ECONOMICS 207
SPRING 2008
PROBLEM SET 7
KEY

Problem 1. Find the second derivative of each of the following functions with respect to x

a. $f(x) \;=\; 5x^3 - 4x^2 + 15x + 10$

$$f'(x) = 15x^2 - 8x + 15$$

$$f''(x) = 30x - 8$$

b. $f(x) \;=\; 200x^{5/6} + 3x^{-4} + 30x^{-1/3}$

$$\begin{aligned} f'(x) &= \frac{1000}{6}x^{-1/6} - 12x^{-5} - 10x^{-4/3} \\ &= \frac{500}{3}x^{-1/6} - 12x^{-5} - 10x^{-4/3} \\ f''(x) &= -\frac{500}{18}x^{-7/6} + 60x^{-6} + \frac{40}{3}x^{-7/3} \\ &= -\frac{250}{9}x^{-7/6} + 60x^{-6} + \frac{40}{3}x^{-7/3} \end{aligned}$$

c. $f(x) \;=\; \left(5x^3 + 4x^2 + 3\log(2x^2 + 5x)\right)^3$

$$f'(x) = 3\left(5x^3 + 4x^2 + 3\log(2x^2+5x)\right)^2 \left(15x^2 + 8x + \frac{3(4x+5)}{2x^2+5x}\right)$$

$$\begin{aligned} f''(x) = {} & 6\left(5x^3 + 4x^2 + 3\log(2x^2+5x)\right)\left(15x^2 + 8x + \frac{3(4x+5)}{2x^2+5x}\right)^2 \\ & + 3\left(5x^3 + 4x^2 + 3\log(2x^2+5x)\right)^2\left(30x + 8 + \frac{12(2x^2+5x) + 3(4x+5)^2}{(2x^2+5x)^2}\right) \end{aligned}$$

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d. $f(x) = 3xe^{2x^2}$

$$\begin{aligned} f'(x) &= 3e^{2x^2} + 3xe^{2x^2} \cdot (4x) \\ &= e^{2x^2}\left(3 + 12x^2\right) \end{aligned}$$

$$\begin{aligned} f'' &= \left(e^{2x^2} \cdot (4x)\right)\left(3 + 12x^2\right) + e^{2x^2}(24x) \\ &= e^{2x^2}\left(48x^3 + 36x\right) \end{aligned}$$

Problem 2. Find the definite integral of each of the following functions.

a. $\int_1^4 \left(12x^2 + 6x + 5\right)\, dx$

$$\begin{aligned}\int_1^4 \left(12x^2 + 6x + 5\right)\, dx &= \left(4x^3 + 3x^2 + 5x\right)\Big|_1^4 \\ &= \left(4 \times 4^3 + 3 \times 4^2 + 5 \times 4\right) - \left(4 \times 1^3 + 3 \times 1^2 + 5 \times 1\right) \\ &= 324 - 12 \\ &= 312\end{aligned}$$

b. $\int_{16}^{81} \left(45x^{-3/4}z^{2/5} - 15\right)\, dx, \quad z = 1024.$

$$\begin{aligned}\int_{16}^{81} \left(45x^{-3/4}z^{2/5} - 15\right) dx &= \int_{16}^{81} \left(45x^{-3/4}1024^{2/5} - 15\right) dx \\ &= \int_{16}^{81} \left(720x^{-3/4} - 15\right) dx \\ &= \left(2880x^{1/4} - 15x\right)\Big|_{16}^{81} \\ &= \left(2880 \times 81^{1/4} - 15 \times 81\right) - \left(2880 \times 16^{1/4} - 15 \times 16\right) \\ &= 7425 - 5520 \\ &= 1905\end{aligned}$$

c. $\int_0^{10} \left(300 - 100x + 9x^2\right)\, dx$

$$\begin{aligned}\int_0^{10} \left(300 - 100x + 9x^2\right)\, dx &= \left(300x - 50x^2 + 3x^3\right)\Big|_0^{10} \\ &= \left(300 \times 10 - 50 \times 10^2 + 3 \times 10^3\right) - 0 \\ &= 1000\end{aligned}$$

d. $\int_0^{8} \left(-320 + 280x - 30x^2\right)\, dx$

$$\begin{aligned}\int_0^{8} \left(-320 + 280x - 30x^2\right)\, dx &= \left(-320x + 140x^2 - 10x^3\right)\Big|_0^{8} \\ &= \left(-320 \times 8 + 140 \times 8^2 - 10 \times 8^3\right) - 0 \\ &= 1280\end{aligned}$$

