1. Consider a monopolist that serves 2 groups of consumers. The marginal cost of production is 1.
   a. Group 1 has an inverse demand function \( p_1 = 10 - q_1 \) thus
      \[
      MR_1 = MC \\
      10 - 2q_1 = 1 \\
      q_1^m = 4.5 \\
      p_1^m = 5.5
      \]
   Group 2 has a demand \( p_2 = 5 - \frac{1}{2} q_2 \)
      \[
      MR_2 = MC \\
      5 - q_2 = 1 \\
      q_2^m = 4 \\
      p_2^m = 3
      \]
   b. Consumers’ surplus, producer’ surplus, total welfare.
      \[
      CS_1 = \frac{10 - 5.5}{2} \times 4.5 = 10.125 \\
      PS_1 = (5.5 - 1) \times 4.5 = 20.25 \\
      W_1 = 10.125 + 20.25 = 30.375 \\
      CS_1 = \frac{5 - 3}{2} \times 4 = 4 \\
      PS_2 = (3 - 1) \times 4 = 8 \\
      W_2 = 4 + 8 = 12
      \]
   c. No price discrimination, aggregate demand is
      \[
      Q = \begin{cases} 
      20 - 3p & \text{if } 0 < p < 5 \\
      10 - p & \text{if } 5 \leq p < 10 
      \end{cases}
      \]
      and
      \[
      MR = \frac{20}{3} - \frac{2}{3}Q = 1 \\
      Q^m = \frac{17}{2} = 8.5 \\
      p^m = \frac{20}{3} - \frac{117}{3} = 3.8333
      \]
And

\[ CS = \frac{10 - 5}{2} \times 5 + (5 - 3.8) \times 5 + \frac{5 - 3.8}{2} \times (8.5 - 5) = 20.6 \]
\[ PS = (3.8 - 1) \times 8.5 = 23.8 \]
\[ W = 20.6 + 23.8 = 44.4 > W_1 + W_2 = 30.375 + 12 = 42.375 \]

2. The standard monopoly solution is \( Q^* = 40 \), \( p^* = $200 \), which generates profits of $8,000.

With perfect price discrimination, the monopolist sets \( MC = p \) to determine the best output level. Costs are $4,808. Revenue is the entire area under the demand curve from \( Q = 0 \) to 48, or $10,368 (area abcd in Figure 1). Profits are $5,560, and consumer surplus is zero.

\[ 4Q = 240 - Q \]
\[ Q^* = 48 \]
\[ R = (192 \times 48) + (24 \times 48) = 10,368 \]

![Figure 1](image)

3. At the original price of $5 per rental, Tuan rents 6 videos per month. His total expenditure per month is $30. Firm cost for these rentals is $12, leaving profits of $18 per month. Under the new pricing strategy, Tuan will continue to purchase as long as the flat fee is less than or equal to his consumer surplus. Figure 2 shows that, at a price of $2, consumer surplus is equal to \( \frac{6 \times 12}{2} = $36 \). Thus the firm can charge a monthly fee or $36, plus a $2 fee per movie, and earn profits of $36, or twice the profits of the $5 price with no fee.

Set $MC = MR$ and solve.

\[ MR = 5Q^{1/2} \]
\[ MC = 5. \]
\[ Q^* = 1 \]
\[ p^* = 10 \]
\[ \pi = 10 - 5 = 5. \]

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The total profit is total revenue minus total cost and tax.

\[ \pi = TR - TC = P(Q)Q - \tau Q - C(Q) \]
\[ d\pi / dQ = P + (dp/dQ)Q - \tau - dc / dQ = 0 \]
\[ P + (dp / dQ)Q = \tau + dc / dQ \]

Set marginal revenue in each market equal to marginal cost to determine the quantities. Plug the quantities into the demand functions to determine prices.

\[ MR_1 = 100 - 2Q_1 = 30 = MC \]
\[ MR_2 = 120 - 4Q_2 = 30 = MC \]

\[ Q_1 = 35; p_1 = 65 \]
\[ Q_2 = 22.5; p_2 = 75 \]