Problem 1. Solve the following equations for x.

a. $8x^2 - 22x + 15 = 0$

$$
\begin{aligned}
8x^2 - 22x + 15 &= 0 \\
\Rightarrow \quad (2x - 3)(4x - 5) &= 0 \\
\Rightarrow \qquad\qquad x = 3/2 \ \text{ or } \ \ x &= 5/4
\end{aligned}
$$

b. $15x^2 - 145x + 90 = 0$

$$
\begin{aligned}
15x^2 - 145x + 90 &= 0 \\
\Rightarrow \quad 3x^2 - 29x + 18 &= 0 \\
\Rightarrow \quad (x - 9)(3x - 2) &= 0 \\
\Rightarrow \qquad\qquad x = 9 \ \text{ or } \ \ x &= 2/3
\end{aligned}
$$

Date: August 25, 2008.

Problem 2. Solve the following equations for x_1.

a. $81x_1^{-1/3} - 27 = 0$

$$
\begin{aligned}
81x_1^{-1/3} - 27 &= 0 \\
\Rightarrow \qquad 81x_1^{-1/3} &= 27 \\
\Rightarrow \qquad x_1^{-1/3} &= 27/81 = 3^{-1} \\
\Rightarrow \qquad x_1^{1/3} &= 3 \\
\Rightarrow \qquad x_1 &= 27
\end{aligned}
$$

b. $50x_1^{-2/5} - 2 = 0$

$$
\begin{aligned}
50x_1^{-2/5} - 2 &= 0 \\
\Rightarrow \qquad 50x_1^{-2/5} &= 2 \\
\Rightarrow \qquad x_1^{-2/5} &= 2/50 = 5^{-2} \\
\Rightarrow \qquad x_1^{1/5} &= 5 \\
\Rightarrow \qquad x_1 &= 5^5 = 3125
\end{aligned}
$$

Problem 3. Solve the following equations for x_1.

a. $2x_1^{2/3} = x_1^{3/4}$

$$2x_1^{2/3} = x_1^{3/4}$$
$$\Rightarrow \quad 2x_1^{2/3} - x_1^{3/4} = 0$$
$$\Rightarrow \quad x_1^{2/3}(2 - x_1^{3/4-2/3}) = 0$$
$$\Rightarrow \quad x_1^{2/3}(2 - x_1^{1/12}) = 0$$
$$\Rightarrow \quad x_1^{2/3} = 0 \quad \text{or} \quad x_1^{1/12} = 2$$
$$\Rightarrow \quad x_1 = 0 \quad \text{or} \quad x = 2^{12} = 4096$$

b. $2x_1^{1/3} = 3x_1^{1/2}$

$$2x_1^{1/3} = 3x_1^{1/2}$$
$$\Rightarrow \quad x_1^{1/3}(2 - 3x_1^{1/6}) = 0$$
$$\Rightarrow \quad x_1^{1/3} = 0 \quad \text{or} \quad 2 - 3x_1^{1/6} = 0$$
$$\Rightarrow \quad x_1 = 0 \quad \text{or} \quad x_1^{1/6} = 2/3$$
$$\Rightarrow \quad x_1 = 0 \quad \text{or} \quad x_1 = (2/3)^6 = 64/729$$

Problem 4. Solve the following systems of equations for x_1 and x_2 using the method of elimination.

a.

$$x_1 + x_2 = 1$$
$$3x_1 + 2x_2 = 4$$

Add the first equation multiplied by -2 to the second equation.

$$3x_1 + 2x_2 + (x_1 + x_2) \times (-2) = 4 + 1 \times (-2)$$
$$\Rightarrow \qquad\qquad x_1 = 2$$

Add the first equation multiplied by -3 to the second equation.

$$3x_1 + 2x_2 + (x_1 + x_2) \times (-3) = 4 + 1 \times (-3)$$
$$\Rightarrow \qquad\qquad -x_2 = 1$$
$$\Rightarrow \qquad\qquad x_2 = -1$$

So the solution is

$$x_1 = 2, \quad x_2 = -1$$

b.

$$x_1 - 3x_2 = -9$$
$$2x_1 - 7x_2 = -22$$

Add the first equation multiplied by -2 to the second equation.

$$2x_1 - 7x_2 + (x_1 - 3x_2) \times (-2) = -22 - 9 \times (-2)$$
$$\Rightarrow \qquad\qquad -x_2 = -4$$
$$\Rightarrow \qquad\qquad x_2 = 4$$

Add the first equation multiplied by $\dfrac{-7}{3}$ to the second equation.

