Chapter 3. Applying the Supply-and-Demand Model

*Shape matters*

*How responsive are the consumers?*
*How responsive are the producers?*
*Tax incidence – who is paying for the tax?*

**Tip:** Pictures always help. Start drawing pictures when you don’t know how to start solving the problems.

1. **Shape matters**

How much more will the quantity demanded increase if we reduce the price by 10%? Pictorial analyses are not enough here. We need a quantitative analysis! The information is all on the demand curve, the task is to extract it.

Let’s see some different shapes of the demand curve. The following figures show a steep and a flat demand curves. The flatter the curve is, the more responsive the quantity is to a change in price.

**Figure 3.01a** How the Effect of a Supply Shock Depends on the Shape of the Demand Curve

![Diagram of demand curve](image-url)
Figure 3.01b  How the Effect of a Supply Shock Depends on the Shape of the Demand Curve

Figure 3.01c  How the Effect of a Supply Shock Depends on the Shape of the Demand Curve
2. How responsive are the consumers?
- How to summarize the information we need? Well we can use “Elasticity”
- It is definition as the percentage change in quantity over that in price.

\[ \varepsilon = \frac{(\Delta Q/Q)}{(\Delta P/P)} \]

- Why define it this way?
  1. We knew that slope = \( \Delta P/\Delta Q \), but it changes with the choice of units. We want to know the ratio of their percentage changes.
  2. Usually we control price to influence quantity. So we want to know the response of quantity to price.

- Relate elasticity to slope:

\[ \varepsilon = \frac{(\Delta Q/Q)}{(\Delta P/P)} \]
\[ = \frac{(\Delta Q/\Delta P)(P/Q)}{P/Q} \]

This is just “the inverse slope” times the ratio or price and quantity.

**Example:**
If the demand curve is

\[ Q = a - bP \]

Then the elasticity is

\[ \varepsilon = - b(P/Q) \]

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**Figure 3.02 Elasticity Along the Pork Demand Curve**

![Diagram of Elasticity Along the Pork Demand Curve](image)
• Upper part: $\varepsilon < -1$, elastic
• Middle point: $\varepsilon = -1$, unitary elastic
• Lower part: $\varepsilon > -1$, inelastic

It is easy to see why it is more elastic when you move up the demand curve and it is more inelastic when you move down the curve. (Show two movements along the curve.)

• Why is it equal to $-1$ in the middle?

\[ Q = a - bP \]

Intercepts: $a$ and $a/b$
Slope = $-b$
Middle point is $(a/2, a/2b)$
So, $\varepsilon = -b ((a/2b)/b/2) = -1$.

• Some extreme demand curves: Horizontal, perfectly elastic where good can be perfectly substitute by another: same kind of apples from different states.

Figure 3.03a  Vertical and Horizontal Demand Curves

(a) Perfectly Elastic Demand

$p$, Price per unit

$p^*$

$Q$, Units per time period

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Vertical, perfectly inelastic demand, e.g. essential goods for survival: daily drug versus the price for a surgery or gene therapy.

Applications

Case 1. “Should we have a sale to boost revenue?”
Schnuck’s has bought 1,000lb of strawberries. Note that these are perishable and cannot be resold. The got a fixed stock. The price of strawberries are set at $4.99 /lb. The revenue is steady but can they do better? The manager is now thinking about a 5% off sale. (Yes, managers do think sometimes, especially those who went to business schools.) Can the sale increase their revenue? The only information we have is strawberries’ price elasticity $\varepsilon = -0.8$. But this is enough to help us make a decision.

$\text{TR} = P \cdot Q$

The sale has two effects: gain from a quantity increase and loss from a price drop.

$\Delta \text{TR} = -\Delta P \cdot Q + \Delta Q \cdot P$

The first one is a negative and the second is a positive change to the total revenue. When $-\Delta P \cdot Q + \Delta Q \cdot P > 0$, we should have a sale. This is because the gain form a quantity increase dominates the loss from a price drop. That is
\[ \Delta Q^*P > - \Delta P^*Q \]
\[ \Delta Q/\Delta P < -Q/P \]
\[ (\Delta Q/\Delta P)(P/Q) < -1 \]
\[ \varepsilon < -1 \]

**Conclusion:**
*When* \( \varepsilon < -1 \), a sale can increase the total revenue. (*Gain from quantity increase dominates.*)
*When* \( \varepsilon > -1 \), a sale will decrease the total revenue. (*Loss from price drop dominates.*)

**Case 2. “Sometimes farmers destroy their crop to prevent loss.”**
Why? If they sell the crop, won’t they get more money?
Ans: The reason is that food is a necessity; this means it the demand for food is in elastic. They cannot sell all their crop at a high price. If they sell the remaining at a lower price, they market price will go down and the total revenue goes down. – The effect of a price drop dominates the effect of a quantity increase.

