Problem Set 4 Suggested Solution.

1. The utility function is given by \( U(w) = \sqrt{w} \).

(a) The expected wealth of each lottery can be found by averaging the wealth levels associated with each lottery:

\[
E_{wA} = 0.5 \times 1 + 0.5 \times 9 = $5 \\
E_{wB} = 0.5 \times 2 + 0.5 \times 8 = $5 \\
E_{wC} = 0.5 \times 0 + 0.5 \times 11 = $5.5
\]

(b) The expected utility of each lottery is the average of the utilities of each wealth level associated with the lottery:

\[
E_{UA} = 0.5 \times U(1) + 0.5 \times U(9) = 0.5 \times \sqrt{1} + 0.5 \times \sqrt{9} = 0.5 \times 1 + 0.5 \times 3 = 2 \\
E_{UB} = 0.5 \times U(2) + 0.5 \times U(8) = 0.5 \times \sqrt{2} + 0.5 \times \sqrt{8} = 0.5 \times 1.41 + 0.5 \times 2.83 = 2.12 \\
E_{UC} = 0.5 \times U(0) + 0.5 \times U(11) = 0.5 \times \sqrt{0} + 0.5 \times \sqrt{11} = 0.5 \times 0 + 0.5 \times 3.37 = 1.66
\]

(c) The certainty equivalent of a lottery is defined as the level of wealth that would give this individual the same utility as the expected utility of the lottery: \( U(ce) = EU \). In other words, this person would be as well off having the certainty equivalent with certainty as he would be facing the risk in the form of lottery with uncertain level of wealth. Here we have \( ce = EU \), then \( ce = (EU)^2 \). We, therefore, have the following certainty equivalents:

\[
ce_A = 2^2 = $4 \\
ce_B = (2.12)^2 = $4.49 \\
ce_C = (1.66)^2 = $2.76
\]

(d) Risk Premium is defined as a difference between expected wealth and certainty equivalent (risk premium = \( Ew - ce \)):

\[
\text{risk premium}_A = E_{wA} - ce_A = 5 - 4 = $1 \\
\text{risk premium}_B = E_{wB} - ce_B = 5 - 4.49 = $0.51 \\
\text{risk premium}_C = E_{wC} - ce_C = 5.5 - 2.76 = $2.74
\]

The risk premium is the amount of money (wealth) that an individual would be willing to pay in order to avoid facing the risk associated with the lottery. Therefore, you can see that lottery B is the least unpleasant for him to take because he'd only be willing to pay $0.51 to avoid this lottery and have $5 (expected wealth) with certainty.

(e) Recall that according to von Neumann-Morgenstern theory of decision making under uncertainty, people choose the lotteries based on expected utility that they derive from
each lottery. Therefore, this person will choose the lottery B because it gives him the highest expected utility of 2.12. Observe that the lottery A gives exactly the same expected wealth of $5 as lottery B does, but A is riskier than B because the values wealth takes in A are farther from the average (expected wealth). Also, lottery C gives higher expected wealth of $5.5, but it is inferior to both A and B from the standpoint of this risk-averse individual because it involves too much risk (there is 50% chance of getting nothing).

(f) Risk-neutral decision makers judge the lotteries based on their expected wealth $E_w$, therefore, risk-neutral individual would choose lottery C because it offers highest payoff on average. The risk is irrelevant for such a decision maker.

2. Problem 6 (page 600). No, a risk-neutral individual will never buy insurance, which is not fair. Risk-neutral people base their decisions on expected wealth, and unfair insurance has expected wealth lower than zero compared to exactly zero if a person doesn’t buy an insurance policy. Risk-averse people may buy an unfair insurance, as long as the amount that they overpay (over the fair insurance premium) is less than or equal to the risk premium of the specific risky situation.

3. Problem 19 (page 176). We fix the capital $K=2$ in the short run, but we can vary the labor input $L$.
   (a) This production function doesn’t exhibit the diminishing marginal returns in the short run. Recall that the marginal product of labor (MP$_L$) is the slope of the total product curve ($q=f(K,L)$) with respect to labor input (partial derivative with respect to labor input). It is easy to show that the slope of the TPL is constant and is equal to 10 (because the production function is linear with respect to labor). In other words, no matter how much labor we are already using, one more unit of it will always increase the output by exactly 10 units. You can just plug any value of $L$ into the function and then increase $L$ by one. Therefore the marginal returns here are constant, not diminishing.

(b) This function $q=\sqrt{L*K} = \sqrt{K} \cdot \sqrt{L}$ does exhibit the diminishing marginal returns in the short run. Without using calculus, it is easy to check that the output will grow by smaller and smaller amounts as we use more and more labor and keep capital fixed at $K$. For example, if we start off with $L=2$, the output $q=2$, if we increase $L$ to 3, the output will grow to $\sqrt{6} = 2.45$, i.e. the output will increase by 0.45 units. Now suppose that we use $L=8$, the output $q=4$. If we use one more unit of labor ($L=9$), the output will be $\sqrt{18} = 4.24$, which means that output went up by only 0.24<0.45. We can continue and see that higher initial levels of $L$ will produce lower values for MP$_L$.

Using calculus it is easy to see that the $MP_L = \frac{\sqrt{K}}{2\sqrt{L}}$. Then, if we keep capital fixed at $K$ and we increase $L$, we will have MP$_L$ decrease as we use more and more labor. In other words, MP$_L$ is a decreasing function of labor use $L$.

4. 
(a) We showed in class that this function is IRS. Let the factor by which we increase all the inputs be \( \delta > 1 \). Then,

\[
f(\delta K, \delta L) = (\delta K)(\delta L) = \delta^2 K \cdot L = \delta^2 f(K,L) > \delta f(K,L)
\]

because \( \delta > 1 \).

(b) Again, showed in class that this function is CRS

\[
f(\delta K, \delta L) = \sqrt{(\delta K)(\delta L)} = \sqrt{\delta^2 K \cdot L} = \delta \sqrt{K \cdot L} = \delta f(K,L)
\]

(c) This function is also CRS

\[
f(\delta K, \delta L) = \min(\delta K, \delta L) = \delta \min(K,L) = \delta f(K,L)
\]

(d) This function is also CRS

\[
f(\delta K, \delta L) = \delta K + \delta L = \delta(K + L) = \delta f(K,L)
\]

(e) If you were thinking about expanding your production, you would wish you had production in (a), because you would for example be able to increase your output by a factor of 4 while only doubling your inputs. All other productions will just double the output if you double your inputs. This is not to say that CEOs have any real choice of technology, this is determined by the specifics of the industry and should be decided by engineers (at least in this model). But whether your technology is IRS, CRS or DRS may influence your decision whether or not to expand the production.