$$2x_1 - 7x_2 + (x_1 - 3x_2) \times \tfrac{-7}{3} = -22 - 9 \times \tfrac{-7}{3}$$
$$\Rightarrow \qquad\qquad -\frac{x_1}{3} = -1$$
$$\Rightarrow \qquad\qquad x_1 = 3$$

So the solution is

$$x_1 = 3, \quad x_2 = 4$$

c.

$$-x_1 + 5x_2 = 13$$
$$-x_1 + 4x_2 = 10$$

Add the first equation multiplied by -1 to the second equation.

$$-x_1 + 4x_2 + (-x_1 + 5x_2) \times (-1) = 10 + 13 \times (-1)$$
$$\Rightarrow \qquad\qquad -x_2 = -3$$
$$\Rightarrow \qquad\qquad x_2 = 3$$

Add the first equation multiplied by $-\dfrac{4}{5}$ to the second equation.

$$-x_1 + 4x_2 + (-x_1 + 5x_2) \times \tfrac{-4}{5} = 10 + 13 \times \tfrac{-4}{5}$$
$$\Rightarrow \qquad\qquad \frac{-x_1}{5} = -\frac{2}{5}$$
$$\Rightarrow \qquad\qquad x_1 = 2$$

So the solution is

$$x_1 = 2, \quad x_2 = 3$$

d.

$$2x_1 + 3x_2 = 4$$
$$5x_1 + 7x_2 = 9$$

Add the first equation multiplied by $-\dfrac{5}{2}$ to the second equation.

$$5x_1 + 7x_2 + (2x_1 + 3x_2) \times \tfrac{-5}{2} = 9 + 4 \times \tfrac{-5}{2}$$
$$\Rightarrow \qquad\qquad \frac{-x_2}{2} = -1$$
$$\Rightarrow \qquad\qquad x_2 = 2$$

Add the first equation multiplied by $-\dfrac{7}{3}$ to the second equation.

$$5x_1 + 7x_2 + (2x_1 + 3x_2) \times \tfrac{-7}{3} = 9 + 4 \times \tfrac{-7}{3}$$
$$\Rightarrow \qquad\qquad \frac{x_1}{3} = -\frac{1}{3}$$
$$\Rightarrow \qquad\qquad x_1 = -1$$

So the solution is

$$x_1 = -1, \quad x_2 = 2$$

e.

$$2x_1 + 2x_2 = 8$$
$$5x_1 + 4x_2 = 17$$

Add the first equation multiplied by $-\dfrac{5}{2}$ to the second equation.

$$5x_1 + 4x_2 + (2x_1 + 2x_2) \times \tfrac{-5}{2} = 17 + 8 \times \tfrac{-5}{2}$$
$$\Rightarrow \qquad -x_2 = -3$$
$$\Rightarrow \qquad x_2 = 3$$

Add the first equation multiplied by -2 to the second equation.

$$5x_1 + 4x_2 + (2x_1 + 2x_2) \times (-2) = 17 + 8 \times (-2)$$
$$\Rightarrow \qquad x_1 = 1$$

So the solution is

$$x_1 = 1, \quad x_2 = 3$$

Problem 5. Solve the following systems of equations for x_1, x_2, and x_3 using the method of elimination.

a.

$$\{x_1 = 1,\ x_2 = 3,\ x_3 = -2\}$$

$$x_1 + 2x_2 - 2x_3 = 11$$

$$3x_1 + 7x_2 - 10x_3 = 44$$

$$3x_1 + 4x_2 + x_3 = 13$$

Add the first equation multiplied by -3 to the second equation.

$$3x_1 + 7x_2 - 10x_3 + (x_1 + 2x_2 - 2x_3) \times (-3) = 44 + 11 \times (-3)$$

$$\Rightarrow \qquad x_2 - 4x_3 = 11 \tag{1}$$

Add the first equation multiplied by -3 to the third equation.

$$3x_1 + 4x_2 + x_3 + (x_1 + 2x_2 - 2x_3) \times (-3) = 13 + 11 \times (-3)$$

$$\Rightarrow \qquad -2x_2 + 7x_3 = -20 \tag{2}$$

Add equation (1) multiplied by 2 to equation (2)

$$-2x_2 + 7x_3 + (x_2 - 4x_3) \times 2 = -20 + 11 \times 2$$

$$\Rightarrow \qquad -x_3 = 2$$

$$\Rightarrow \qquad x_3 = -2 \tag{3}$$

Add equation (1) multiplied by $\frac{7}{4}$ to equation (2)

$$-2x_2 + 7x_3 + (x_2 - 4x_3)\frac{7}{4} = -20 + 11 \times \frac{7}{4}$$

$$\Rightarrow \qquad -\frac{1}{4}x_2 = -\frac{3}{4}$$

$$\Rightarrow \qquad x_2 = 3$$

Add $x_2 = 3$ multiplied by -2 to the first equation; and add $x_3 = -2$ multiplied by 2 to the first equation.

