Key Answers

Exercise 1 [30]: \( f(L, K) = KL^{\frac{1}{3}} \), \( r = 10 \) \( w = 16 \). \( K = 4 \). (Short Run)

1. What is the quantity of labor that minimizes the cost of producing any given input? \( q = KL^{\frac{1}{3}} \) thus \( L^*(q) = \frac{1}{4}q^3 \).

2. What is the minimum cost of producing \( q \) units of output? \( c = wL + rK \), thus \( c(q) = 16L^*(q) + 40 = \frac{1}{4}q^3 + 40 \)

3. What are the marginal cost of production and the average total cost, average variable cost and the average fixed cost?
   \( MC = \frac{\partial c}{\partial q} = \frac{3}{4}q^2 \), \( AC(q) = \frac{1}{4}q^2 + \frac{40}{q} \), \( AVC(q) = \frac{1}{4}q^2 \), \( AFC(q) = \frac{40}{q} \).

4. Derive the firm’s short run supply curve. \( q \) such that \( p = MC(q) \) for \( p \geq \min AVC(q) \) Thus, \( p = MC(q) \) gives \( p = \frac{3}{4}q^2 \) (inverse supply function), and \( q = \frac{2\sqrt{3}}{p} \) is the supply function. \( \min AVC(q) = 0 \) thus supply curve is \( q = \frac{2\sqrt{3}}{p} \) for \( p \geq 0 \).

Exercise 2 [40]: \( f(K, L) = K + L \), \( r = 2 \) and \( w = 4 \).

1. Derive the marginal product and the average product for labor. \( MP_L = \frac{\partial TP}{\partial L} \) where \( TP = K + L \), thus \( MP_L = 1 \). \( AP = \frac{TP}{L} = \frac{K+L}{L} \).

2. Derive the marginal product and the average product for capital. \( MP_K = \frac{\partial TP}{\partial K} = 1 \), \( AP = \frac{TP}{K} = \frac{K+L}{K} \).

3. Long Run analysis
   (a) What combination of labor and capital minimizes the cost of producing any given output? Because perfect substitute technology and \( w > r \), \( K = q \) and \( L = 0 \).
   (b) What is the minimum cost of producing \( q \) units of output? \( c(q) = 2q \)
   (c) What are the marginal cost of production and the average cost? \( MC(q) = 2 \) and \( AC(q) = 2 \)
   (d) Derive the firm’s long run supply curve. Supply is such that \( p = MC(q) \) Thus \( p = 2 \) is the inverse supply function.

Exercise 3 [30]: The domestic supply \( P = 10 + Q \) and domestic demand \( P = 100 - 2Q \). The world supply is \( P = 10 \).

1. In absence of government policy, the U.S. supply is the world supply. What is the consumers’ surplus at the equilibrium price? What is the producer surplus? \( S = D \) where supply is the world supply. Thus \( Q^*_S = 45 \) and \( P^*_S = 10 \). \( CS = \frac{(100-10)45}{2} = 2025 \) and \( PS = 0 \).

2. The only relevant supply is now the domestic supply. What is the consumers’ surplus? What is the producers’ surplus? New equilibrium price and quantity: \( Q^* = 30 \) and \( P^* = 40 \). Thus \( CS = \frac{(100-40)30}{2} = 900 \) and \( PS = \frac{(40-10)30}{2} = 450 \).

3. What is the effect of this policy on the total welfare? what is the deadweight loss? Welfare before is \( W_{before} = 2025 \) and welfare after is \( W_{after} = 900 + 450 = 1350 \). Thus the \( DWL = 2025 - 1350 = 675 \).