Problem 3. Solve the following systems of equations.

$$\begin{aligned} 180x_1^{-2/3}x_2^{2/5} \ - \ 80 \ &= 0 \\ 216x_1^{1/3}x_2^{-3/5} \ - \ 81 \ &= 0 \end{aligned}$$

Note that $x_1 = 27$ and $x_2 = 32$.

Rearrange the second equation.

$$\begin{aligned} 216x_1^{1/3}x_2^{-3/5} - 81 &= 0 \\ \Rightarrow \qquad x_1^{1/3}x_2^{-3/5} &= 81/216 = 3/8 \\ \Rightarrow \qquad x_1^{1/3} &= \tfrac{3}{8}x_2^{3/5} \\ \Rightarrow \qquad x_1^{-1/3} &= \tfrac{8}{3}x_2^{-3/5} \\ \Rightarrow \qquad x_1^{-2/3} &= \tfrac{64}{9}x_2^{-6/5} \end{aligned}$$

Substitute $x_1^{-2/3} = \frac{64}{9}x_2^{-6/5}$ into the first equation.

$$\begin{aligned} 180x_1^{-2/3}x_2^{2/5} - 80 &= 0 \\ \Rightarrow \quad 180\left(\tfrac{64}{9}x_2^{-6/5}\right)x_2^{2/5} - 80 &= 0 \\ \Rightarrow \qquad (20 \times 64)x_2^{-4/5} &= 80 \\ \Rightarrow \qquad x_2^{-4/5} &= 1/16 = 2^{-4} \\ \Rightarrow \qquad x_2^{1/5} &= 2 \\ \Rightarrow \qquad x_2 &= 2^5 = 32 \end{aligned}$$

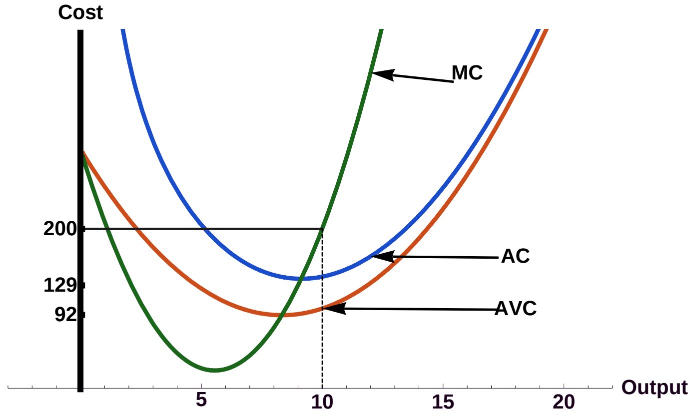
Substitute $x_2 = 32$ into $x_1^{1/3} = \frac{3}{8}x_2^{3/5}$.

$$\begin{aligned} x_1^{1/3} &= \tfrac{3}{8}x_2^{3/5} \\ \Rightarrow \quad x_1^{1/3} &= \tfrac{3}{8} \times 32^{3/5} \\ \Rightarrow \quad x_1^{1/3} &= 3 \\ \Rightarrow \quad x_1 &= 3^3 = 27 \end{aligned}$$

So the solution is $x_1 = 27, \ x_2 = 32$.