$$x_1 + 2x_2 - 2x_3 - 2x_2 + 2x_3 = 11 + 3 \times (-2) - 2 \times 2$$

$$\Rightarrow \qquad x_1 = 1$$

So the solution is

$$x_1 = 1, \quad x_2 = 3, \quad x_3 = -1$$

b.
$$\{x_1 = 2, \ x_2 = 2, \ x_3 = -1\}$$

$$\frac{1}{2}x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + 10x_2 + 4x_3 = 20$$
$$3x_1 + 6x_2 + 7x_3 = 11$$

Add the first equation multiplied by -4 to the second equation.

$$2x_1 + 10x_2 + 4x_3 + \left(\frac{1}{2}x_1 + 2x_2 + x_3\right) \times (-4) = 20 + 4 \times (-4)$$
$$\Rightarrow \qquad\qquad\qquad\qquad 2x_2 = 4$$
$$\Rightarrow \qquad\qquad\qquad\qquad x_2 = 2$$

Add the first equation multiplied by -6 to the second equation.

$$3x_1 + 6x_2 + 7x_3 + \left(\frac{1}{2}x_1 + 2x_2 + x_3\right) \times (-6) = 11 + 4 \times (-6)$$
$$\Rightarrow \qquad\qquad\qquad\qquad -6x_2 + x_3 = -13 \qquad\qquad\qquad (4)$$

Add $x_2 = 2$ multiplied by 6, i.e., $6x_2 = 12$, to equation (4).

$$-6x_2 + x_3 + 6x_2 = -13 + 12$$
$$\Rightarrow \qquad\qquad x_3 = -1$$

Add $x_2 = 2$ multiplied by -2 to the first equation; and add $x_3 = -1$ multiplied by -1 to the first equation.

$$\frac{1}{2}x_1 + 2x_2 + x_3 - 2x_2 - x_3 \ = 4 - 4 + 1$$
$$\Rightarrow \qquad\qquad \frac{1}{2}x_1 = 1$$
$$\Rightarrow \qquad\qquad x_1 = 2$$

So the solution is
$$x_1 = 2, \quad x_2 = 2, \quad x_3 = -1$$

c.

$$\{x_1 = 1, \ x_2 = -1, \ x_3 = 2\}$$

$$3x_1 \ + \ \frac{1}{3}x_2 \ + \ 2x_3 \ = \ \frac{20}{3}$$

$$6x_1 \ + \ x_2 \ + \ 4x_3 \ = \ 13$$

$$-3x_1 \ - \ 2x_2 \ - \ 3x_3 \ = \ -7$$

Add the first equation multiplied by -2 to the second equation.

$$6x_1 + x_2 + 4x_3 + \left(3x_1 + \frac{1}{3}x_2 + 2x_3\right) \times (-2) = 13 + \frac{20}{3} \times (-2)$$

$$\Rightarrow \qquad\qquad\qquad\qquad \frac{1}{3}x_2 = -\frac{1}{3}$$

$$\Rightarrow \qquad\qquad\qquad\qquad x_2 = -1$$

Add the first equation to the third equation.

$$-3x_1 - 2x_2 - 3x_3 + \left(3x_1 + \frac{1}{3}x_2 + 2x_3\right) = -7 + \frac{20}{3}$$

$$\Rightarrow \qquad\qquad\qquad\qquad -\frac{5}{3}x_2 - x_3 = -\frac{1}{3} \qquad\qquad\qquad (5)$$

Add $x_2 = -1$ multiplied by $\frac{5}{3}$ to equation (5).

$$-\frac{5}{3}x_2 - x_3 + \frac{5}{3}x_2 = -\frac{1}{3} - \frac{5}{3}$$

$$\Rightarrow \qquad\qquad\qquad -x_3 = -2$$

$$\Rightarrow \qquad\qquad\qquad x_3 = 2$$

Add $x_2 = -1$ multiplied by -1 to the second equation; and add $x_3 = 2$ multiplied by -4 to the second equation.

