

**ECONOMICS 207**  
**SPRING 2008**  
**PROBLEM SET 2**  
**KEY**

**Problem 1.** Do the following problems from the book.

- a. Section 2.3
  - 1) 2a
  - 2) 2d
  - 3) 2e
- b. Section 2.3
  - 1) 3a
  - 2) 3b
  - 3) 4a
- c. Section 3.1
  - 1) 1a
  - 2) 1b
  - 3) 1c
- d. Section 3.2
  - 1) 1
  - 2) 2
- e. section 4.2
  - 1) 1a
  - 2) 3a
  - 3) 6

**Problem 2.** Carry out the following long division operations.

a.  $18 \overline{)234}$

$$\begin{array}{r} 13 \\ 18 \overline{)234} \\ \underline{180} \\ 54 \\ \underline{54} \\ 0 \end{array}$$

b.  $19 \overline{)228}$

$$\begin{array}{r} 12 \\ 19 \overline{)228} \\ \underline{190} \\ 38 \\ \underline{38} \\ 0 \end{array}$$

c.  $143 \overline{)65245}$

$$\begin{array}{r} 456 \\ 143 \overline{)65245} \\ \underline{57200} \\ 8045 \\ \underline{7150} \\ 895 \\ \underline{858} \\ 37 \end{array}$$

d.  $49 \overline{)117649}$

$$\begin{array}{r} 2401 \\ 49 \overline{)117649} \\ \underline{98000} \\ 19649 \\ \underline{19600} \\ 49 \\ \underline{49} \\ 0 \end{array}$$

**Problem 3.** Complete the square in the following and then write in the following form  $(x + a)^2 + c$ . For example the first problem would be written  $(x + 1)^2 - 1$ .

a.  $x^2 + 2x$

$$(x + 1)^2 - 1.$$

b.  $x^2 - 10x$

$$(x - 5)^2 - 25.$$

c.  $5x^2 + 20x$

$$5(x + 2)^2 - 20$$

d.  $7x^2 - 28x$

$$7(x - 2)^2 - 28$$

e.  $10x^2 - 20x + 90$

$$10(x - 1)^2 + 80$$

f.  $x^2 + 5x$

$$(x + 5/2)^2 - 25/4$$

**Problem 4.** Simplify, add, subtract, multiply or divide the following fractions. Express all answers in reduced form.

a.  $\frac{35}{80} + \frac{7}{16}$

$$\frac{35}{80} + \frac{7}{16} = \frac{7 \times 5}{16 \times 5} + \frac{7}{16} = \frac{7}{16} + \frac{7}{16} = \frac{7}{8}$$

b.  $\frac{13}{35} + \frac{3}{49}$

$$\frac{13}{35} + \frac{3}{49} = \frac{13 \times 7}{35 \times 7} + \frac{3 \times 5}{49 \times 5} = \frac{91 + 15}{245} = \frac{106}{245}$$

c.  $\left(\frac{455}{910}\right) \left(\frac{\frac{18}{5}}{\frac{9}{10}}\right)$

$$\begin{aligned} \left(\frac{455}{910}\right) \left(\frac{\frac{18}{5}}{\frac{9}{10}}\right) &= \frac{455}{910} \times \frac{18}{5} \times \frac{10}{9} = \frac{455}{455 \times 2} \times \frac{9 \times 2}{5} \times \frac{5 \times 2}{9} \\ &= \frac{1}{2} \times 4 = 2 \end{aligned}$$

d.  $\frac{1}{11} + \frac{1}{3} + \frac{44}{77} - \frac{11}{21}$

$$\begin{aligned} \frac{1}{11} + \frac{1}{3} + \frac{44}{77} - \frac{11}{21} &= \frac{1 \times 21}{11 \times 21} + \frac{1 \times 77}{3 \times 77} + \frac{44 \times 3}{77 \times 3} - \frac{11 \times 11}{21 \times 11} \\ &= \frac{21 + 77 + 132 - 121}{231} = \frac{109}{231} \end{aligned}$$