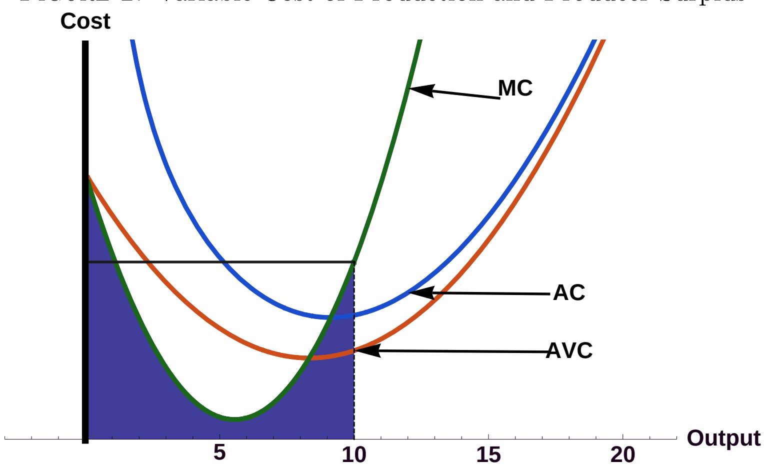
Problem 4. The cost function for a firm is a rule or mapping that tells the total cost of production of any output level produced by the firm. If the variable y represents the output of the firm, then the cost function is given by c(y). Marginal cost represents the change in the cost of production for the firm as output changes and is given by the derivative of the cost function with respect to output,i.e., Marginal Cost(MC) = $\frac{dc(y)}{dy}$. A competitive firm facing a fixed output price maximizes profit at the output level where marginal cost is equal to price as in the figure 1.

Figure 1. Profit Maximization



The area below the cost curve is a measure of variable cost and can be found by integrating the marginal cost curve from 0 to any given output level y. The shaded area in figure 2 represents the variable cost of production for the cost function $c(y) = 400 + 300y - 50\,y^2 + 3y^3$.

Figure 2. Variable Cost of Production and Producer Surplus



Producer surplus is the area below a given price and above the marginal cost curve. Producer surplus is the unshaded area below the horizontal line at 200 in figure 2. Producer surplus can be computed by subtracting the shaded area from total revenue.

a. Find the profit maximizing level of output for the following firm. Demonstrate that the level you choose maximizes profit.

$$\begin{aligned} price \;&=\; p \;=\; \$200 \\ cost \;&=\; c(y) \;=\; 400 \;+\; 300y \;-\; 50y^2 \;+\; 3y^3 \end{aligned}$$

The profit is given by

$$\begin{aligned} Profit &= Revenue - Cost \\ &= py - c(y) \\ &= 200y - \left(400 + 300y - 50y^2 + 3y^3\right) \\ &= -\left(400 + 100y - 50y^2 + 3y^3\right) \end{aligned}$$

Then the derivatives of profit are given by

$$\frac{d\,Profit}{d\,y} = -\left(100 - 100y + 9y^2\right) \tag{1}$$

$$\frac{d^2\,Profit}{d\,y^2} = -(-100 + 18y) = 100 - 18y \tag{2}$$

Set the first derivative, equation (1), to zero.

$$\begin{aligned} -\left(100 - 100y + 9y^2\right) \;&=\; 0 \\ \Rightarrow \quad -(y-10)(9y-10) \;&=\; 0 \\ \Rightarrow \quad y = 10 \qquad \text{or} \qquad & y = 10/9 \end{aligned}$$

Check the second derivative, equation (2).
At $y = 10/9$, $100 - 18y = 100 - 18 \times (10/9) = 80 > 0$.
At $y = 10$, $100 - 18y = 100 - 18 \times 10) = -80 < 0$.

So the optimal level is that $y = 10$.

b. What is revenue minus variable cost for this firm when price is \$400?

When price is \$400, the revenue minus variable cost is given by

$$\begin{aligned} Revenue - variable\ cost &= 400y - \left(100y - 50y^2 + 3y^3\right) \\ &= 300y + 50y^2 - 3y^3 \end{aligned}$$

c. Find producer surplus for this firm assuming you are only given the following marginal cost function: $MC(y) = 300 - 100y + 9y^2$ and a price of \$200.

