^3}{3} + \frac{25x^2}{2} + 150x \)

You need not find the points of inflection for this problem as they are all imaginary numbers.

Hint: You will need to use long division twice.

The derivatives of \( f(x) = \frac{x^5}{5} - \frac{x^4}{4} - \frac{31x^3}{3} + \frac{25x^2}{2} + 150x \) are given by

\[
\begin{align*}
    f'(x) &= x^4 - x^3 - 31x^2 + 25x + 150 \quad (10) \\
    f''(x) &= 4x^3 - 3x^2 - 62x + 25 \quad (11)
\end{align*}
\]

Set the first derivative, equation (10), to be zero.

\[
f'(x) = x^4 - x^3 - 31x^2 + 25x + 150 = 0 \quad (12)
\]

By trying, it can be found out that \( x = 3 \) is a solution. So \( f'(x) = x^4 - x^3 - 31x^2 + 25x + 150 \) can be factorized using \( (x - 3) \) as follows.

\[
\begin{align*}
x - 3 & \Big| x^4 - x^3 - 31x^2 + 25x + 150 \\
& = x^4 + 3x^3 - 2x^3 - 6x^2 + 25x^2 + 75x - 50x - 150 \\
& \quad = 2x^3 - 31x^2 - 2x^3 + 6x^2 + 25x + 25x^2 - 75x \\
& \quad \quad = -25x^2 - 25x - 50x + 150 \\
& \quad \quad \quad = 50x - 150 \\
& \quad \quad \quad \quad = 0
\end{align*}
\]
Still by trying, it can be found that \( x^3 + 2x^2 - 25x - 50 \) can be factorized by \((x + 2)\) as follows.

\[
\begin{array}{c}
x + 2) x^3 + 2x^2 - 25x - 50 \\
\quad - x^2 - 2x^2 \\
\hline
\quad - 25x - 50 \\
\quad 25x + 50 \\
\hline
\quad 0
\end{array}
\]

Then we can solve the equation \( f'(x) = 0. \)

\[
f'(x) = x^4 - x^3 - 31x^2 + 25x + 150 = 0
\]

\[
\Rightarrow \quad (x - 3)(x + 2)(x^2 - 25) = 0
\]

\[
\Rightarrow \quad x = 3 \quad \text{or} \quad x = -2 \quad \text{or} \quad x = \pm 5
\]

Check the second derivative, equation (11), for \( x = 3, x = -2, \) and \( x = \pm 5 \) respectively.

\[
f''(3) = 4 \times 3^3 - 3 \times 3^2 - 62 \times 3 + 25 = -80
\]

\[
f''(-2) = 4 \times (-2)^3 - 3 \times (-2)^2 - 62 \times (-2) + 25 = 105
\]

\[
f''(5) = 4 \times 5^3 - 3 \times 5^2 - 62 \times 3 + 25 = 140
\]

\[
f''(-5) = 4 \times (-5)^3 - 3 \times (-5)^2 - 62 \times (-5) + 25 = -240
\]

Therefore, \( x = 3 \) and \( x = -5 \) are local maximum points; \( x = -2 \) and \( x = 5 \) are local minimum points.
Problem 4. Solve the following system of equations.

\[1764x_1^{-2/3}x_2^{1/4} - 108 = 0\]
\[1323x_1^{1/3}x_2^{-3/4} - 343 = 0\]

Hint: \(x_1 = 343\).

Rearrange the first equation.

\[1764x_1^{-2/3}x_2^{1/4} - 108 = 0\]
\[\Rightarrow x_1^{-2/3}x_2^{1/4} = 108/1764 = 3/49\]
\[\Rightarrow x_2^{1/4} = \frac{3}{49}x_1^{2/3}\]
\[\Rightarrow x_2^{-3/4} = \left(\frac{3}{49}x_1^{2/3}\right)^{-3}\]

Substitute \(x_2^{-3/4} = \left(\frac{3}{49}x_1^{2/3}\right)^{-3}\) into the second equation.

\[1323x_1^{1/3}x_2^{-3/4} - 343 = 0\]
\[\Rightarrow 1323x_1^{1/3}\left(\frac{3}{49}x_1^{2/3}\right)^{-3} - 343 = 0\]
\[\Rightarrow x_1^{-5/3} = \frac{343}{1323 \cdot \frac{3}{49}} = 7^{-5}\]
\[\Rightarrow x_1 = 7^3 = 343\]