e.  $\frac{8a}{3b} + \frac{3b}{a} + \frac{5}{b}$

$$\frac{8a}{3b} + \frac{3b}{a} + \frac{5}{b} = \frac{8a^2}{3ab} + \frac{9b^2}{3ab} + \frac{15a}{3ab} = \frac{8a^2 + 9b^2 + 15a}{3ab}$$

f.  $\frac{a}{ab-b^2} - \frac{2}{a-b} + \frac{ab+b^2}{a^3-ab^2}$

$$\begin{aligned} \frac{a}{ab-b^2} - \frac{2}{a-b} + \frac{ab+b^2}{a^3-ab^2} &= \frac{a}{b(a-b)} - \frac{2}{a-b} + \frac{b(a+b)}{a(a+b)(a-b)} \\ &= \frac{a^2}{ab(a-b)} - \frac{2ab}{ab(a-b)} + \frac{b^2}{ab(a-b)} \\ &= \frac{a^2 - 2ab + b^2}{ab(a-b)} = \frac{(a-b)^2}{ab(a-b)} = \frac{a-b}{ab} \end{aligned}$$

**Problem 5.** Factor the following.

a.  $2x^2 - x - 10$

$$2x^2 - x - 10 = (2x - 5)(x + 2)$$

b.  $3x^2 - 17x - 28$

$$3x^2 - 17x - 28 = (3x + 4)(x - 7)$$

c.  $9x^2 - 17x - 30$

$$9x^2 - 17x - 30 = (x - 3)(9x + 10)$$

d.  $18x^2 - 123x - 21$

$$18x^2 - 123x - 21 = 3(6x^2 - 41x - 7) = 3(x - 7)(6x + 1)$$

e.  $8x^2 + 22x + 15$

$$8x^2 + 22x + 15 = (2x + 3)(4x + 5)$$

f.  $20x^2 + 64x - 21$

$$20x^2 + 64x - 21 = (2x + 7)(10x - 3)$$

**Problem 6.** Solve the following equations for  $x$ .

a.  $2x + 3 = 15$

$$\begin{aligned} 2x + 3 &= 15 \\ \Rightarrow 2x &= 15 - 3 = 12 \\ \Rightarrow x &= 6 \end{aligned}$$

b.  $7x + 3 = 39 - 5x$

$$\begin{aligned} 7x + 3 &= 39 - 5x \\ \Rightarrow 7x + 5x &= 39 - 3 \\ \Rightarrow 12x &= 36 \\ \Rightarrow x &= 3 \end{aligned}$$

c.  $\frac{x+84}{2x-3} = 10$

$$\begin{aligned} \frac{x+84}{2x-3} &= 10 \\ \Rightarrow x + 84 &= 10(2x - 3) \\ \Rightarrow x + 84 &= 20x - 30 \\ \Rightarrow 84 + 30 &= 20x - x \\ \Rightarrow 114 &= 19x \\ \Rightarrow x &= 114/19 = 6 \end{aligned}$$

d.  $\frac{2x-7}{3x+2} = \frac{3}{17}$

$$\begin{aligned} \frac{2x-7}{3x+2} &= \frac{3}{17} \\ \Rightarrow 17(2x - 7) &= 3(3x + 2) \\ \Rightarrow 34x - 119 &= 9x + 6 \\ \Rightarrow 34x - 9x &= 119 + 6 \\ \Rightarrow 25x &= 225 \\ \Rightarrow x &= 225/25 = 9 \end{aligned}$$

$$\text{e. } \frac{x^2+x-6}{(x+3)(x-2)} = \frac{1}{4}$$

$$\begin{aligned} & \frac{x^2+x-6}{(x+3)(x-2)} = \frac{1}{4} \\ \Rightarrow & \frac{x^2+x-6}{x^2+x-6} = \frac{1}{4} \\ \Rightarrow & 1 = \frac{1}{4} \end{aligned}$$

Since  $1 \neq \frac{1}{4}$ , there is no solution to this equation.

