Chapter 7. Costs

Short-run costs
Long-run costs
Lowering costs in the long-run

0. Economic cost and accounting cost
“Opportunity cost”: the highest value of other alternative activities forgone.
To determine the opportunity of a resource, you need to compare the “other uses” of it. Although this cost may not show up on the balance sheet.

“Opportunity cost” is important for firms to make decisions among several choices. For example, a person has two choices: run his own business or work for other firms. If he run his own business, the explicit cost is $40,000, which includes wage paid to labors, rent for office, etc. By running his own business, this person can earn $60,000 profits. So in total, he will have $20,000 in hand. If he just considers this profit, he will make the decision to operate his own firm. However, if he thinks about the other choice: working for other firms, the decision will be different. If what he earned by working from other firms is larger than $20,000, he will run his own business. Otherwise, he will earn more to work for other firms.

1. Short-run Costs

Fixed cost vs. variable cost (changing with output):
Fixed cost includes things firms cannot change, for example, capital costs and rent for land. Other costs than can be changed with output are variable. Another way to see it: the cost you have to pay when you produce nothing (but still in business).

\[ C = FC + VC \]

Marginal cost
\[ MC = \Delta C/\Delta q \]
Note that FC will not change, so marginal cost also means marginal variable cost.

Average costs
\[ AC = C/q \]

There are average (total) cost, average fixed cost, and average variable cost.
\[ AC = AFC + AVC \]
Let’s see a numerical example (Table 7.1)

<table>
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<tr>
<th>Output, q</th>
<th>Fixed Cost, F</th>
<th>Variable Cost, VC</th>
<th>Total Cost, C</th>
<th>Marginal Cost, MC</th>
<th>Average Fixed Cost, AFC = ( F/q )</th>
<th>Average Variable Cost, AVC = ( VC/q )</th>
<th>Average Cost, AC = ( C/q )</th>
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Next, let’s see the curves (Figure 7.1).

Panel (a) plots the total cost, fixed cost and variable cost. Panel (b) shows the marginal cost, average cost, average fixed cost and average variable cost.

Properties of these curves:
1. Fixed cost will not change; a horizontal line.
2. 1). Total cost is a parallel shift up from variable cost.
   2). The difference between total cost curve and variable cost curve is equal to fixed cost at any output level.
   3). Variable cost is 0 when output is 0.
   4). Total cost is the vertical sum of variable cost and fixed cost.
3. 1). Average cost at a particular 1 is the slope of a line from origin to the corresponding point on the cost curve.
   2). Marginal cost at a particular 1 is the slope of either total cost curve or variable cost curve at that point because TC, VC are parallel.
   3). Average variable cost at a particular q is the slope of a line from origin to the corresponding point on variable cost curve.
   4). When the line from origin is tangent to total cost at point a, then Average cost is equal to Marginal cost at that point a. Also, this equality occurs at the corresponding point in panel (b), where the Marginal curve intersects the Average cost curve.
4. AC and MC
   i. MC < AC → AC decreases
   ii. MC > AC → AC increases
   iii. MC = AC (crossing point) → AC at minimum. I.e., MC intersect AC and AVC at the minimum.
5. AFC declines as output goes up.
6. AC = AVC + AFC, it’s relationship with MC is the same as in 4.
[2] Cost and product curves
[2.1] In the short-run, capital is fixed, then the variable cost is just the cost of labor. And variable cost equals to wage rate times labor input. Total product curve of labor and variable cost curve are the same thing (Figure 7.2)!
Let’s look at this graph. The horizontal axis is unit of labor inputs. And the output is on the vertical axis. Since variable cost = w*L, we can multiply every point on the x-axis by this wage rate w. By doing so, the labor axis is changed into variable cost. However, y-axis is not changed. So we can see that the total output curve and variable cost curve overlaps each other. But we have to notice that normally when we draw the variable cost curve, we put variable cost on y-axis, and output on x-axis. But the variable cost in this graph is the opposite case.
Marginal cost can be derived from marginal product as well. For output to increase by $\Delta q$, we need $\Delta L$ units of labor. It will cost us $w^*\Delta L$ and this is a change in VC.

$$MC = \frac{\Delta C}{\Delta q} = \frac{\Delta VC}{\Delta q} = \frac{w^*\Delta L}{\Delta q}$$

Moreover, the relationship between $\Delta q$ and $\Delta L$ are described by $MPL = \frac{\Delta q}{\Delta L}$. The inverse is $\frac{1}{MPL} = \frac{\Delta L}{\Delta q}$. So,

$$MC = \frac{w}{MPL}$$

Let’s compare the curves
i. Specialization at the beginning stage of production: MP increases; MC decreases.
ii. Diminishing marginal returns: MP decreases; MC increases.