Substitute \(x_1 = 343\) into \(x_2^{1/4} = \frac{3}{49}x_1^{2/3}\).

\[x_2^{1/4} = \frac{3}{49}x_1^{2/3}\]
\[\Rightarrow x_2^{1/4} = \frac{3}{49} \cdot 343^{2/3} = 3\]
\[\Rightarrow x_2 = 3^4 = 81\]

So the solution are \(x_1 = 343, x_2 = 81\).
Problem 5. Find all first and second partial derivatives of each of the following
a. \( f(x_1, x_2) = 60x_1 + 21x_2 - x_1^2 + x_1x_2 - x_2^2 \)

\[
\begin{align*}
\frac{\partial f}{\partial x_1} &= 60 - 2x_1 + x_2 \\
\frac{\partial f}{\partial x_2} &= 21 + x_1 - 2x_2 \\
\frac{\partial^2 f}{\partial x_1 \partial x_1} &= -2 \\
\frac{\partial^2 f}{\partial x_1 \partial x_2} &= 1 \\
\frac{\partial^2 f}{\partial x_2 \partial x_1} &= 1 \\
\frac{\partial^2 f}{\partial x_2 \partial x_2} &= -2
\end{align*}
\]
\[ f(x_1, x_2) = 80x_1 + 22x_2 - x_1^2 + 2x_1x_2 - 2x_2^2 \]

\[ \frac{\partial f}{\partial x_1} = 80 - 2x_1 + 2x_2 \]

\[ \frac{\partial f}{\partial x_2} = 22 + 2x_1 - 4x_2 \]

\[ \frac{\partial^2 f}{\partial x_1^2} = -2 \]

\[ \frac{\partial^2 f}{\partial x_1 \partial x_2} = 2 \]

\[ \frac{\partial^2 f}{\partial x_2^2} = -4 \]
c. \( f(x_1, x_2) = 10x_1 + 9x_2 - x_1^2 + x_1x_2 - 2x_2^2 \)

<table>
<thead>
<tr>
<th>( \frac{\partial f}{\partial x_1} )</th>
<th>( = 10 - 2x_1 + x_2 )</th>
<th>( \frac{\partial f}{\partial x_2} )</th>
<th>( = 9 + x_1 - 4x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial^2 f}{\partial x_1 \partial x_1} )</td>
<td>( = -2 )</td>
<td>( \frac{\partial^2 f}{\partial x_1 \partial x_2} )</td>
<td>( = 1 )</td>
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<tr>
<td>( \frac{\partial^2 f}{\partial x_2 \partial x_1} )</td>
<td>( = 1 )</td>
<td>( \frac{\partial^2 f}{\partial x_2 \partial x_2} )</td>
<td>( = -4 )</td>
</tr>
</tbody>
</table>
d. \( f(x_1, x_2, x_3) = 100x_1^{1/2}x_2^{1/5}x_3^{1/4} - 6x_1 - 5x_2 - 2x_3 \)

\[
\frac{\partial f}{\partial x_1} = 50x_1^{-1/2}x_2^{1/5}x_3^{1/4} - 6 \\
\frac{\partial f}{\partial x_2} = 20x_1^{1/2}x_2^{-4/5}x_3^{1/4} - 5 \\
\frac{\partial f}{\partial x_3} = 25x_1^{1/2}x_2^{1/5}x_3^{-3/4} - 2
\]

\[
\frac{\partial^2 f}{\partial x_1 \partial x_1} = -25x_1^{-3/2}x_2^{1/6}x_3^{1/4} \\
\frac{\partial^2 f}{\partial x_1 \partial x_2} = 10x_1^{-1/2}x_2^{-4/5}x_3^{1/4} \\
\frac{\partial^2 f}{\partial x_1 \partial x_3} = \frac{25}{2}x_1^{-1/2}x_2^{1/5}x_3^{-3/4}
\]

\[
\frac{\partial^2 f}{\partial x_2 \partial x_1} = -25x_1^{-3/2}x_2^{1/6}x_3^{1/4} \\
\frac{\partial^2 f}{\partial x_2 \partial x_2} = -16x_1^{1/2}x_2^{-9/5}x_3^{1/4} \\
\frac{\partial^2 f}{\partial x_2 \partial x_3} = 5x_1^{1/2}x_2^{-4/5}x_3^{-3/4}
\]