$$\text{f. } \frac{x+7}{(x-5)(x+2)} = \frac{1}{2}$$

$$\begin{aligned} & \frac{x+7}{(x-5)(x+2)} = \frac{1}{2} \\ \Rightarrow & 2(x+7) = (x-5)(x+2) = x^2 - 3x - 10 \\ \Rightarrow & x^2 - 3x - 10 = 2x + 14 \\ \Rightarrow & x^2 - 5x - 24 = 0 \\ \Rightarrow & (x-8)(x+3) = 0 \\ \Rightarrow & x = 8 \text{ or } x = -3 \end{aligned}$$

**Problem 7.** Solve the following equations for  $x$ .

a.  $x^2 - 3x - 28 = 0$

$$\begin{aligned}x^2 - 3x - 28 &= 0 \\ \Rightarrow (x - 7)(x + 4) &= 0 \\ \Rightarrow x = 7 \text{ or } x = -4\end{aligned}$$

b.  $3x^2 - 17x - 28 = 0$

$$\begin{aligned}3x^2 - 17x - 28 &= 0 \\ \Rightarrow (x - 7)(3x + 4) &= 0 \\ \Rightarrow x = 7 \text{ or } x = -4/3\end{aligned}$$

c.  $3x^2 - 7x - 20 = 0$

$$\begin{aligned}3x^2 - 7x - 20 &= 0 \\ \Rightarrow (x - 4)(3x + 5) &= 0 \\ \Rightarrow x = 4 \text{ or } x = -5/3\end{aligned}$$



d.  $18x^2 - 9x - 20 = 0$

$$\begin{aligned} & 18x^2 - 9x - 20 = 0 \\ \Rightarrow & (3x - 4)(6x + 5) = 0 \\ \Rightarrow & x = 4/3 \text{ or } x = -5/6 \end{aligned}$$

e.  $20x^2 + 17x - 10 = 0$

$$\begin{aligned} & 20x^2 + 17x - 10 = 0 \\ \Rightarrow & (4x + 5)(5x - 2) = 0 \\ \Rightarrow & x = -5/4 \text{ or } x = 2/5 \end{aligned}$$

f.  $9x^2 - 36x - 28 = 0$

$$\begin{aligned} & 9x^2 - 36x - 28 = 0 \\ \Rightarrow & (3x + 2)(3x - 14) = 0 \\ \Rightarrow & x = -2/3 \text{ or } x = 14/3 \end{aligned}$$

**Problem 8.** Solve the following equations for  $x_1$ .

a.  $9x_1^{-1/4} - 3 = 0$

$$\begin{aligned} 9x_1^{-1/4} - 3 &= 0 \\ \Rightarrow 9x_1^{-1/4} &= 3 \\ \Rightarrow x_1^{-1/4} &= 3/9 = 3^{-1} \\ \Rightarrow x_1^{1/4} &= 3 \\ \Rightarrow x_1 &= 81 \end{aligned}$$

b.  $24x_1^{-1/4} - 6 = 0$

$$\begin{aligned} 24x_1^{-1/4} - 6 &= 0 \\ \Rightarrow 24x_1^{-1/4} &= 6 \\ \Rightarrow x_1^{-1/4} &= 6/24 = 4^{-1} \\ \Rightarrow x_1^{1/4} &= 6/24 = 4 \\ \Rightarrow x_1 &= 4^4 = 256 \end{aligned}$$

c.  $243x_1^{-4/7} - 3 = 0$

$$\begin{aligned} 243x_1^{-4/7} - 3 &= 0 \\ \Rightarrow x_1^{-4/7} &= 3/243 \\ \Rightarrow x_1^{-4/7} &= 3^{-4} \\ \Rightarrow x_1^{1/7} &= 3 \\ \Rightarrow x_1 &= 3^7 = 2187 \end{aligned}$$

d.  $125x_1^{-2/3} - 5 = 0$

$$\begin{aligned} 125x_1^{-2/3} - 5 &= 0 \\ \Rightarrow x_1^{-2/3} &= 5/125 \\ \Rightarrow x_1^{-2/3} &= 5^{-2} \\ \Rightarrow x_1^{1/3} &= 5 \\ \Rightarrow x_1 &= 125 \end{aligned}$$

