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Econ 301
Fall 2009

Intermediate Microeconomics
Second Midterm
Sample Test

Your

Name____

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Directions

1. This is a 50-minute exam.

2. Answer all questions in the space provided.

3. Next to each section is the number of points allocated to that section.

   Total number of points on this exam is 30 points

GOOD LUCK!

1. Expected utility of a portfolio with two risky assets can be written as:

$$EU = ER - k\alpha^2\sigma^2 = \alpha\bar{X} + \beta\bar{Y} - k\left(\alpha^2\sigma_x^2 + \beta^2\sigma_y^2\right), \qquad (1)$$

where k (>1) is a risk aversion measure, σ_i^2 is the variance of the risky asset i, and α and β are the fractions of the portfolio held in the risky assets, X and Y, respectively. Thus, $\alpha + \beta = 1$.

A. Express the expected utility maximization problem with the above constraint as a Lagrangian function.

$$\mathcal{L} = \alpha\bar{X} + \beta\bar{Y} - k\left(\alpha^2\sigma_x^2 + \beta^2\sigma_y^2\right) + \lambda[1 - \alpha - \beta]. \qquad (2)$$

B. Derive the first order conditions.

$$\bar{X} - 2k\alpha\sigma_x^2 - \lambda = 0, \qquad (3)$$

$$\bar{Y} - 2k\beta\sigma_y^2 - \lambda = 0, \qquad (4)$$

$$1 - \alpha - \beta = 0. \qquad (5)$$

C. Find the optimal value of α.

From (3) and (4), we get

$$\alpha^* = \frac{2k\sigma_y^2 + \bar{X} - \bar{Y}}{2k(\sigma_x^2 + \sigma_y^2)}. \qquad (6)$$

D. Suppose the variance of the two risky assets are the same, i.e., $\sigma_x^2 = \sigma_y^2 = \sigma^2 = 1.$ and that expected returns on X and Y are 300% and 200% respectively, i.e., $\bar{X} = 3, \bar{Y} = 2.$ In order to maximize expected utility, would you invest everything in *X* because its expected return is higher (3 > 2)?

No, because

$$\alpha^* = \frac{2k + 3 - 2}{2k(2)} = \frac{2k + 1}{4k} < 1.$$

E. Suppose expected returns are the same, $\bar{X} = \bar{Y} = 2,$ but X is twice as risky as Y ($\sigma_x^2 = 2, \sigma_y^2 = 1$). Would you eliminate the risky asset X from your portfolio altogether?

No, because the optimal portfolio reduces to:

$$\alpha^* = \frac{2k}{2k(2+1)} > 0, \tag{7}$$

2. In Ames there is only one newspaper, the Daily Tribune. The demand for the paper depends on the price and the amount of scandals reported. The demand function for newspapers has a constant elasticity of demand, and is given by $Q(P) = tP^{-3},$ or

$$t = P^3 Q, \tag{8}$$

where Q is the number of issues sold per day, and t is a positive constant.

a. Calculate the Total and Marginal revenue for the Daily Tribune.

Note that in (8), P is a function of Q. Differentiating (8) with respect to Q gives

$$3P^2 Q(dP/dQ) + P^3 = 0.$$

Or,

$$P' \equiv \frac{dP}{dQ} = -\frac{P^3}{3P^2 Q} = -\frac{P}{3Q}. \tag{9}$$

Write the demand function as $Q(P) = tP^{-3}$. Then

$$TR = PQ = PtP^{-3} = tP^{-2}.$$

$$MR = -2 \times tP^{-3}(dp / dQ) = -2Q(dp / dQ) \qquad (10)$$

Substituting (9) into (10) gives

$$MR = \frac{2QP}{3Q} = \frac{2P}{3}.$$

b. Calculate the price elasticity of the demand for the Daily Tribune.

$$PED = -\frac{dQ}{dP}\frac{P}{Q} = \frac{3Q}{P}\frac{P}{Q} = 3.$$

c. Suppose marginal cost is MC = $2. Write down the profit maximization condition for the Daily Tribune in terms of the price elasticity of demand and its marginal revenue. (You only need to write the formula!)

$$MR = \frac{2P}{3} = MC = 2. \qquad (11)$$

d. Solve for the profit-maximizing price for the Tribune to charge per newspaper.

Solving this above equation, we get

$$P = 3$$

e. If the Daily Tribune charges its profit-maximizing price and t = 1, how many copies would it sell?

$$Q = (3)^{-3} = \frac{1}{(3)^3} = \frac{1}{27}.$$

3. Suppose the average cost of a representative competitive firm is given by:

$$AC(q) = 2q^2 - 16q + 90,$$

where, q is the quantity.

Answer the following questions

(a) Write the expressions for Total Cost, Marginal Cost, and Variable Cost.

$$TC = q \times AC = 2q^3 - 16q^2 + 90q.$$

(b) Derive the firm's supply function.

$$MC = 6q^2 - 32q + 90.$$

Since p = MC for a competitive firm, the firm's supply function is:

$$p = 6q^2 - 32q + 90. \qquad (12)$$

(c) Suppose the price in the market is \$80. At this price, is the firm making a profit or incurring a loss? Calculate the amount of profit (or loss) the firm is making at the equilibrium price.

An optimal output satisfies the condition:

$$80 = 6q^2 - 32q + 90.$$

or $3q^2 - 16q + 5 = (q-5)(3q-1) = 0.$ Choose the larger output, q = 5. pq = 400.
$TC = 2q^3 - 16q^2 + 90q = 2 \times 125 - 16 \times 25 + 90 \times 5 = 250 - 400 + 450 = 300.$
Profit is \$100.

(d) If all the firms in the market are the same, what do you expect to happen in the long run in this industry?

Since the representative firm is making profits, the number of firms will increase.