The producer surplus is equal to the revenue minus variable cost which is the integral of marginal cost. Then it is given by

$$\begin{aligned} Producer\ surplus &= revenue - \int_0^y MC(y)d\,y \\ &= py - \int_0^y \left(300 - 100y + 9y^2\right) d\,y \\ &= py - \left(300y - 50y^2 + 3y^3\right) \\ &= 200y - \left(300y - 50y^2 + 3y^3\right) \\ &= -\left(100y - 50y^2 + 3y^3\right) \end{aligned}$$

And when $y = 10$, the producer surplus is then given by

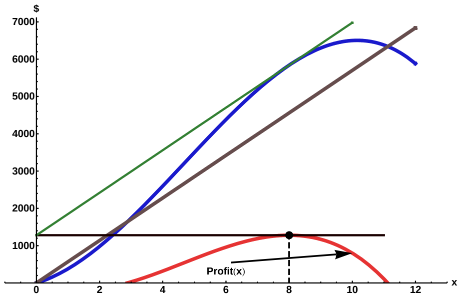
$$\begin{aligned} -\left(100y - 50y^2 + 3y^3\right) &= -\left(100 \times 10 - 50 \times 10^2 + 3 \times 10^3\right) \\ &= 1000 \end{aligned}$$

Problem 5. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input.

a. Write down an equation that represents profit for the firm.
b. Maximize this function by taking its derivative with respect to the variable input x and setting the derivative equal to zero.
c. Solve the equation in part 5b for the potentially profit maximizing level of x.
d. Determine using the second order conditions which of the roots represents maximum profit.

$$\begin{aligned} \textit{output price} &= p = 5 \\ \textit{input price} &= w = 570 \\ y = \textit{output} &= f(x) = 50x + 28x^2 - 2x^3 \end{aligned}$$

Figure 3. Revenue, Cost and Profit



a.

The profit is given by

$$\begin{aligned} Profit &= Revenue - cost \\ &= py - wx \\ &= 5\left(50x + 28x^2 - 2x^3\right) - 570x \\ &= 5\left(-64x + 28x^2 - 2x^3\right) \end{aligned}$$

b.

The derivatives of the profit are given by

$$\frac{d\,Profit}{d\,x} = 5\left(-64 + 56x - 6x^2\right) \tag{3}$$

$$\frac{d^2\,Profit}{d\,x^2} = 5\,(56 - 12x) \tag{4}$$

Set the first derivative, equation (3), to zero.

$$5\left(-64 + 56x - 6x^2\right) = 0 \tag{5}$$

c.

Solve equation (5) in part b of this problem.

$$\begin{aligned} 5\left(-64+56x-6x^2\right) &= 0 \\ \Rightarrow \quad -32+28x-3x^2 &= 0 \\ \Rightarrow \quad -(x-8)(3x-4) &= 0 \\ \Rightarrow \quad x=8 \quad &\text{or} \quad x=4/3 \end{aligned}$$

d.

Check the second derivative for $x = 8$ and $x = 4/3$ respectively.

At $x = 4/3$, $\dfrac{d^2\,Profit}{d\,x^2} = 5\,(56 - 12 \times (4/3)) = 5 \times 40 > 0$.

At $x = 5$, $\dfrac{d^2\,Profit}{d\,x^2} = 5\,(56 - 12 \times 8) = 5 \times (-40) < 0$.

So $x = 5$ is the optimal level of maximizing profit.

Problem 6. Solve the following system of equations for x_1, x_2, and x_3.

$$\{x_1 = -2,\ x_2 = 2,\ x_3 = 1\}$$

$$\begin{aligned} x_1 + 3x_2 - 3x_3 &= 1 \\ 4x_1 + 13x_2 - 9x_3 &= 9 \\ 2x_1 + 5x_2 - 8x_3 &= -2 \end{aligned}$$

Add the first equation multiplied by -4 to the second equation.

$$\begin{aligned} & 4x_1 + 13x_2 - 9x_3 + (x_1 + 3x_2 - 3x_3) \times (-4) = 9 - 4 \\ \Rightarrow \quad & x_2 + 3x_3 = 5 \\ \Rightarrow \quad & x_2 = 5 - 3x_3 \end{aligned} \tag{6}$$

Add the first equation multiplied by -2 to the second equation.

