

ECONOMICS 207
SPRING 2008
PROBLEM SET 1
KEY

Problem 1. Do the following problems from the book.

a. Section 1.2

- 1) 4a
- 2) 4b
- 3) 4c
- 4) 13a

b. Section 1.3

- 1) 1b
- 2) 2a
- 3) 4a
- 4) 4c
- 5) 9a
- 6) 12a
- 7) 12c
- 8) 15c
- 9) 15f

c. Section 1.4

- 1) 2b
- 2) 3a
- 3) 3c
- 4) 5b
- 5) 6f

d. Section 1.5

- 1) 1d
- 2) 1f
- 3) 3d
- 4) 4a
- 5) 4g
- 6) 5b
- 7) 6a
- 8) 6b
- 9) 6c
- 10) 9a
- 11) 9d

Problem 2. Carry out the following operations.

a. $2^2 \times 2^4$

$$2^2 \times 2^4 = 2^{2+4} = 2^6 = 64$$

b. $(2^2)^4$

$$(2^2)^4 = 4^4 = 256$$

c. $(2 + 2)^3$

$$(2 + 2)^3 = 4^3 = 64$$

d. $2^3 + 2^4$

$$2^3 + 2^4 = 8 + 16 = 24$$

e. $(2 \times 2)^3$

$$(2 \times 2)^3 = 4^3 = 64$$

f. $(2^2)^3$

$$(2^2)^3 = 4^3 = 64$$

g. $(6^2) \times (2^2)$

$$(6^2) \times (2^2) = 36 \times 4 = 144$$

Problem 3. Carry out the following operations.

a. $3^2 \times 3^3$

$$3^2 \times 3^3 = 3^{2+3} = 3^5 = 243$$

b. $(3^2)^3$

$$(3^2)^3 = 9^3 = 729$$

c. $(3 + 3)^3$

$$(3 + 3)^3 = 6^3 = 108$$

d. $3^3 + 3^4$

$$3^3 + 3^4 = 27 + 81 = 108$$

e. $(3 \times 3)^2 + 36^{\frac{1}{2}}$

$$(3 \times 3)^2 + 36^{\frac{1}{2}} = 9^2 + (6^2)^{\frac{1}{2}} = 81 + 6^{2 \times \frac{1}{2}} = 87$$

f. $(3^2)^2 + 216^{\frac{1}{3}}$

$$(3^2)^2 + 216^{\frac{1}{3}} = 9^2 + (6^3)^{\frac{1}{3}} = 81 + 6 = 87$$

g. $(7^2)^2$

$$(7^2)^2 = 49^2 = 2401$$

Problem 4. Carry out the following operations.

a. $(5^2)^2$

$$(5^2)^2 = 25^2 = 625$$

b. $(a^2)^2$

$$(a^2)^2 = a^{2 \times 2} = a^4$$

c. $2^2 \times 2^3 \times 2^4$

$$2^2 \times 2^3 \times 2^4 = 2^{2+3+4} = 2^9 = 512$$

d. $2^2 \times 2^x \times 2^4$

$$2^2 \times 2^x \times 2^4 = 2^{2+x+4} = 2^{x+6}$$

e. $(2^3 \times 2^x)^2 \times 2^4$

$$(2^3 \times 2^x)^2 \times 2^4 = (2^{3+x})^2 \times 2^4 = 2^{6+2x} \times 2^4 = 2^{6+2x+4} = 2^{10+2x}$$

f. $(5 \times x_1^{\alpha_1} \times 2^3)^2$

$$(5 \times x_1^{\alpha_1} \times 2^3)^2 = (5 \times 2^3 \times x_1^{\alpha_1})^2 = (40 \times x_1^{\alpha_1})^2 = 160x_1^{2\alpha_1}$$

g. $(Ax_1^{\alpha_1} x_2^{\alpha_2})^2$

$$(Ax_1^{\alpha_1} x_2^{\alpha_2})^2 = A^2 x_1^{2\alpha_1} x_2^{2\alpha_2}$$

Problem 5. Simplify the following expressions.

