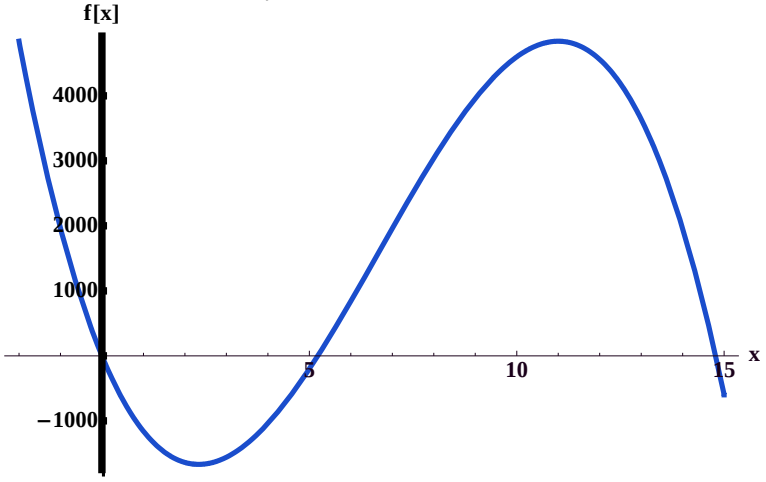
ECONOMICS 207
SPRING 2008
PROBLEM SET 9

Problem 1. For each of the following problems, find the critical points. For each critical point state whether the function is at a relative maximum, relative minimum, or otherwise. Also find the points of inflection for each function.

a. $f(x) = -20x^3 + 400x^2 - 1540x$
Hint: The inflection point is $x = \frac{20}{3}$

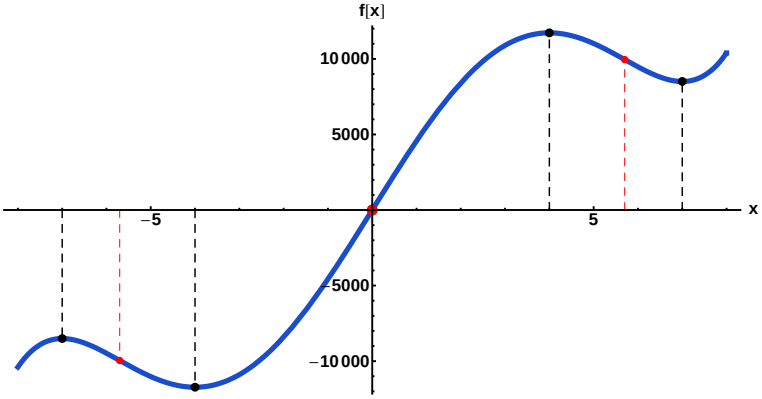
FIGURE 1. $f(x) = -20x^3 + 400x^2 - 1540x$



Date: March 12, 2008.

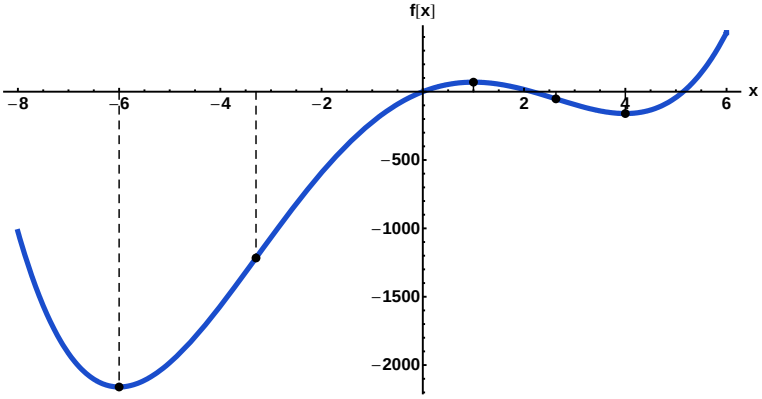
b. $f(x) = \frac{6}{5}x^5 - 130x^3 + 4704x$
Hint: The inflection points are $x = 0, \pm\frac{\sqrt{65}}{2}$

FIGURE 2. $f(x) = \frac{6}{5}x^5 - 130x^3 + 4704x$



c. $f(x) = \frac{3}{2}x^4 + 2x^3 - 78x^2 + 144x$
Hint: The inflection points are $x = \frac{-1 \pm \sqrt{79}}{3}$

