

A. Assume a production process (terms defined previously) where:

$$q = K^{1/2}L^{1/2}$$
$$r = \$80$$
$$w = \$20$$
$$MP_L/MP_K = K/L$$
$$\text{LR cost minimizing } K = \frac{1}{4}L$$

1. What is LR optimal L^* as a function of q ? (Hint: plug $\frac{1}{4}L$ into production function for K .)

$$\Rightarrow q = \left(\frac{1}{4}L\right)^{1/2}L^{1/2}$$
$$\Rightarrow q = \frac{1}{2}L \quad \Rightarrow L^* = 2q$$

2. What is LR optimal K^* as a function of q ? (Hint: $K^* = \frac{1}{4}L^*$)

$$\Rightarrow K^* = \frac{1}{4}L^* = \frac{1}{4}(2q) \Rightarrow K^* = \frac{1}{2}q$$

3. What is the LR TC as a function of q ?

$$= rK^* + wL^* = 80\left(\frac{1}{2}q\right) + 20(2q)$$
$$= 40q + 40q = 80q$$

4. How many units of K should be used to produce $q = 200$ in the LR?

plug optimal $K = \frac{1}{4}L$ into isoquant

$$\Rightarrow 200 = \left(\frac{1}{4}L\right)^{1/2}L^{1/2} \Rightarrow 200 = \frac{1}{2}L \Rightarrow L^* = 400$$
$$\Rightarrow K^* = \frac{1}{4}L^* = \frac{1}{4}(400) = 100$$

5. What is minimum LR TC of producing $q = 200$?

$$\text{LR TC} = 80q = 80(200) = 16,000$$

6. Assume the firm is using $K = L = 9$. If it quadruples input usage to $K = L = 36$, what 'returns to scale' are observed?

$$K=9=L \Rightarrow q = (9)^{1/2}(9)^{1/2} = (3)(3) = 9$$

$$K=36=L \Rightarrow q = (36)^{1/2}(36)^{1/2} = (6)(6) = 36$$

\Rightarrow input quadrupled and output quadrupled

\Rightarrow constant returns to scale

Refer to the information below to answer questions #7-#9.

Assume marketing specialists at Quaker Oats are deciding how to allocate monies in the company's Gatorade advertising budget (= \$1 million). Gatorade currently accounts for about 40% of company sales and earnings. Marketing researchers within the company have provided you with the following estimated relationship:

$$Q = 20 + 5B - 1.25 B^2 + 4P - .5 P^2, \text{ where}$$

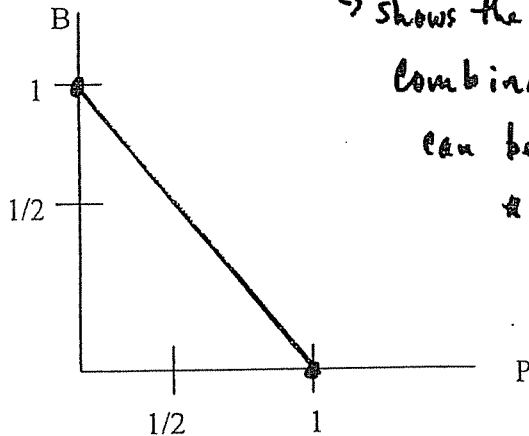
Q = gallons of Gatorade sold (millions of gallons per year)

B = millions of dollars spent on broadcast media (e.g. radio, tv) advertising

P = millions of dollars spent on print media advertising

Assume that each 'unit' of B and P costs \$1 each.

7. Draw in the graph below the company's isocost line for the given advertising budget (label axis intercepts). Explain in words what this isocost line shows.



8. Derive and explain what the slope of this isocost line is.

$$= \frac{\Delta B}{\Delta P} = \frac{-P_P}{P_B} = \frac{-1}{1} \Rightarrow \text{1 unit of B can be exchanged for 1 unit of P holding total cost constant (at $1 mil.)}$$

9. In this case, it can be shown that $MP_B = 5 - 2.5B$, where MP = marginal product. Derive and explain what MP_P means.

$$MP_P = \frac{\partial Q}{\partial P} = (1)(+)P^{1-1} - (2)(.5)P^2 = (4 - P)$$

= additional Q of Gatorade sold for a 1 unit increase in P holding B constant

10. Derive and explain the slope of an isoquant in this case (in general, meaning specific P and B values are not given).

$$= \frac{MP_P}{MP_B} = \frac{4 - P}{5 - 2.5B}$$