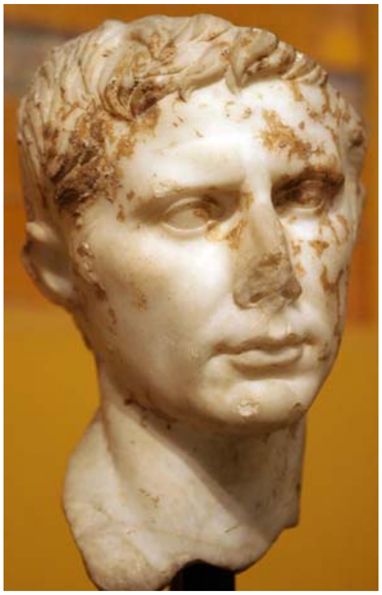
Chapter 16. Part B
Insurance

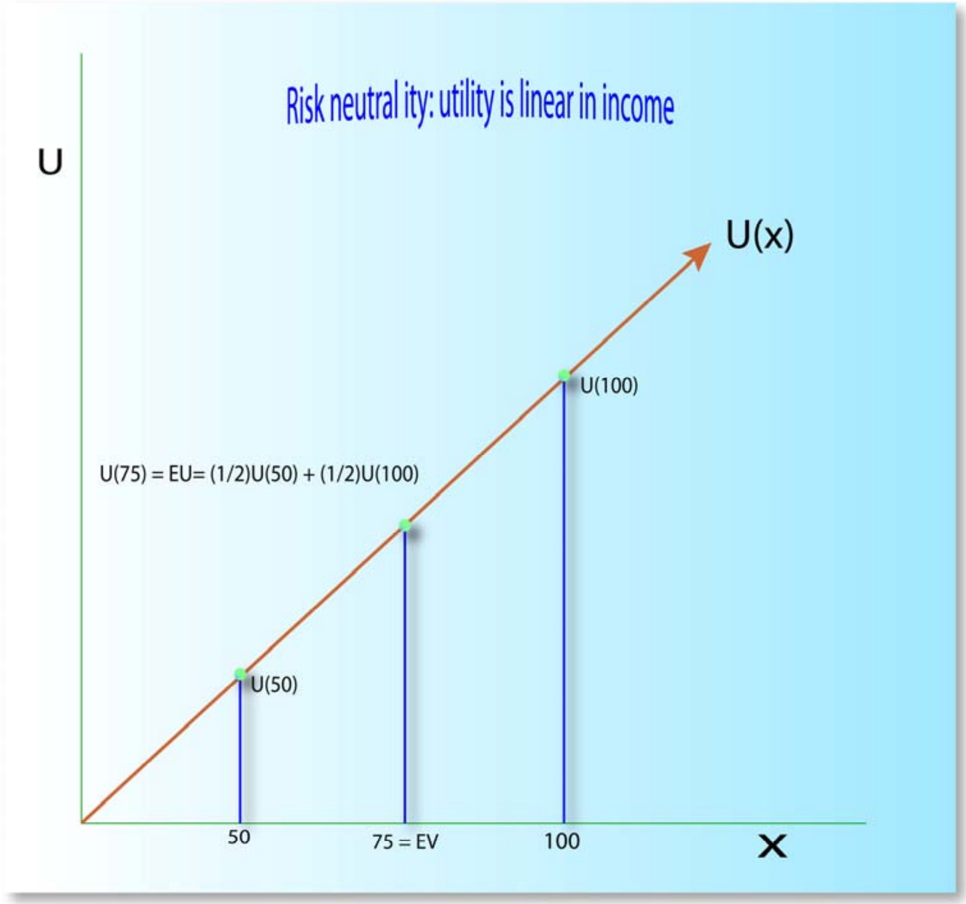
Why do people buy insurance? National health insurance is a hotly debated topic in this country.  August Caesar	Insurance is needed when consumers face a special lottery. In real life, the consumers rarely faces an even chance of winning two different amounts. Instead of a fair gamble, an individual faces a small chance of a disaster. For instance, Job 3:25 states: What I feared has come upon me; what I dreaded has happened to me. We often attribute it to "Acts of God." As a preventive measure, August Caesar first organized fire fighting force, which consisted of about 600 slaves, in 23 B.C. But insurance scheme was not invented until after the great London fire of 1666.
Can we represent such disasters by a lottery?	Sure. There are only two possibilities or events. Health or fire insurance can be represented by $$EU = (1-p)U[w] + pU[w-L], \quad (1)$$ where w is the current income and L is the amount of loss arising from illness, accidents, or fire, and p is the probability of loss (accidents or illness).
Often consumers seem to lose everything when a disaster hits. For instance, when a typhoon whips one's neighborhood or a fire engulfs one's house, often the consumer's wealth is wiped out, i.e., $L = w$.	If an individual faces a risk of losing everything (L = w), expected utility is $$EU = (1-p)U[w] + pU[0],$$ And the expected value of the risk is