The average variable cost is the inverse of average product.

$$VC = w^*L$$

$$AVC = \frac{VC}{q} = \frac{w^*L}{q} = \frac{w}{AP_L}$$ (Inverse relationship)

With a constant wage, the AVE moves in the opposite direction of the Average product of Labor. Because $AP_L$ tends to rise and fall, $AC$ tends to fall and then rise.

How do cost changes shift the cost curves?
A component of the cost may be variable or fixed. We will see examples for both.

1. A specific tax – variable cost
We know a tax will shift the supply curve. And the cost curves determine a firm’s supply in the market. What happens to the cost curves when a specific tax is imposed?
If we treat tax as part of the cost, it belongs to the variable category. Suppose there is a $10 specific tax, then for every product that this firm produces, he has to pay an extra cost: tax. $\text{TC}^a = \text{TC}^b + 10q$. But be careful of the fixed cost: fixed cost is the cost that firm has to pay even this firm doesn’t produce anything; if firm produce 0 of output, then the tax this firm pays is 0; so the fixed cost is the same as before. Fixed cost will not change.

VC moves up by $10q$

AFC moves up by $10$ exactly.

AC and MC also move up by $10$ (Figure 7.3), but they still intersect at the AC’s minimum and quantity produced at the minimum is still 8.

2. A lump-sum fee – fixed cost (A lump-sum that a firm pays for the right to operate a business).

If you want to join a chain store, you need to pay a franchise fee to the headquarters. This is a fixed amount per year and is part of the fixed cost.

FC moves up by the fee amount. ($\text{FC}^a = \text{FC}^b + \text{fee}$)

AFC moves up by $\text{fee}/q$.

VC, AVC, MC will not change.

AC moves up by $\text{fee}/q$, but $\text{fee}/q$ is decreasing when q increase, which implies that the difference between after-tax average cost and before-tax average cost is getting closer to 0.

**Practice Question:** What is the effect of a lump-sum fee on the quantity at which a firm’s after-tax average cost curve reaches its minimum?
Answer:

2. Long-run cost
Long-run is “when firms can change all their inputs.” Plant size and machinery can be varied. So, there is no fixed cost. Fixed cost in the long-run is equal to 0. We will talk about only C, AC and MC (no distinction between FC and VC).

Cost is a function of output q. To produce any given output q, firms can now vary inputs in the long-run. So how is the cost curve decided for a given q? Isoquant is the curve on which all points produce same output. But those different combinations of inputs will incur different costs. Firms are going to choose one which produces a given output q but incurs the minimum cost.

Optimal input combination
[1] Just as consumers choose among “bundles of goods” to maximize utility given a budget constraint, firms choose “combinations of inputs” to minimize cost given a output level. (Firms will determine the projected output and then determine the inputs.) For consumers, we have indifference curves. For firms, we have isoquants and “isocost lines.” Note here an isoquant is the technology constraint for a firm.

Compared to a consumer’s choice.
Preferences (maximize utility) ~ cost (minimize cost)
Budget constraint ~ technology constraint (isoquants)
Consumers and firms are all maximizing (minimizing) under constraints.

If the wage rate is w and the capital rent is r, the cost is
\[ C = wL + rK \]

(Note that the capital rent is like you borrow money from the bank to buy machines and you pay mortgage monthly. So, the total cost can be calculated as periodic payments.)

We can easily see that, in a K-L graph, the combinations of capital and labor that cost you the same amount of money (isocost) form a straight line (Figure 7.4). For a specific amount \( C \), the isocost line is

\[ K = \frac{C}{r} - \frac{w}{r}L \]

Say, we want to spend cost $100, \( w = $5 \), \( r = $10 \) (per hour), the line would have

- slope = \(- \frac{w}{r} = \Delta K/\Delta L = - \frac{1}{2} \)
- and intercepts \( K = \frac{100}{r} = 10 \) and \( L = \frac{100}{w} = 20 \).

The outer lines mean higher costs.

[2] Now, let’s see a firm making the optimal choice of inputs.

Suppose the output level is determined to be at \( q = 100 \). The firm chooses combinations along this 100-isocost.

\( \rightarrow \) It will choose the lowest isocost on the isoquant (Figure 7.5).

We have three rules to decide the optimal inputs:

1) Lowest-isocost rule: pick the bundle of inputs where the lowest isocost line touches the isoquant.

Note: This should work for all shapes of isoquant if you are asked to find the optimal inputs graphically.

2) Tangency rule: pick the bundle where the isoquant is tangent to isocost line.

\[ \frac{MPL}{MPK} = \frac{w}{r} \]

3) Last-dollar rule:

\[ \frac{MPL}{w} = \frac{MPK}{r} \]
There are two cases: corner solution (not happening) and interior solution. For interior solution (as in Figure 7.5),

\[ \Rightarrow \text{isocost tangent to isoquant} \]

The same reason as for IC and BC: If not tangent, we can find another cheaper combination.