$$\begin{aligned} & 2x_1 + 5x_2 - 8x_3 + (x_1 + 3x_2 - 3x_3) \times (-2) = -2 - 2 \\ \Rightarrow \quad & -x_2 - 2x_3 = -4 \\ \Rightarrow \quad & x_2 = 4 - 2x_3 \end{aligned} \tag{7}$$

From equation (6) and equation (7), we obtain.

$$\begin{aligned} 5 - 3x_3 &= 4 - 2x_3 \\ \Rightarrow \quad 5 - 4 &= 3x_3 - 2x_3 \\ \Rightarrow \quad x_3 &= 1 \end{aligned}$$

Substitute $x_3 = 1$ into equation (6).

$$\begin{aligned} x_2 &= 5 - 3x_3 = 5 - 3 \\ &= 2 \end{aligned}$$

Substitute $x_2 = 2$ and $x_3 = 1$ into the first equation.

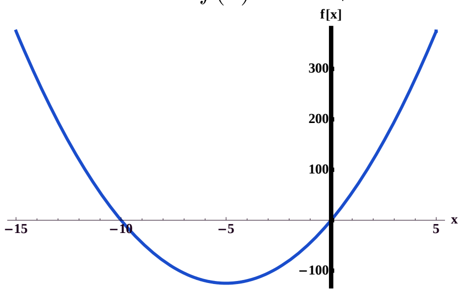
$$\begin{aligned} x_1 + 3x_2 - 3x_3 &= 1 \\ \Rightarrow \quad x_1 + 3 \times 2 - 3 \times 1 &= 1 \\ \Rightarrow \quad x_1 + 3 &= 1 \\ \Rightarrow \quad x_1 &= -2 \end{aligned}$$

So the solutions is $x_1 = -2,\ x_2 = 2,\ x_3 = 1$.

Problem 7. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Find the inflection points.

a. $y = 5x^2 + 50x$

FIGURE 4. $f(x) = 5x^2 + 50x$



The derivatives of $f(x) = 5x^2 + 50x$ are given by

$$f'(x) = 10x + 50 \quad (8)$$

$$f''(x) = 10 \quad (9)$$

Set the first derivative, equation (8), to be zero.

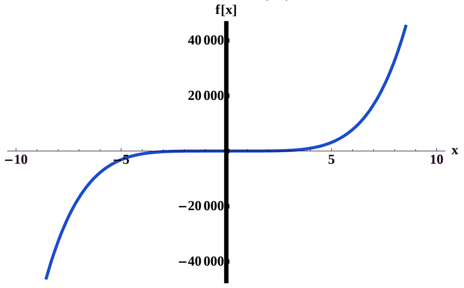
$$\begin{aligned} f'(x) = 10x + 50 &= 0 \\ \Rightarrow \qquad x &= -5 \end{aligned}$$

Since the second derivative, equation (9), is greater than zero, $x = -5$ is a local minimum point. And it is a global minimum point.

Also because the second derivative, equation (9), is greater than zero, there is no inflection point for this function.

b. $y = x^5$

FIGURE 5. $f(x) = x^5$



The derivatives of $f(x) = x^5$ are given by

$$f'(x) = 5x^4 \tag{10}$$

$$f''(x) = 20x^3 \tag{11}$$

$$f^{(3)}(x) = 60x^2 \tag{12}$$

$$f^{(4)}(x) = 120x \tag{13}$$

$$f^{(5)}(x) = 120 \tag{14}$$

Set the first derivative, equation (10), to be zero.

$$\begin{aligned} f'(x) = 5x^4 &= 0 \\ \Rightarrow \qquad x &= 0 \end{aligned}$$

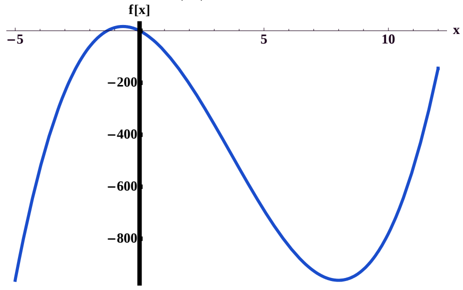
Also set the second derivative, equation (11), to be zero.