\[
\frac{\partial^2 f}{\partial x_3 \partial x_1} = 25x_1^{1/2}x_2^{1/5}x_3^{-3/4} \\
\frac{\partial^2 f}{\partial x_3 \partial x_2} = 5x_1^{1/2}x_2^{4/5}x_3^{-3/4} \\
\frac{\partial^2 f}{\partial x_3 \partial x_3} = -\frac{75}{4}x_1^{1/2}x_2^{1/5}x_3^{-7/4}
\]
Problem 6. For each of the following problems, write an equation that represents profit as a function of the two inputs $x_1$ and $x_2$. Write it in the form $\pi = pf(x_1, x_2) - w_1 x_1 - w_2 x_2$ and then simplify the expression. Then find all first and second partial derivatives of the function at the specified point.

a. 

\[ f(x_1, x_2) = 60x_1 + 21x_2 - x_1^2 - x_2^2 \]

\[ p = 4 \quad w_1 = 60 \quad w_2 = 12 \]

\[ x_1 = 36 \quad x_2 = 27 \]

\[ \pi = 4(60x_1 + 21x_2 - x_1^2 - x_2^2) - 60x_1 \cdot 12x_2 \]

\[ = 180x_1 + 72x_2 - 4x_1^2 + 4x_2^2 \]

<table>
<thead>
<tr>
<th>[ \frac{\partial \pi}{\partial x_1} ]</th>
<th>[ \frac{\partial \pi}{\partial x_2} ]</th>
<th>[ \frac{\partial^2 \pi}{\partial x_1 \partial x_1} ]</th>
<th>[ \frac{\partial^2 \pi}{\partial x_1 \partial x_2} ]</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$180 - 8x_1$</td>
<td>$72 + 4x_1 - 8x_2$</td>
<td>$-8$</td>
<td>$4$</td>
<td>$4$</td>
<td>$8$</td>
</tr>
</tbody>
</table>
b.

\[ f(x_1, x_2) = 80x_1 + 22x_2 - x_1^2 + 2x_1x_2 - 2x_2^2 \]

\[ p = 4 \]

\[ w_1 = 24, \quad w_2 = 16 \]

\[ x_1 = 83, \quad x_2 = 46 \]

\[ \pi = 4(80x_1 + 22x_2 - x_1^2 + 2x_1x_2 - 2x_2^2) - 24x_1 - 16x_2 \]

\[ = 356x_1 + 72x_2 - 4x_1^2 + 8x_1x_2 - 8x_2^2 \]

\[
\frac{\partial \pi}{\partial x_1} = 356 - 8x_1 + 8x_2
\]

\[
\frac{\partial \pi}{\partial x_2} = 72 + 8x_1 - 16x_2
\]

\[
\frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -8
\]

\[
\frac{\partial^2 \pi}{\partial x_2 \partial x_2} = 8
\]

\[
\frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 8
\]

\[
\frac{\partial^2 \pi}{\partial x_2 \partial x_1} = -16
\]
\[ f(x_1, x_2) = 10x_1 + 9x_2 - x_1^2 + x_1x_2 - 2x_2^2 \]

\[ p = 5 \]
\[ w_1 = 30 \]
\[ w_2 = 20 \]
\[ x_1 = 3 \]
\[ x_2 = 2 \]

\[ \pi = 5(10x_1 + 9x_2 - x_1^2 + x_1x_2 - 2x_2^2) - 30x_1 - 20x_2 \]
\[ = 20x_1 + 25x_2 - 5x_1^2 + 5x_1x_2 - 10x_2^2 \]

\[ \frac{\partial \pi}{\partial x_1} = 20 - 10x_1 + 5x_2 \]

\[ \frac{\partial \pi}{\partial x_2} = 25 - 30x_1 - 20x_2 \]

\[ \frac{\partial^2 \pi}{\partial x_1 \partial x_1} = -10 \]

\[ \frac{\partial^2 \pi}{\partial x_1 \partial x_2} = 5 \]

\[ \frac{\partial^2 \pi}{\partial x_2 \partial x_1} = 5 \]

\[ \frac{\partial^2 \pi}{\partial x_2 \partial x_2} = -20 \]
d.

