Intermediate Microeconomics 301

Final Exam

May, 6 2002

Time: 2 hours

Instructions. To obtain credit, you must give arguments to support your answer. The numbers in brackets at the start of each question are the numbers of points the questions are worth.

Exercise 1 [15]: Consumer Theory
Caroll’s utility function is
\[ U = 5XY^2 \]
The price of \(X\) is \(p_X = \$5\) and the price of \(Y\) is \(p_Y = \$1\), and her income is \(m = \$30\). What is her optimal consumption bundle? Show in a graph.

Exercise 2 [20]: Producer Theory
Suppose a production function is given by \(f(K, L) = K^{0.5}L^{0.5}\), and that the price of capital is \(\$2\) and the price of labor is \(\$4\). (Long Run analysis)

1. What combination of labor and capital minimizes the cost of producing any given output?
2. What is the minimum cost of producing \(q\) units of output?
3. What are the marginal cost of production and the average cost?

Exercise 3 [20]: Demand and Supply
Consider that demand for Barbie dolls is described by equation \(Q_D = 200 - 4p\), and supply is \(Q_S = 50 + 2p\).

1. What is the elasticity of the demand?
2. Find the equilibrium price and quantity (numerically and graphically). At the equilibrium, is the demand elastic or inelastic?
3. What is the new equilibrium (price,quantity) if a decrease in consumer income shifted the demand curve to \(Q'_D = 150 - 4p\)?

Exercise 4 [15]: Nash Equilibrium
In the following game, players must move simultaneously. How many Nash equilibria are there? Which will occur without collusion? Which will occur if collusion is allowed?

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<tr>
<td>L</td>
<td>3,1</td>
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<td>M</td>
<td>2,4</td>
<td>5,3</td>
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Exercise 5 [30]: Oligopoly

Two firms, Sweetwater and Goodwater produce (4 gallons) bottled water. The demand curve for bottled water is

\[ P = 100 - 2Q \]

where \( P \) is the price (in dollars) of a bottled water, \( Q = q_1 + q_2 \) is the total amount of bottled waters produced, \( q_1 \) is the number of bottled waters produces and sold per month by Sweetwater, and \( q_2 \) is the number of bottled waters produced and sold per month by Goodwater. Each firm has the same marginal cost of 20 and fixed costs of zero.

1. If each of these two firms sets its own output level to maximize its profits, assuming that the other firm holds constant its output level, what will be the output for each firm? (hint: determine the Cournot Equilibrium). What will be the equilibrium price? What will be the profit for each firm?

2. Suppose now a Stackelberg model in which Sweetwater decides first its quantity, and then, Goodwater observes the quantity supplied by Sweetwater, and chooses its own quantity. Determine the quantity that each firm will supply. At what price? What will be the profit for each firm?

3. Compare and contrast the output levels and profits for the Cournot and Stackelberg models.

4. Suppose now that only one firm serves this market (Sweetwater). What is the quantity supplied by Sweetwater? What is the price charged to consumers? What is the profit of Sweetwater?