$$\begin{aligned} f''(x) = 20x^3 &= 0 \\ \Rightarrow \qquad x &= 0 \end{aligned}$$

At $x = 0$, the second, third, and fourth derivatives are all zero. However, the fifth derivative is not zero. As a result, $x = 0$ is an inflection point.

c. $f(x) = 3x^3 - 33x^2 - 48x$

FIGURE 6. $f(x) = 3x^3 - 33x^2 - 48x$



The derivatives of $f(x) = 3x^3 - 33x^2 - 48x$ are given by

$$f'(x) = 9x^2 - 66x - 48 \quad (15)$$

$$f''^{(x)} = 18x - 66 \quad (16)$$

$$f^{(3)}(x) = 18 \quad (17)$$

Set the first derivative,equation (15), to be zero.

$$\begin{aligned} f'(x) = 9x^2 - 66x - 48 &= 0 \\ \Rightarrow \quad f'(x) = 3x^2 - 22x - 16 &= 0 \\ \Rightarrow \quad (x-8)(3x+2) &= 0 \\ \Rightarrow \quad x = 8 \quad &\text{or} \quad x = -2/3 \end{aligned}$$

Check the second derivatives, equation (16), for $x = 8$ and $x = -2/3$ respectively.

$$f''(8) = 18 \times 8 - 66 = 78 > 0$$

$$f''(-2/3) = 18 \times (-2/3) - 66 = -78 < 0$$

So $x = 8$ is a local minimum point; $x = -2/3$ is a local maximum point;

Set the second derivative, equation (16), to be zero.

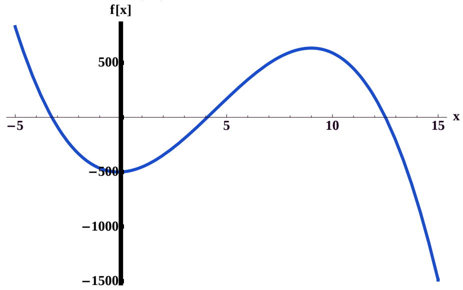
$$f''(x) = 18 \times x - 66 = 0$$

$$\Rightarrow \quad x = 11/3$$

Since $f^{(3)}(x) = 18 \neq 0$, $x = 11/3$ is an inflection point.

d. $f(x) = -3x^3 + 40x^2 + 9x - 500$

FIGURE 7. $f(x) = -3x^3 + 40x^2 + 9x - 500$



The derivatives of $f(x) = -3x^3 + 40x^2 + 9x - 500$ are given by

$$f'(x) = -9x^2 + 80x + 9 \tag{18}$$

$$f''(x) = -18x + 80 \tag{19}$$

$$f^{(3)}(x) = -18 \tag{20}$$

Set the first derivative, equation (18), to be zero.

$$\begin{aligned} f'(x) = -9x^2 + 80x + 9 &= 0 \\ \Rightarrow \quad -(x-9)(9x+1) &= 0 \\ \Rightarrow \quad x = 9 \quad &\text{or} \quad x = -1/9 \end{aligned}$$

Check the second derivative, equation (19), for $x = 9$ and $x = -1/9$ respectively.

$$f''(9) = -18 \times 9 + 80 = -82 < 0$$

$$f''(-1/9) = -18 \times (-1/9) + 80 = 82 > 0$$

So $x = 9$ is a local maximum point; $x = -1/9$ is a local minimum point.

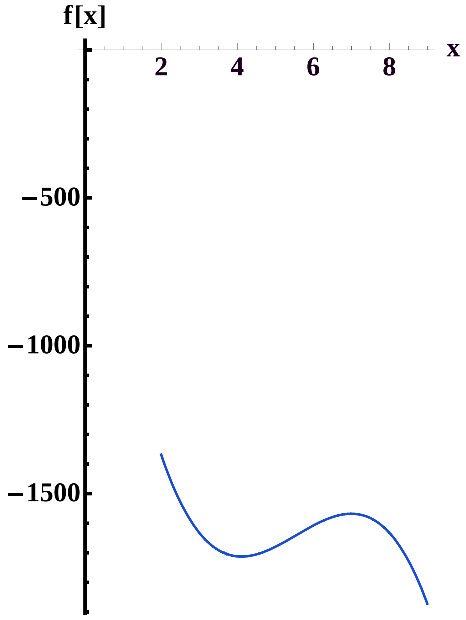
Set the second derivative, equation (19), to be zero.

