

Assume

q	=	200L ^{1/2} , where
q	=	number of bottles of pop sold by a concession stand at an athletic event
L	=	number of workers hired for the day for the event
P	=	the price of pop per bottle = \$2.00
w	=	the daily wage or cost per worker = \$40.00
TFC	=	total fixed costs = \$100.00

1. Show mathematically that the amount of L that needs to be hired depends on the desired level of output (q) [i.e., solve for L in the given production function].

$$q = 200L^{1/2} \Rightarrow L^{1/2} = \frac{q}{200} \Rightarrow L = \frac{q^2}{40,000} = .000025q^2$$

2. What is the mathematical equation that shows SR TC is a function of q given the assumptions above?

$$= TFC + TVC = TFC + WL = TFC + W(.000025q^2) \\ = 100 + 40(.000025q^2) = (100 + .001q^2)$$

3. Given your answer to Q. #2, what is the mathematical equation that shows MC as a function of q?

$$= \frac{dTFC}{dq} = (2)(.001)q^{2-1} = .002q$$

4. Given your answer to Q. #2, what is the mathematical equation that shows AVC as a function of q?

$$= \frac{TVC}{q} = \frac{.001q^2}{q} = .001q$$

5. Given your answer to Q. #2, what is the mathematical equation that shows AFC as a function of q?

$$= \frac{TFC}{q} = \frac{100}{q}$$

6. What is the TC of producing $Q = 1000$?

$$\begin{aligned} &= 100 + .001 q^2 = 100 + (.001)(1000)(1000) \\ &= 100 + 1000 = 1100 \end{aligned}$$

7. What is the MC of producing and selling another bottle of pop at $Q = 1000$? (Use equation from Q. #3 to calculate).

$$= .002q = (.002)(1000) = \$2.00$$

8. What is the AVC of producing at $Q = 1000$?

$$= .001q = (.001)(1000) = \$1.00$$

9. What is the value of w/MP_L at $Q = 1000$?

$$\begin{aligned} w &= 40 \\ MP_L &= \frac{\partial q}{\partial L} = \left(\frac{1}{2}\right)(200)q^{\frac{1}{2}-1} = \frac{100}{\sqrt{L}} \\ &\Rightarrow \frac{w}{MP_L} = \frac{40}{100/\sqrt{L}} \\ &= \frac{40}{100/5} = \frac{40}{20} = 2.00 \end{aligned}$$

$\rightarrow 1000 = 200\sqrt{L} \Rightarrow \sqrt{L} = 5$

10. What is the AVC of producing at $Q = 1000$ if w increases to \$50 (from \$40)?

$$\begin{aligned} TVC &= w \cdot L = 50(2000025q^2) \\ &= .00125q^2 \end{aligned}$$

$$\begin{aligned} \Rightarrow AVC &= \frac{TVC}{q} = \frac{.00125q^2}{q} = .00125q \\ &= .00125(1000) \\ &= \$1.25 \end{aligned}$$