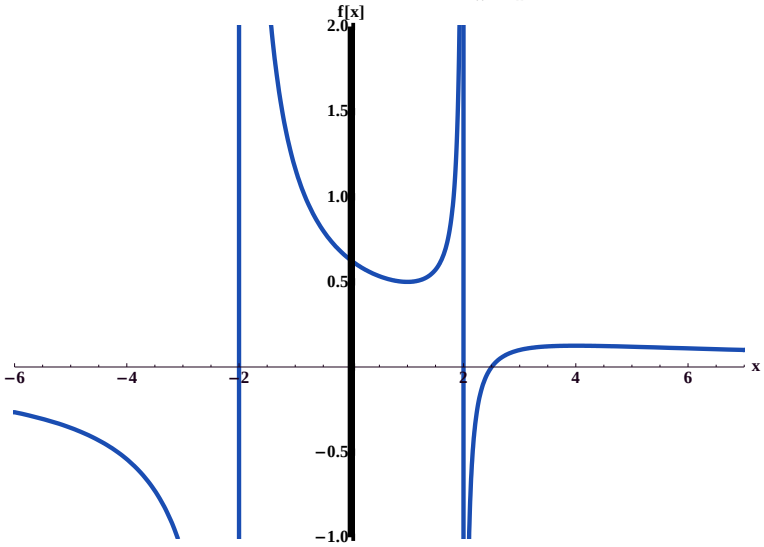
FIGURE 3. $f(x) = \frac{3}{2}x^4 + 2x^3 - 78x^2 + 144x$



d. $f(x) = \frac{x-5/2}{x^2-4}$

Hint: The second derivative evaluated at the x = 1 is $\frac{1}{3}$. The second derivative evaluated at the x = 4 is $\frac{-1}{48}$. You need not find the inflection points.

FIGURE 4. $f(x) = \frac{x-5/2}{x^2-4}$



Problem 2. a. Use elementary row operations to solve the following system of equations. The answers are $x_1 = -1$, $x_2 = 4$, $x_3 = 5$.

$$Ax = b$$

$$\begin{pmatrix} 1 & -1 & 2 \\ -4 & 5 & -2 \\ 3 & -4 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 14 \\ -24 \end{pmatrix}$$

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More Space for Part 2a

b. Use elementary row operations to solve the following system of equations. The answers are x_1 = -2, x_2 = 1, x_3 = 4.

$$Fx = c$$

$$\begin{pmatrix} 2 & -1 & 2 \\ -3 & 1 & -2 \\ 4 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -11 \end{pmatrix}$$

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More Space for Part 2b

c. What is the following product?

$$\begin{pmatrix} 13 & 9 & 8 \\ 10 & 7 & 6 \\ -1 & -1 & -1 \end{pmatrix} \quad \begin{pmatrix} 5 \\ 14 \\ -24 \end{pmatrix}$$

d. What is the following product?

$$\begin{pmatrix} -1 & -1 & 0 \\ 11 & 10 & 2 \\ 7 & 6 & 1 \end{pmatrix} \quad \begin{pmatrix} 3 \\ -1 \\ -11 \end{pmatrix}$$

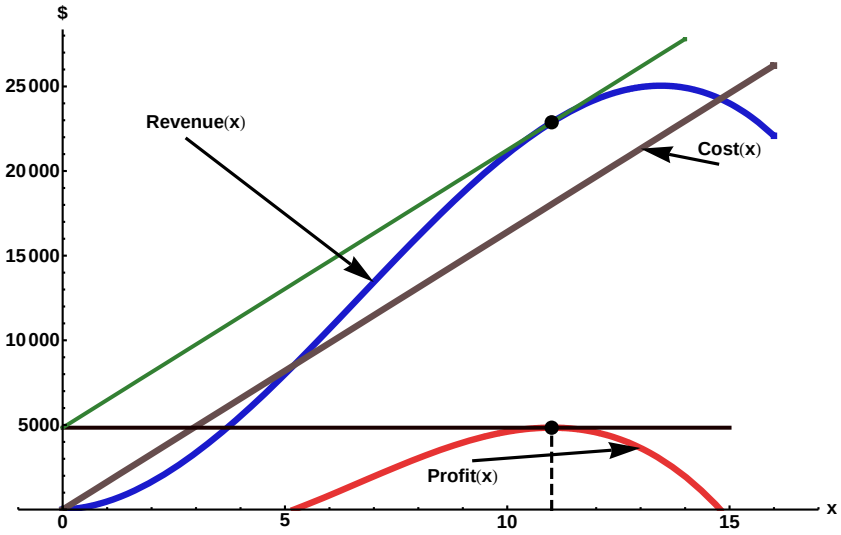
Problem 3. In the following problem you are given a production function for a firm where y is the level of output and x is the level of the variable input. You are given the price (p) of the output and the price (w) of the single variable input.

$$output\ price = p = 10$$

$$input\ price = w = 1640$$

$$y = output = f(x) = 10x + 40x^2 - 2x^3$$

FIGURE 5. Revenue, Cost and Profit



a. Write down an equation that represents profit for the firm.

b. Maximize this function by taking its derivative with respect to the variable input x and setting the resulting equation equal to zero.

c. If you identify more than one critical value from setting the first derivative of profit equal to zero, show which ones, if any, maximize profit.

d. What is the optimal level of input for this for this firm?

e. What is the optimal level of output for this for this firm?

f. Explain in words why the value of the marginal product for this firm is equal to the price of the single variable input at the profit maximizing level of input use. You can use the following information in explaining this phenomenon.

$$\begin{aligned} Output &= y = f(x) \\ MP &= \text{Marginal Product} = \frac{df(x)}{dx} = f'(x) = \frac{\Delta y}{\Delta x} \\ Revenue &= pf(x) \\ Cost &= wx \\ Profit &= Revenue - Cost = pf(x) - wx \end{aligned}$$