$$\begin{aligned} f''(x) = -18x + 80 &= 0 \\ \Rightarrow \quad x &= 40/9 \end{aligned}$$

Since $f^{(3)}(x) = -18 \neq 0$, $x = 40/9$ is an inflection point.

e. $f(x) = -12x^3 + 200x^2 - 1036x$

FIGURE 8. $f(x) = -12x^3 + 200x^2 - 1036x$



The derivatives of $f(x) = -12x^3 + 200x^2 - 1036x$ are given by

$$f'(x) = -36x^2 + 400x - 1036 \tag{21}$$

$$f''(x) = -72x + 400 \tag{22}$$

$$f^{(3)}(x) = -72 \tag{23}$$

Set the first derivative, equation (21), to be zero.

$$\begin{aligned} f'(x) = -36x^2 + 400x - 1036 &= 0 \\ \Rightarrow \quad 9x^2 - 100x + 259 &= 0 \\ \Rightarrow \quad (x-7)(9x-37) &= 0 \\ \Rightarrow \quad x = 7 \quad &\text{or} \quad x = 37/9 \end{aligned}$$

Check the second derivative, equation (22), for $x = 7$ and $x = 37/9$ respectively.

$$f''(7) = -72 \times 7 + 400 = -104 < 0$$

$$f''(37/9) = -72 \times (37/9) + 400 = 104 > 0$$

So $x = 7$ is a local maximum point; $x = 37/9$ is a local minimum point.

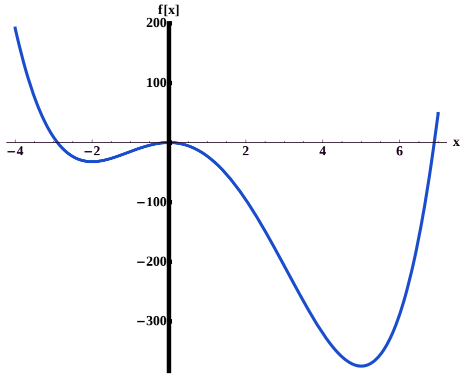
Set the second derivative, equation (22), to be zero.

$$f''(x) = -72x + 400 = 0 \qquad \Rightarrow \qquad x = 50/9$$

Since $f^{(3)}(x) = -72 < 0$, $x = 50/9$ is an inflection point.

f. $f(x) = x^4 - 4x^3 - 20x^2$

FIGURE 9. $f(x) = x^4 - 4x^3 - 20x^2$



The derivatives of $f(x) = x^4 - 4x^3 - 20x^2$ are given by

$$f'(x) = 4x^3 - 12x^2 - 40x \tag{24}$$

$$f''(x) = 12x^2 - 24x - 40 \tag{25}$$

$$f^{(3)}(x) = 24x - 24 \tag{26}$$

Set the first derivative, equation (24), to be zero.

$$\begin{aligned} f'(x) = 4x^3 - 12x^2 - 40x &= 0 \\ \Rightarrow \quad 4x(x^2 - 3x - 10) &= 0 \\ \Rightarrow \quad 4x(x-5)(x+2) &= 0 \\ \Rightarrow \quad x = 0 \quad \text{or} \quad x = 5 \quad &\text{or} \quad x = -2 \end{aligned}$$

Check the second derivative, equation (25), for $x = 0$, $x = 5$, and $x = -2$ respectively.

$$f''(0) = 12 \times 0^2 - 24 \times 0 - 40 = -40 < 0$$

$$f''(5) = 12 \times 5^2 - 24 \times 5 - 40 = 140 > 0$$

$$f''(-2) = 12 \times (-2)^2 - 24 \times (-2) - 40 = 56 > 0$$

So $x = 0$ is a local maximum point; $x = 5$ is a local minimum point; $x = -2$ is a local minimum point.