a. $\frac{4^2}{4}$

$$\frac{4^2}{4} = 4^{2-1} = 4$$

b. $\frac{2^2 a^2}{2a^3}$

$$\frac{2^2 a^2}{2a^3} = \frac{2}{a}$$

c. $\left(\frac{3^2 a^2}{3a^3}\right)^2$

$$\left(\frac{3^2 a^2}{3a^3}\right)^2 = \left(\frac{3}{a}\right)^2 = \frac{9}{a^2}$$

d. $\frac{4^2 a^2 a^4}{2a^3 4^2}$

$$\frac{4^2 a^2 a^4}{2a^3 4^2} = \frac{a^6}{2a^3} = \frac{a^3}{2}$$

Problem 6. Simplify, add, subtract, multiply or divide the following fractions. Express all answers in reduced form.

a. $\frac{21}{35}$

$$\frac{21}{35} = \frac{3 \times 7}{5 \times 7} = \frac{3}{5}$$

b. $\left(\frac{252}{462}\right) \left(\frac{4}{\frac{1}{2}}\right)$

$$\begin{aligned} \left(\frac{252}{462}\right) \left(\frac{4}{\frac{1}{2}}\right) &= \frac{252}{462} \times 8 = \frac{42 \times 6}{42 \times 11} \times 8 = \frac{6 \times 8}{11} \\ &= \frac{48}{11} \end{aligned}$$

c. $\frac{\frac{686}{245}}{\frac{441}{294}}$

$$\begin{aligned} \frac{\frac{686}{245}}{\frac{441}{294}} &= \frac{686 \times 294}{245 \times 441} = \frac{(49 \times 14) \times (21 \times 14)}{(49 \times 5) \times 21^2} = \frac{14 \times 14}{5 \times 21} = \frac{2 \times 7 \times 14}{5 \times 3 \times 7} = \frac{2 \times 14}{5 \times 3} \\ &= \frac{28}{15} \end{aligned}$$

d. $\frac{3}{4} + \frac{5}{10} + \frac{2}{5}$

$$\frac{3}{4} + \frac{5}{10} + \frac{2}{5} = \frac{15}{20} + \frac{10}{20} + \frac{8}{20} = \frac{15 + 10 + 8}{20} = \frac{33}{20}$$

e. $\frac{7}{4} + \frac{5}{11} + \frac{17}{22}$

$$\frac{7}{4} + \frac{5}{11} + \frac{17}{22} = \frac{7}{4} + \frac{5}{11} + \frac{17}{22} = \frac{77}{44} + \frac{20}{44} + \frac{34}{44} = \frac{77 + 20 + 34}{44} = \frac{131}{44}$$

f. $\frac{5}{16} + \frac{1}{3} + \frac{1}{12} - \frac{5}{6}$

$$\frac{5}{16} + \frac{1}{3} + \frac{1}{12} - \frac{5}{6} = \frac{15}{48} + \frac{16}{48} + \frac{4}{48} - \frac{40}{48} = \frac{15 + 16 + 4 - 40}{48} = -\frac{5}{48}$$

g. $\frac{5}{8} \frac{16}{5} + \frac{1}{\frac{1}{2}} - \frac{7}{35} + \left(\frac{5}{7}\right)^{-2} = \frac{99}{25}$

$$\begin{aligned} \frac{5}{8} \frac{16}{5} + \frac{1}{\frac{1}{2}} - \frac{7}{35} + \left(\frac{5}{7}\right)^{-2} &= \frac{16}{8} + \frac{2}{10} - \frac{7}{35} + \left(\frac{25}{49}\right)^{-1} = 2 + \frac{1}{5} - \frac{1}{5} + \frac{49}{25} \\ &= 2 + \frac{49}{25} = \frac{50 + 49}{25} = \frac{99}{25} \end{aligned}$$

h. $\frac{\frac{4}{13}}{\frac{1}{39}} + \frac{\frac{3}{13}}{\frac{4}{52}} - \frac{\frac{5}{13}}{\frac{5}{65}}$

