Intermediate Microeconomics 301

Final Exam: KEY ANSWERS
May, 6 2002

Time: 2 hours

Instructions. To obtain credit, you must give arguments to support your answer. The numbers in brackets at the start of each question are the numbers of points the questions are worth.

Exercise 1 [15]: Program of Caroll is
\[
\begin{align*}
\max_{X,Y} & \quad X^2Y^2 \\
\text{s.t.} & \quad 5X + Y = 30
\end{align*}
\]
\[
\Rightarrow \max_X (30 - 5X)^2
\]
\[
\text{FOC} : \quad 30 - 15X = 0 \\
\Rightarrow \quad X^* = 2
\]
\[
\text{and } Y^* = 20
\]

+ graph: Equation of Indifference curve is \( Y = (\frac{X}{5X})^{1/2} \) and equation of budget constraint is \( Y = 30 - 5X \).

Exercise 2 [20]: \( f(K, L) = K^{0.5}L^{0.5} \), \( p_K = $2 \) and \( p_L = $4 \).

1. 
\[
\begin{align*}
\min_{K,L} & \quad 2K + 4L \\
\text{s.t.} & \quad q = K^{0.5}L^{0.5}
\end{align*}
\]
\[
\Rightarrow \min_L \left( \frac{2q^2}{L^2} + 4 \right) \\
\Rightarrow \quad L^* = \frac{1}{\sqrt{2}}q \\
\text{and } K^* = \sqrt{2}q
\]

2. \( C(q) = 2K^* + 4L^* = 4\sqrt{2}q \)

3. \( MC(q) = 4\sqrt{2} = AC(q) \)

Exercise 3 [20]: Demand \( Q_D = 200 - 4p \), and supply is \( Q_S = 50 + 2p \).

1. Elasticity of the demand: \( \varepsilon = \frac{\partial q}{\partial p} |_{p} = -4 \) and \( \varepsilon = -4 \frac{p}{200 - 4p} \).

2. \( D=S, \quad 200 - 4p = 50 + 2p \Rightarrow p^* = 25 \) and \( Q^* = 100. \) + graph

3. New demand curve to \( Q_D' = 150 - 4p \), thus \( S+D', \quad 150 - 4p = 50 + 2p \Rightarrow p^{**} = \frac{50}{3} = 16.667 \) and \( Q^* = 150 - \frac{4 \times 50}{3} = 83.333 \).

Exercise 4 [15]:

<table>
<thead>
<tr>
<th>( \frac{1}{2} )</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>3,1</td>
<td>7,0</td>
</tr>
<tr>
<td>M</td>
<td>2,4</td>
<td>5,3</td>
</tr>
</tbody>
</table>
There is only 1 Nash equilibrium (L,L), and the payoff associated is (3,1). It is a dominant strategy for each player. With collusion they can choose (M,M) but this would be the outcome only if the game is repeated an infinite number of times.

**Exercise 5 [30]:** 2 firms, the demand curve for bottled water is

\[ P = 100 - 2Q \]

\[ Q = q_1 + q_2 \]

is the total amount of bottled waters produced, the same marginal cost of 20 and fixed costs of zero.

1. Cournot. The profit of each firm \( i = 1, 2 \) is \( \Pi_i = (100 - 2(q_i + q_j) - 20)q_i \) where \( j \neq i \). Thus firm 1 solves

\[
\begin{align*}
\text{Max}_q (100 - 2(q_1 + q_2) - 20)q_1 \\
\text{FOC} : & \quad 100 - 4q_1 - 2q_2 - 20 = 0 \\
q_1 &= 20 - \frac{1}{2}q_2 = R_1(q_2)
\end{align*}
\]

where \( R_1(q_2) \) is the best response function of firm 1 to quantity of 2. Symmetric for firm 2, thus \( q_2 = 20 - \frac{1}{2}q_1 = R_2(q_1) \). Thus we have a system of 2 equations to solve

\[
\begin{align*}
q_1 &= 20 - \frac{1}{2}q_2 \\
q_2 &= 20 - \frac{1}{2}q_1
\end{align*}
\]

Replace one equation in the other gives \( q_1 = 20 - \frac{1}{2}(20 - \frac{1}{2}q_1) \), and thus \( q_1^* = \frac{40}{3} = 13.333 = q_2^* \). The price is \( p^* = 100 - 2(q_1^* + q_2^*) = 46.667 \) and the profit of each firm is \( \Pi_i^* = (46.667 - 20)13.333 = 355.55 \) for \( i = 1, 2 \).

2. Stackelberg model.

- First firm 1 chooses its quantity by maximizing its profit \( \text{Max}_q (100 - 2(q_1 + q_2) - 20)q_1 \)
- Second, firm 2 observe \( q_1 \) and chooses \( q_2 \) that maximizes its profit.
- By backward induction, we start by the end, and solve for firm 2 first for any given \( q_1 \)

\[
\begin{align*}
\text{Max}_q (100 - 2(q_1 + q_2) - 20)q_2 \\
\text{Max}_q (100 - 2(q_1 + R_2(q_1)) - 20)q_1 \\
\text{Max}_q (100 - 2(q_1 + 20 - \frac{1}{2}q_1) - 20)q_1 \\
\text{Max}_q (40 - q_1)q_1
\end{align*}
\]

Thus the best response is \( q_2 = 20 - \frac{1}{2}q_1 = R_2(q_1) \). That we plug in the program of firm 1

\[
\begin{align*}
\text{Max}_q (100 - 2(q_1 + R_2(q_1)) - 20)q_1 \\
\text{Max}_q (100 - 2(q_1 + 20 - \frac{1}{2}q_1) - 20)q_1 \\
\text{Max}_q (40 - q_1)q_1
\end{align*}
\]

The solution of this program is \( 40 - 2q_1 = 0 \), and thus \( q_1^* = 20 > 13.333 \). Thus, we can determine \( q_2^* = 20 - \frac{1}{2}q_1^* = 10 < 13.333 \). The price is \( p^* = 100 - 2(q_1^* + q_2^*) = 40 < 46.667 \). And the profit of each firm is now \( \Pi_1^* = (p^* - 20)q_1^* = (40 - 20)20 = 400 > 355.55 \) and \( \Pi_2^* = (p^* - 20)q_2^* = (40 - 20)10 = 200 < 355.55 \).
3. Thus, $\Pi_1^s > \Pi_1^c$, $\Pi_2^s < \Pi_2^c$, $q_1^s > q_1^c$ and $q_2^s < q_2^c$. There is a first-mover advantage.

4. Monopoly’s program is

$$\begin{align*}
\text{Max} & \quad (100 - 2q - 20)q \\
\text{FOC} & \quad 100 - 4q - 20 = 0 \\
q^m & = 20
\end{align*}$$

and $p^m = 100 - 2q^m = 60$ and profit is $\Pi^m = (p^m - 20)q^m = (60 - 20)20 = 800.0$. 