	$$EZ = (1-p)w.$$ Expected loss is: *pw*. If the consumer is risk averse, his risk premium will be greater than the expected loss, $$\pi > pL = pw.$$
If an insurance firm is risk neutral, it can probably make money.	Right. If this risk premium consumers are willing to pay is less than the operating cost of an insurer (insurance firm), i.e., $$\pi - pL > \text{Operating cost.} \quad (2)$$ then there exists an opening for a successful insurance company. This is the origin of all insurance firms. Insurance premiums are often several times the expected value of losses from disasters, and explains why insurance firms have built a financial empire. For example, If 1000 houses were insured, insurance premiums from 100-200 houses may cover the expected losses and the remainder will be profits. Often these insurance premiums are invested in stocks and sometimes they lose and become bankrupt.

RISK NEUTRALITY

If an individual is risk neutral, his utility is linear in income.

Risk Neutrality
Utility is linear in income. Risk premium is zero!



If people are risk neutral, their behavior is inconsistent with the fact that they are willing to pay less than $20 for the lottery in the St. Petersburg Paradox.

Why don't people pay more for the lottery ticket to play the game, when the expected value is infinite?

Daniel Bernouilli resolved this paradox by suggesting that individuals are risk averse and value the game at less than the expected value. Therefore, they need a risk premium to be induced to play.

Risk premium is the amount an individual is willing to pay to avoid a fair risk.

Concave Function

A function U(X) is concave if $U'' < 0$, i.e., if its second derivative is negative. A concave function defined on a finite space has a maximum.

A function U(X) is convex if $U'' > 0$, i.e., if its second derivative is positive. A convex function has a minimum.

Maximum First order (necessary) condition: $U' = 0$. Second order (necessary) condition: $U'' < 0$.	Minimum First order condition: $U' = 0$. Second order condition: $U'' > 0$.
To find a maximum, find the location of X where the first derivative is zero. If the second derivative is negative at that point, a maximum is found.	Again, to find a minimum, find the place where the first derivative is zero. That spot is a candidate for a minimum. If the second derivative is negative at that point, a maximum is found.
Global maximum If the objective function is concave everywhere, there is a global maximum. Local maximum For a local maximum, the second derivative must be negative at the point where the first derivative is zero.	Global minimum If the objective function is convex everywhere, there is a global minimum. Local minimum For a local minimum, the second derivative must be positive at the point where the first derivative is zero. If the first derivative is never zero, there is no maximum or minimum.

Jensen's inequality	Risk Premium
If $U(\bullet)$ is concave, $EU(X) < U(EX).$	Risk premium π is defined by: $EU(X) = U(EX - \pi).$ π is positive (negative) if the individual is risk averse (loving).

Risk Aversion

In Figure 2, income is measured on the horizontal axis. Expected value of the gamble is the midpoint between 50 and 100, which is 75.

Utility is measured on the vertical axis. Expected utility is also the midpoint between U(50) and U(100), which is NOT U(75). Expected utility is much less than utility of receiving $75 for sure, U(75). That expected utility could also be guaranteed by receiving some amount with certainty, and that amount is called Certainty Equivalent of the gamble.

The difference between EV and CE is the risk premium, as defined earlier.

Risk Preference

If an individual has a convex utility function, CE will be greater than Expected Value. Hence, his risk premium is negative.

Derivatives of Polynomial functions	Example
$Y = f(X) = AX^n.$ **First derivative** First derivative is often denoted by prime ('). $\frac{dY}{dX} = Y' = f' = nAX^{n-1}.$	$Y = \frac{1}{X} = X^{-1}.$ $Y' = -X^{-2}.$ $Y'' = 2X^{-3}.$ In this example, one cannot set Y ' = 0.

Second Derivative Second derivative is often denoted by double prime ("). $\frac{d^2Y}{dX^2} = Y'' = f'' = n(n-1)AX^{n-2}$. (5)	True, $Y' = -\frac{1}{X^2}$ approaches zero as X approaches infinity ($X \to \infty$)! However, at no point Y ' = 0. This means that the function Y does not have a minimum.

Derivative of a Product $Y = f(X)g(X)$. (6) $Y' = f'g + fg'$. (7)	
Derivative of a Ratio $Y = \frac{f(X)}{g(X)}$. (8) $Y' = \frac{f'g - fg'}{g^2}$. (9)	A special case (when f = 1): $Y = \frac{1}{g(X)}$. Then $Y' = -\frac{g'}{g^2}$.
Partial Derivatives Some objective functions have two or more variables. A partial derivative with respect to X is obtained by applying the above rules, while treating Y or any other variable as constant.	