$$6x_1 + x_2 + 4x_3 - x_2 - 4x_3 = 13 + 1 - 8$$

$$\Rightarrow \qquad\qquad\qquad 6x_1 = 6$$

$$\Rightarrow \qquad\qquad\qquad x_1 = 1$$

So the solution is

$$x_1 = 1, \quad x_2 = -1, \quad x_3 = 2$$

Problem 6. Solve the following systems of equations for x_1 and x_2 using the method of substitution.

a.

$$24x_1^{-3/4}x_2^{1/2} - 9 = 0$$
$$48x_1^{1/4}x_2^{-1/2} - 32 = 0$$

From the first equation,

$$
\begin{aligned}
24x_1^{-3/4}x_2^{1/2} - 9 &= 0 \\
\Rightarrow \quad 24x_1^{-3/4}x_2^{1/2} &= 9 \\
\Rightarrow \quad \frac{24}{9}x_1^{-3/4} &= x_2^{-1/2} \\
\Rightarrow \quad x_2^{-1/2} &= \frac{8}{3}x_1^{-3/4}
\end{aligned}
$$

Substitute $x_2^{-1/2} = \frac{8}{3}x_1^{-3/4}$ into the second equation.

$$
\begin{aligned}
48x_1^{1/4}x_2^{-1/2} - 32 &= 0 \\
\Rightarrow \quad 48x_1^{1/4}\left(\frac{8}{3}x_1^{-3/4}\right) - 32 &= 0 \\
\Rightarrow \quad x_1^{1/4}\left(\frac{8}{3}x_1^{-3/4}\right) &= 32/48 \\
\Rightarrow \quad x_1^{-1/2} &= \frac{2}{3} \times \frac{3}{8} = 4^{-1} \\
\Rightarrow \quad x_1^{1/2} &= 4 \\
\Rightarrow \quad x_1 &= 16
\end{aligned}
$$

Substitute $x_1 = 16$ into $x_2^{-1/2} = \frac{8}{3}x_1^{-3/4}$.

$$
\begin{aligned}
x_2^{-1/2} &= \frac{8}{3}x_1^{-3/4} \\
\Rightarrow \quad x_2^{-1/2} &= \frac{8}{3}16^{-3/4} \\
\Rightarrow \quad x_2^{-1/2} &= \frac{1}{3} \\
\Rightarrow \quad x_2 &= 9
\end{aligned}
$$

So the solutions is

$$x_1 = 16, \quad x_2 = 9$$

b.

$$128x_1^{-3/5}x_2^{1/4} - 64 = 0$$
$$80x_1^{2/5}x_2^{-3/4} - 5 = 0$$

From the first equation,

$$
\begin{aligned}
128x_1^{-3/5}x_2^{1/4} - 64 &= 0 \\
\Rightarrow \qquad x_1^{-3/5}x_2^{1/4} &= 64/128 \\
\Rightarrow \qquad x_2^{1/4} &= 2^{-1}x_1^{3/5} \\
\Rightarrow \qquad x_2^{-3/4} &= 8x_1^{-9/5}
\end{aligned}
$$

Substitute $x_2^{-3/4} = 8x_1^{-9/5}$ into the second equation.

$$
\begin{aligned}
80x_1^{2/5}x_2^{-3/4} - 5 &= 0 \\
\Rightarrow \quad 80x_1^{2/5}\left(8x_1^{-9/5}\right) &= 5 \\
\Rightarrow \qquad x_1^{-7/5} &= \tfrac{1}{128} = 2^{-7} \\
\Rightarrow \qquad x_1 &= 2^5 = 32
\end{aligned}
$$

Substitute $x_1 = 32$ into $x_2^{1/4} = 2^{-1}x_1^{3/5}$.

$$
\begin{aligned}
x_2^{1/4} &= 2^{-1}x_1^{3/5} \\
\Rightarrow \quad x_2^{1/4} &= 2^{-1}32^{3/5} = 4 \\
\Rightarrow \qquad x_2 &= 256
\end{aligned}
$$

So the solution is

$$x_1 = 32, \quad x_2 = 256$$

Problem 7. Do the following problems from the book.

a. Section 3.6
   1) 1a
   2) 1b
   3) 3
   4) 5
b. Section 4.2
   1) 3b
   2) 7a
   3) 7b
   4) 13
c. Section 4.4
   1) 1a
   2) 1b
   3) 3
   4) 7a
   5) 7b
   6) 7c
   7) 7d
d. Section 4.5
   1) 3
   2) 5
e. Section 4.6 (Besides equation 2 on page 105 of the text, equation 5 on page 106 of the text is useful.)
   1) 3a
   2) 3c
   3) 3d
   4) 3e
   5) 3f