\[ f(x_1, x_2) = x_1^{1/3} x_2^{1/4} \]
\[ p = 5292 \]
\[ w_1 = 108, \quad w_2 = 343 \]
\[ x_1 = 343, \quad x_2 = 81 \]

\[ \pi = 5292 x_1^{1/3} x_2^{1/4} - 108 x_1 - 343 x_2 \]

\[
\begin{align*}
\frac{\partial \pi}{\partial x_1} &= 1764 x_1^{-2/3} x_2^{1/4} - 108 \\
\frac{\partial \pi}{\partial x_2} &= 1323 x_1^{1/3} x_2^{-3/4} - 343 \\
\frac{\partial^2 \pi}{\partial x_1 \partial x_1} &= -1176 x_1^{-5/3} x_2^{1/4} \\
\frac{\partial^2 \pi}{\partial x_1 \partial x_2} &= 441 x_1^{-2/3} x_2^{-3/4} \\
\frac{\partial^2 \pi}{\partial x_2 \partial x_1} &= 441 x_1^{-2/3} x_2^{-3/4} \\
\frac{\partial^2 \pi}{\partial x_2 \partial x_2} &= -3969 x_1^{1/3} x_2^{-7/4}
\end{align*}
\]
In this table fill in values of $x_1$ and $x_2$ given to obtain numerical answers.

<table>
<thead>
<tr>
<th>$\frac{\partial^2 \pi}{\partial x_1 \partial x_1}$</th>
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<td>$\frac{\partial \pi}{\partial x_1}$ = 1764 $\times$ 343 $\times$ $\frac{1}{3}$ $\times$ $\frac{1}{4}$ $\times$ $\frac{1}{3}$ $\times$ $\frac{1}{4}$ $\times$ 81 $\times$ 108 = -108</td>
<td>$\frac{\partial \pi}{\partial x_2}$ = 441 $\times$ 343 $\times$ $\frac{1}{3}$ $\times$ $\frac{1}{4}$ $\times$ $\frac{1}{3}$ $\times$ $\frac{1}{4}$ $\times$ 81 $\times$ 343 = 0</td>
<td>$\frac{\partial \pi}{\partial x_1}$ = 1764 $\times$ 343 $\times$ $\frac{1}{3}$ $\times$ $\frac{1}{4}$ $\times$ $\frac{1}{3}$ $\times$ $\frac{1}{4}$ $\times$ 81 $\times$ 108 = -108</td>
<td>$\frac{\partial \pi}{\partial x_2}$ = 441 $\times$ 343 $\times$ $\frac{1}{3}$ $\times$ $\frac{1}{4}$ $\times$ $\frac{1}{3}$ $\times$ $\frac{1}{4}$ $\times$ 81 $\times$ 343 = 0</td>
</tr>
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</table>
\[
\begin{align*}
\text{e. } f(x_1, x_2) &= x_1^{1/2} x_2^{1/4} \\
p &= 288, \\
w_1 &= 32, \\
w_2 &= 81 \\
x_1 &= 81, \\
x_2 &= 16 \\
\pi &= \frac{288}{1} x_1^{1/2} x_2^{1/4} - 32x_1 - 81x_2 \\
\frac{\partial \pi}{\partial x_1} &= 144 \\
\frac{\partial \pi}{\partial x_2} &= 72 \\
\frac{\partial^2 \pi}{\partial x_1^2} &= -54 \\
\frac{\partial^2 \pi}{\partial x_1 \partial x_2} &= 36 \\
\frac{\partial^2 \pi}{\partial x_2^2} &= 36
\end{align*}
\]
In this table fill in values of $x_1$ and $x_2$ given to obtain numerical answers.

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<tr>
<th>$\frac{\partial \pi}{\partial x_1}$</th>
<th>$\frac{\partial ^2 \pi}{\partial x_1 \partial x_1}$</th>
<th>$\frac{\partial ^2 \pi}{\partial x_1 \partial x_2}$</th>
<th>$\frac{\partial ^2 \pi}{\partial x_2 \partial x_1}$</th>
<th>$\frac{\partial ^2 \pi}{\partial x_2 \partial x_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$144 \times 81^{1/2} \times 16^{1/4} - 32 = 0$</td>
<td>$36 \times 81^{1/2} \times 16^{1/4} = \frac{3}{2}$</td>
<td>$18 - 4 \times 81^{1/2} \times 16^{1/4} = \frac{3}{2}$</td>
<td>$18 - 4 \times 81^{1/2} \times 16^{1/4} = \frac{3}{2}$</td>
<td>$-54 \times 81^{1/2} \times 16^{1/4} = -\frac{243}{64}$</td>
</tr>
</tbody>
</table>

In this table fill in values of $x_1$ and $x_2$ given to obtain numerical answers.