e.  $81x_1^{-3/5} - 3 = 0$

$$\begin{aligned} & 81x_1^{-3/5} - 3 = 0 \\ \Rightarrow & x_1^{-3/5} = 3/81 \\ \Rightarrow & x_1^{-3/5} = 3^{-3} \\ \Rightarrow & x_1^{1/5} = 3 \\ \Rightarrow & x_1 = 3^5 = 243 \end{aligned}$$

f.  $1250x_1^{-4/3} - 2 = 0$

$$\begin{aligned} & 1250x_1^{-4/3} - 2 = 0 \\ \Rightarrow & x_1^{-4/3} = 2/1250 \\ \Rightarrow & x_1^{-4/3} = 5^{-4} \\ \Rightarrow & x_1^{1/3} = 5 \\ \Rightarrow & x_1 = 5^3 = 125 \end{aligned}$$

**Problem 9.** Consider the following quadratic equation in  $x_1$ .

$$p(3a_3x_1^2 + 2a_2x_1 + a_1) = w_1$$

In each of the problems below solve the equation for  $x_1$  for the given values of  $a_3, a_2, a_1, p$  and  $w_1$ .

a.  $a_3 = -1, a_2 = 20, a_1 = 100, p = 20$  and  $w_1 = 500$

$$\begin{aligned} p(3a_3x_1^2 + 2a_2x_1 + a_1) &= w_1 \\ \Rightarrow 20(-3x_1^2 + 40x_1 + 100) &= 500 \\ \Rightarrow -3x_1^2 + 40x_1 + 100 &= 500/20 = 25 \\ \Rightarrow -3x_1^2 + 40x_1 + 75 &= 0 \\ \Rightarrow (x_1 - 15)(-3x_1 - 5) &= 0 \\ \Rightarrow x_1 = 15 \quad \text{or} \quad x_1 = 5/3 \end{aligned}$$

b.  $a_3 = -1, a_2 = 5, a_1 = 50, p = 10$  and  $w_1 = 20$

$$\begin{aligned} p(3a_3x_1^2 + 2a_2x_1 + a_1) &= w_1 \\ \Rightarrow 10(-3x_1^2 + 10x_1 + 50) &= 20 \\ \Rightarrow -3x_1^2 + 10x_1 + 50 &= 20/10 = 2 \\ \Rightarrow -3x_1^2 + 10x_1 + 48 &= 0 \\ \Rightarrow (x_1 - 6)(-3x_1 - 8) &= 0 \\ \Rightarrow x_1 = 6 \quad \text{or} \quad x_1 = 8/3 \end{aligned}$$

c.  $a_3 = -2$ ,  $a_2 = 50$ ,  $a_1 = 200$ ,  $p = 10$  and  $w_1 = 4860$

$$\begin{aligned} p(3a_3x_1^2 + 2a_2x_1 + a_1) &= w_1 \\ \Rightarrow 10(-6x_1^2 + 100x_1 + 200) &= 4860 \\ \Rightarrow -6x_1^2 + 100x_1 + 200 &= 486 \\ \Rightarrow -6x_1^2 + 100x_1 - 286 &= 0 \\ \Rightarrow 3x_1^2 - 50x_1 + 143 &= 0 \\ \Rightarrow (x_1 - 13)(3x_1 - 11) &= 0 \\ \Rightarrow x_1 = 13 \text{ or } x_1 = 11/3 \end{aligned}$$

d.  $a_3 = -2$ ,  $a_2 = 50$ ,  $a_1 = 150$ ,  $p = 15$  and  $w_1 = 4500$

$$\begin{aligned} p(3a_3x_1^2 + 2a_2x_1 + a_1) &= w_1 \\ \Rightarrow 15(-6x_1^2 + 100x_1 + 150) &= 4500 \\ \Rightarrow -6x_1^2 + 100x_1 + 150 &= 300 \\ \Rightarrow -6x_1^2 + 100x_1 - 150 &= 0 \\ \Rightarrow 3x_1^2 - 50x_1 + 75 &= 0 \\ \Rightarrow (x - 15)(3x_1 - 5) &= 0 \\ \Rightarrow x = 15 \text{ or } x_1 = 5/3 \end{aligned}$$