Econ 301
Deiter

R In-class #12 (10 pts.)

Name Key

Assume a production process where:

q	=	$10K^{1/2}L^{1/2}$
q	=	units of output
K	=	units of capital (= vertical axis input)
L	=	units of labor (= horizontal axis input)
r	=	the rental rate per unit of K = $90
w	=	the per unit cost of labor = $10

Show work in all of your answers.

1. Show that the 'inverse MP ratio', i.e. (MP of L)/(MP of K) is K/L.

$$\frac{MP_L}{MP_K} = \frac{dq/dL}{dq/dK} = \frac{(\frac{1}{2})(10K^{1/2})L^{1/2-1}}{(\frac{1}{2})(10L^{1/2})K^{1/2-1}} = \frac{5K^{1/2}L^{-1/2}}{5K^{-1/2}L^{1/2}} = \frac{K}{L}$$

2. Calculate the 'inverse input price ratio'.

$$= \frac{w}{r} = \frac{10}{90} = \frac{1}{9}$$

3. Which of the following is the LR cost-minimizing quantity of labor to use (hint: use answers to #1 and #2 above)?

a. L = 2K
b. L = 4K
c. L = 1/9K
d. L = 9K
e. L = 81K

$\Rightarrow$ inverse MP ratio = inverse input P ratio

$\Rightarrow \frac{K}{L} = \frac{1}{9} \Rightarrow K = \frac{1}{9}L \Rightarrow L = 9K$

4. What is the equation of the q = 90 isoquant and how many L would be required to produce q = 90 in the SR if K is fixed at K = 9?

isoquant $\Rightarrow 90 = 10K^{1/2}L^{1/2}$

$\Rightarrow 9 = K^{1/2}L^{1/2}$

$\Rightarrow K^{1/2} = \frac{9}{L^{1/2}} \Rightarrow K = \frac{81}{L}$

$q = 90,\ \bar{K} = 9$

$\Rightarrow 90 = 10(9)^{1/2}L^{1/2}$

$\Rightarrow 90 = 30L^{1/2}$

$\Rightarrow L^{1/2} = 3 \Rightarrow L = 9$

5. What is the LR cost-minimizing K and L associated with producing 90 units of output?

$\Rightarrow 90 = 10K^{1/2}L^{1/2}$ where $L = 9K$

$\Rightarrow 90 = 10K^{1/2}(9K)^{1/2}$

$\Rightarrow 90 = 30K \Rightarrow K^* = 3 \Rightarrow L^* = 9K = 27$

6. What is the minimum LR TC of producing 90 units of output?

$= rK^* + wL^*$

$= 90(3) + 10(27) = 270 + 270 = \$540$

7. Calculate the SR TC of producing q = 90 for the information given in Q. #4.

$$= r\bar{K} + wL$$
$$= 90(9) + 10(9)$$
$$= 810 + 90 = \$900$$

8. If K is fixed in the SR at K = 4, show that SR TC = 360 + .025q².

$\bar{K} = 4 \Rightarrow q = 10(4)^{1/2} L^{1/2}$

$\Rightarrow q = 20L^{1/2}$

$\Rightarrow L^{1/2} = q/20$

$\Rightarrow L = q^2/400 = .0025q^2$

$\Rightarrow$ SR TC $= r\bar{K} + w(L \text{ as fn of } q)$

$= 90(4) + 10(.0025q^2)$

$= 360 + .025q^2$

9. If K is fixed in the SR at K = 4 and MR = 10, what are the values of q to a) maximize profits and b) break even? (Hint: use SR TC info in Q. #8).

a) π max $\Rightarrow$ MR = MC

$\Rightarrow 10 = .05q$

$\Rightarrow q = 200$

b) B.E. $q \Rightarrow$ TC − TR = 0

$\Rightarrow .025q^2 + 360 - 10q = 0 \Rightarrow \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4(.025)}}{2(.025)}$

$= \dfrac{10 \pm 8}{.05} = 40$ and 360

10. If K is fixed in the SR at K = 4, for what level of q is K = 4 also the LR cost-minimizing amount of K to use in the production process? (Hint: use answer to Q. #3).

$K^* = 4 \Rightarrow L^* = 9K^* = 9(4) = 36$

$\Rightarrow q = 10(4)^{1/2}(36)^{1/2}$

$= 10(2)(6)$

$= 120$