- **Consumers’ response to other factors.**
Other factors affect the quantity demanded as well like we saw in the demand function. Their elasticities are defined in the same way.

- **Income Elasticity (you can use \( \varepsilon_Y \))**
\[ \xi = (\Delta Q/Q)/(\Delta Y/Y) \]
\[ = (\Delta Q/\Delta Y)(Y/Q) \]
Percentage change of quantity demanded over that of income.

**Example:**
\[ Q = 171 - 20P + 20 P_b + 3P_c + 2Y \]
When \( Q = 220 \) and \( Y = 12.5 \), calculate the income elasticity.
\[ \Delta Q = 2\Delta Y \]
\[ \xi = 2 \left( \frac{12.5}{220} \right) = 0.114 \]

**Application:**
“How to predict the demand in ten years?”
Many companies need to tell their stockholders about their business prospect 3 to 5 years into the future. When most companies do long term predictions, the most reliable variable is people’s income. Since other factors are varying all the time, they will take it as fixed. Suppose the economists predict that the economy has a growth rate of 2% per year from 2003 to 2006. We know that our product faces an income elasticity \( \varepsilon_Y = 1.5 \).
How much should we expand our capacity in 2006, 3 year into the future?
\[ \varepsilon_Y = (\Delta Q/Q)/(\Delta Y/Y) = 1.5 \]
\[ \Delta Q/Q /6\% = 1.5 \]
\[ \Delta Q/Q = 9\% \]
• Cross-price elasticity

\[ \varepsilon_C = \frac{\Delta Q}{Q} \frac{\Delta P_0}{P_0} = \frac{\Delta Q}{\Delta P_0}(P_0/Q) \]

When \( \varepsilon_C < 0 \), the other good is a complement of this good.
When \( \varepsilon_C > 0 \), the other good is a substitute of this good.

Example:

\[ Q = 171 - 20P + 20P_b - 3P_c + 2Y \]

For good \( b \), a substitute.

\[ \Delta Q = 20 \Delta P_b \]

\[ \varepsilon_C = \left( \frac{\Delta Q}{\Delta P_b} \right)(P_b/Q) = 20(P_b/Q) \]

When \( Q = 220 \) and \( P_b = 4 \), we have cross-price elasticity = 0.364.

For good \( c \), a complement.

\[ \Delta Q = -3 \Delta P_c \]

\[ \varepsilon_C = \left( \frac{\Delta Q}{\Delta P_c} \right)(P_c/Q) = -3(P_c/Q) \]

When \( Q = 220 \) and \( P_c = 10 \), we have cross-price elasticity = - 0.136.

3. How responsive are the producers?

The information on the supply curve can be summarized concisely in the same way. We define the price elasticity of supply as

\[ \varepsilon_S = \frac{\Delta Q}{Q} \frac{\Delta P}{P} = \frac{\Delta Q}{\Delta P}(P/Q) \]

Note that it has the same formula as the elasticity of demand. But it is calculated along the supply curve. It is the percentage change in quantity supplied over that in price.

Example:

\[ Q = 88 + 40P \]

\[ \Delta Q = 40 \Delta P \]

So, when \( Q = 220 \) and \( P = 3.3 \),

\[ \varepsilon_S = \left( \frac{\Delta Q}{\Delta P} \right)(P/Q) = 40*3.3/220 = 0.6 \]

The same, we call

\( \varepsilon_S < 1 \), inelastic
\( \varepsilon_S = 1 \), unitary elastic
\( \varepsilon_S > 1 \), elastic

Example:

A vertical supply curve, perfectly inelastic: the supply of land.
4. Tax Incidence

Questions
1. What are the effects that a sale tax has on equilibrium price and equilibrium quantity?
2. Who is paying for the entire tax?
3. Do the equilibrium price and quantity depend on whether the tax is assessed on consumers or on producers?