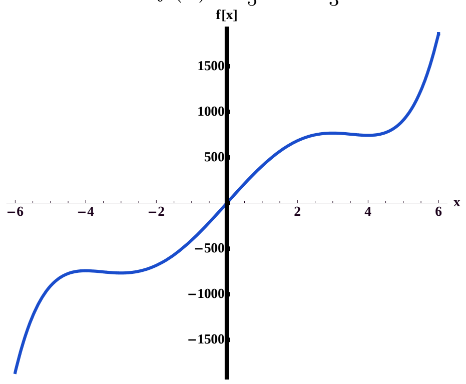
Set the second derivative, equation (25), to be zero.

$$\begin{aligned} f''(x) = 12x^2 - 24x - 40 &= 0 \\ \Rightarrow \quad 3x^2 - 6x - 10 &= 0 \\ \Rightarrow \quad x &= \tfrac{6 \pm \sqrt{36 - 4 \times 3 \times (-10)}}{6} \\ \Rightarrow \quad x &= \tfrac{3 \pm \sqrt{39}}{3} \end{aligned}$$

Since $f^{(3)}(x) = 24x - 24$ equals to zero only when $x = 1$, $x = \frac{3 \pm \sqrt{39}}{3}$ are inflection points.

g. $f(x) = \frac{3}{5}x^5 - \frac{75}{3}x^3 + 432x$

FIGURE 10. $f(x) = \frac{3}{5}x^5 - \frac{75}{3}x^3 + 432x$



The derivatives of $f(x) = \frac{3}{5}x^5 - \frac{75}{3}x^3 + 432x$ are given by

$$f'(x) = 3x^4 - 75x^2 + 432 \tag{27}$$

$$f''(x) = 12x^3 - 150x = 3x(4x^2 - 50) \tag{28}$$

$$f^{(3)}(x) = 36x^2 - 150 \tag{29}$$

Set the first derivative, equation (27), to be zero.

$$\begin{aligned} f'(x) = 3x^4 - 75x^2 + 432 &= 0 \\ \Rightarrow \quad x^4 - 25x^2 + 144 &= 0 \\ \Rightarrow \quad (x^2 - 9)(x^2 - 16) &= 0 \\ \Rightarrow \quad x = \pm 3 \quad &\text{or} \quad x = \pm 4 \end{aligned}$$

Check the second derivative, equation (28), for $x = \pm 3$ and $x = \pm 4$ respectively.

$$f''(3) = 3 \times 3 \left(4 \times 3^2 - 50\right) = 9 \times (-14) < 0$$

$$f''(-3) = 3 \times (-3) \left(4 \times (-3)^2 - 50\right) = -9 \times (-14) > 0$$

$$f''(4) = 3 \times 4 \left(4 \times 4^2 - 50\right) = 12 \times 14 > 0$$

$$f''(-4) = 3 \times (-4) \left(4 \times (-4)^2 - 50\right) = -12 \times 14 < 0$$

So $x = 3$ and $x = -4$ are local maximum points; $x = -3$ and $x = 4$ are local minimum points.

Set the second derivative, equation (22), to be zero.

$$\begin{aligned} f''(x) = 3x(4x^2 - 50) &= 0 \\ \Rightarrow \quad x = 0 \quad \text{or} \quad & x = \pm\sqrt{\tfrac{50}{4}} = \pm\tfrac{5\sqrt{2}}{2} \end{aligned}$$

Since $f^{(3)}(x) = 36x^2 - 150$ equals to zero only when $x^2 = 150/36 = 25/6$, i.e., $x = \pm\frac{5\sqrt{6}}{6}$, $x = 0$ and $x = \pm\frac{5\sqrt{2}}{2}$ are inflection points.

Problem 8. Do the following problems from the book.

a. Section 9.4
   1) 3 (Example 3 is useful to study)
   2) 5

b. Section 8.5
   1) 3
   2) 5a

c. Section 8.6
   Problem 1

d. Section 8.7
   You will need to use Theorem 8.7.1 for these problems.
   1) 1a
   2) 1b
   3) 3a
   4) 3b
   5) 3c