

Assume  $q = 200L^{1/2}$ , where  
 $q$  = number of bottles of pop sold by a concession stand at an athletic event  
 $L$  = number of workers hired for the day for the event  
 $P$  = the price of pop per bottle = \$2.00  
 $w$  = the daily wage or cost per worker = \$40.00  
 $TFC$  = total fixed costs = \$100.00

1. Derive the mathematical equations for  $MP_L$  and  $AP_L$  as functions of  $L$ .

$$MP = \frac{dq}{dL} = \frac{1}{2}(200)L^{-1/2} = 100L^{-1/2} \text{ or } 100/L^{1/2} \text{ or } 100/\sqrt{L}$$

$$AP = q/L = 200L^{1/2}/L = 200/L^{1/2} \text{ or } 200/\sqrt{L}$$

2. Use your equations in Q. #1 to provide the missing data in the following table:

L	TP	MP	AP
1	200	100	200
4	400	50	100
9	600	33.33	66.67
16	800	25	50
25	1000	20	40
36	1200	16.67	33.33
49	1400	14.29	28.57
64	1600	12.5	25
81	1800	11.1	22.2
100	2000	10	20

3. What is the mathematical equation for TRP as a function of  $L$ ?

$$= Pq = (2)(200L^{1/2}) = 400L^{1/2} \text{ or } 400\sqrt{L}$$

4. What is the mathematical equation for MRP as a function of  $L$ ?

$$= P \cdot MP = (2)(100L^{-1/2}) = 200L^{-1/2} \text{ or } 200/L^{1/2} \text{ or } 200/\sqrt{L}$$

5. What is the value of marginal factor cost (MFC)? Explain in words what this means.

$$= w = 40$$

= additional cost per additional unit of  $L$

6. What is the optimal (profit-maximizing) number of workers to hire?

$$\Rightarrow MRP = MFC \Rightarrow 200/\sqrt{L} = 40 \Rightarrow \sqrt{L} = 5 \Rightarrow L^* = 25$$

7. What is the maximum attainable level of profit (TRP - total labor costs - TFC)?

$$\Rightarrow \text{at } L^* = 25, \quad \text{TRP} - \text{TVC} - \text{TFC} = 400\sqrt{25} - 40(25) - 100$$

$$= 2000 - 1000 - 100 = \text{\$}900$$

8. How many dollars of profit are given up (versus maximum attainable) if 16 workers are hired?

$$\Rightarrow \text{TRP} - \text{TVC} - \text{TFC} = 400\sqrt{16} - 40(16) - 100$$

$$= 1600 - 640 - 100 = \text{\$}860 \Rightarrow \text{give up } \text{\$}40$$

9. What is the profit-maximizing number of workers to hire per day if P increases to \$2.40/bottle?

$$\Rightarrow \text{MRP} = \text{MFC} \Rightarrow \frac{240}{\sqrt{L}} = 40 \Rightarrow \sqrt{L} = 6 \Rightarrow L^* = 36$$

10. Complete the table below. Then, show graphically below the impact of the pop price increase in Q. #9 in a graph of MRP and MFC curves (label all curves and axes plainly).

<u>L</u>	<u>Initial MRP</u>	<u>New MRP</u>	<u>\$</u>
1	200	240	
4	100	120	
9	66.67	80	
16	50	60	
25	40	48	
36	33.33	40	
49	28.57	34.30	
64	25	30	
81	22.2	26.67	
100	20	24	