$$\frac{\frac{4}{13}}{\frac{1}{39}} + \frac{\frac{3}{13}}{\frac{4}{52}} - \frac{\frac{5}{13}}{\frac{5}{65}} = \frac{4 \times 39}{13} + \frac{3}{13} \times \frac{52}{4} - \frac{5}{13} \times \frac{65}{5} = 12 + 3 - 5 = 10$$

i. $\frac{\frac{1}{2}}{\frac{4}{12}} - \frac{3}{16} \left(\frac{3}{16}\right)^{-2} = -\frac{23}{6}$

$$\frac{\frac{1}{2}}{\frac{4}{12}} - \frac{3}{16} \left(\frac{3}{16}\right)^{-2} = \frac{1}{2} \times \frac{12}{4} - \frac{3}{16} \times \left(\frac{16}{3}\right)^2 = \frac{3}{2} - \frac{16}{3} = \frac{9}{6} - \frac{32}{6} = -\frac{23}{6}$$

Problem 7. Complete the following operations.

a. $e^{2x}e^{4x}$

$$e^{2x}e^{4x} = e^{2x+4x} = e^{6x}$$

b. $(e^{2x})^3$

$$(e^{2x})^3 = e^{2x \times 3} = e^{6x}$$

c. $e^{\log(x)}$

$$e^{\log(x)} = x$$

d. $\frac{4x_1^{1/3}}{x_1}$

$$\frac{4x_1^{1/3}}{x_1} = 4x_1^{1/3-1} = 4x_1^{-2/3}$$

e. $\frac{1}{3} \frac{Ax_1^{1/3}x_2^{1/2}}{x_1}$

$$\frac{1}{3} \frac{Ax_1^{1/3}x_2^{1/2}}{x_1} = \frac{1}{3} \frac{Ax_1^{5/6}}{x_1} = \frac{1}{3} Ax_1^{5/6-1} = \frac{1}{3} Ax_1^{-1/6}$$

f. $(3 + 2x)(3 - 2x)$

$$(3 + 2x)(3 - 2x) = 9 + 6x - 6x - (2x)^2 = 9 - 4x^2$$

g. $(3 + 2x)(3 - 2x)(3 + 2x)$

$$\begin{aligned} (3 + 2x)(3 - 2x)(3 + 2x) &= (9 - 4x^2)(3 + 2x) = 3(9 - 4x^2) + 2x(9 - 4x^2) = 27 - 12x^2 + 18x - 8x^3 \\ &= 27 + 18x - 12x^2 - 8x^3 \end{aligned}$$

h. $(x - 2)^3$

$$\begin{aligned} (x - 2)^3 &= (x - 2)^2(x - 2) = (x^2 - 4x + 4)(x - 2) \\ &= x(x^2 - 4x + 4) - 2(x^2 - 4x + 4) = x^3 - 4x^2 + 4x - 2x^2 + 8x - 8 \\ &= x^3 - 6x^2 + 12x - 8 \end{aligned}$$

i. $\frac{(x^2 - 6x + 9)(x - 2)}{(x - 3)(x - 3)}$

$$\frac{(x^2 - 6x + 9)(x - 2)}{(x - 3)(x - 3)} = \frac{(x - 3)^2(x - 2)}{(x - 3)^2} = x - 2$$

Problem 8. Factor the following.

a. $x^2 - 6x + 9$

$$x^2 - 6x + 9 = (x - 3)^2$$

b. $x^2 - 25$

$$x^2 - 25 = (x + 5)(x - 5)$$

c. $x^2 - x - 12$

$$x^2 - x - 12 = (x - 4)(x + 3)$$

d. $3x^2 + 15x - 42$

$$3x^2 + 15x - 42 = 3(x^2 + 5x - 14) = 3(x + 7)(x - 2)$$

e. $12x^2 - x - 6$

$$12x^2 - x - 6 = (3x + 2)(4x - 3)$$

f. $x^2 + \sqrt{-1}x + 2$

$$x^2 + \sqrt{-1}x + 2 = x^2 + \sqrt{-1}x - 2(\sqrt{-1})^2 = (x + 2\sqrt{-1})(x - \sqrt{-1})$$