Two types of sales tax
1. Sales tax/ ad valorem tax: a rate on the sales price, i.e., government collect tax for every dollar you spent. (we use $a$)
   Example: a tax rate 0.08 on a 50 purchase is 4. ($\alpha \text{ (alpha)} = 0.08$)
   We call $50 + 50 \times 0.08 = 54$ as the market price. This is different from what we see in the market. In stores, they only mark the price as “what they receive”, but when you actually purchase the good, you are paying the marked price plus the tax. In our example, consumers are paying 54, while producers are receiving 50, and 4 is collected by government.

2. Unit tax/specific tax: a dollar amount on each unit sold (we use $\tau$).
   Example: Government charges 18.4 cents for each gallon of gas. So, when the gas station charges you 1.49 dollars per gallon, the market price is 1.49 where 1.49 has includes the tax already.

Who is paying more of the tax?
Judge it by demand elasticity and supply elasticity.

Formula:

$$dp = \frac{\epsilon_s}{\epsilon_s - \epsilon} d\tau$$

where $\epsilon$ is the demand elasticity, $\epsilon_s$ is the supply elasticity.

Facts:
1. If $|\epsilon|$ larger, then price increase less.
   Tax incidence falls less on consumers (more on producers).
2. If $\epsilon_s$ larger, then price increase more.
   Tax incidence falls more on consumers (less on producers).
Do the equilibrium price and quantity depend on whether the tax is assessed on consumers or on producers?
(there are two cases: “tax from producers” and “tax from consumers”)

Case 1: Collect tax from producers
This basically says that producers pay tax after he receives money from consumers. So if \( p_c \) is the price that consumers pay, \( p_c \) is the price that producers receive before the tax is collected. After the tax is collected, what producers actually get is \[ p_s = p_c - \tau. \] And producers are going to decide how much he is going to supply by this price that he actually receives.

So, instead of \( Q_s(p_s) \), the new supply curve is \( Q_s(p_c - \tau) \), if we denote \( p \) as \( p_c \), the market price.

Example:
\[
Q_s = 88 + 40 \, p_s \\
Q_d = 286 - 20p_c
\]

What is the new supply curve (after taxing producer)?
\[
Q_s = 88 + 40(p_c - \tau) = 88 + 40(p_c - 1.05) = 46 + 40p_c
\]

Then we solve out the new equilibrium:
\[
Q_s = Q_d \\
46 + 40p_c^* = 286 - 20p_c^* \\
p_c^* = 4
\]

This is the equilibrium market price, what the consumers pay. Producers receive
\[
p_s^* = p_c^* - \tau = 4 - 1.05 = 2.95
\]

So, consumers pay 4, producers receive 2.95 and government gets 1.05. And consumers pay 70 cents of the tax, while producers pay 35 cents for the tax. Consumers bear more of the tax burden. (How to get this numbers?)
Case 2: Collect tax from consumers
This says that after consumers pay price to firms/producers, consumers still have to pay tax to government. So what consumers actually spend is $p_c = p_s + \tau$, where $p_s$ is the market price. (Note: still using the same equation). Producers receive $p_s$.

Using the same example above, now the change only happens with demand curve. Consumers are now paying $p_c = p_s + \tau$, so they will decide how much they are going to buy based on price $p_s + \tau$.

What is the new demand curve?
$$Q_d = 286 - 20(p_s + \tau) = 286 - 20(p_s + 1.05) = 265 - 20p_s$$

So, the new equilibrium is
$$Q_s = Q_d$$
$$88 + 40p_s = 265 - 20p_s$$
$$p_s^* = 2.95$$

This is what consumers pay to producers.
So, consumers pay $p_c = p_s + \tau = 2.95 + 1.05 = 4$ to get a good. Producers receive $p_s = 2.95$, while government takes away 1.05.
Summary
We can see from the above example that collecting tax either from consumers or from producers give the same result. Tax revenue also stays the same: $1.05 \times 206 = 216.3$, which is the rectangle area in the following graph:

Figure 3.06 Effect of a $1.05$ Specific Tax on Pork Collected from Consumers
How about the ad valorem tax

- How does the ad valorem tax change the demand curve?

Before tax, consumer pays “p” to buy a good. Now after the tax, consumers have to pay “p + ap” to buy it where “ap” is the tax paid to government.

The new demand curve is \[ Q_d = 286 - 20(1 + \alpha)p \]

The slope of demand curve changes from 20 to 20(1 + \alpha), i.e., the demand curve shrink to the left. The equilibrium price and quantity change in the same way as before when there is a sale tax. But how much exactly might be different.