Note that at the tangent point, the slope on the isocost and the slope on the isoquant are the same.

Slope on isoquant = \( \text{MRTS}_K \text{ for } L = -\frac{\text{MP}_L}{\text{MP}_K} \)

Slope on isocost = \(-\frac{w}{r} \)

(Slopes are negative here.)

This means

\[ \text{MRTS}_K \text{ for } L = -\frac{\text{MP}_L}{\text{MP}_K} = -\frac{w}{r} \]

And also

\[ \frac{\text{MP}_L}{w} = \frac{\text{MP}_K}{r} \]

\[ \Rightarrow \text{This means the last dollar spent on both inputs generates the same marginal product. This is the firms’ cost-minimizing condition.} \]

We can review the consumer’s optimal choice at the same time (B-vertical).

slope on IC = \( \text{MRS}_B \text{ for } Z = -\frac{\text{MU}_Z}{\text{MU}_B} \)

slope on BC = \( \text{MRT}_B \text{ for } Z = -\frac{P_Z}{P_B} \)

Tangency means
MRS_B for Z = P_Z / P_B
MU_Z / MU_B = P_Z / P_B

This means you would want to gain higher utility from one unit of goods that are more expensive than the other.

We have a consumers’ utility maximizing condition (for an interior solution):
MU_Z / P_Z = MU_B / P_B.
The last dollar brings the same utility increment from both goods.

**Effect of a factor price change**

When the wage falls, “all of” the isocost curves change; they become flatter: at any cost level, labor is cheaper relative to capital (Figure 7.6).

![Graph showing isocost curves with wage change]

The slopes of all isocosts respond to the price change. (There is no fixed pivotal point as for a budget constraint.)

What is the optimal choice now?

Using the new isocost curves, we can find the optimal combination y.

Example (Problem 7.2):
Both w and r doubles and the projected output level is the same.
→ Slope does not change, same optimal combination. But, cost doubles.

**Long-run cost curves**

We have seen the short-run cost curves. How would we get the long-run cost curve? We can plot the optimal combination for every q and record the cost spent on these inputs (Figure 7.7). Note that each cost corresponds to a combination of K and L.
→ The curve in Figure 7.7 may not be a straight line (depends on return to scale).
Panel (a) shows the relationship between the lowest-cost factor combinations and various level of output. The curve through the tangency point is the long-run expansion path: the cost-minimizing combination of labor and capital for each output level.

**Practice:**

What is the LR cost curve for a fixed-proportion production function (perfect complements) when it takes one unit of labor and one unit of capital to produce one unit of output? Describe the long-run cost curve.

**Solution:**
Practice:
What is the long-run cost curve if $q=L+K$?

From the Long-Run cost curve, we can calculate the LRAC and LRMC curves. Most commonly, the LRAC and LRMC curves have the same shape as the short-run curves: U-shaped (Figure 7.8).

(a) Cost Curve

(b) Marginal and Average Cost Curves

What is the meaning of the shape of LRAC?
Depending of the slope of the LRAC, we call each portion:

1. Downward sloping LRAC; efficient to produce more → **Economies of scale**
2. Flat LRAC; same efficiency → **No economies of scale**
3. Upward sloping LRAC; inefficient to produce more → **Diseconomies of scale**
Note that this is “scale economies”, which addressed the relationship between AC and q. “Returns to scale” addresses the relationship between q and inputs changed proportionally. For “constant return to scale”, the AC and MC will be constant, and equals to each other. For “increasing return to scale”, AC is decreasing. For “Decreasing return to scale”, we have an increasing return to scale. In Chp 6, we have learned that most productions have IRS when Q is small, CRS for a moderate Q and DRS when q is large and the cost curve in Panel (a) is the one for this kind of production when return to scale could vary with firm’s size. Corresponding to this cost curve, we have a U-shaped AC.

Another common type is the L-shaped AC curves (p.208, Application). Let’s compare the two shapes. There is a efficient range of scales on the U-shaped curves since the cost to produce each unit is the lowest. On the L-shaped AC curves, any large scale is efficient. – L-shaped AC favors bigger firms. Examples are water, electricity.

Practice: A firm has Cobb-Douglas production function, \( Q = AL^\alpha K^\beta \), where \( \alpha + \beta < 1 \). On the basis of this information, what properties does its cost function have?

Solution:

3. Lowing cost in the long-run
Beside the isoquant picture we have used, we can also find the LR cost curves by investigating the SR curves. We can see a firm’s optimal combination by letting it choose capital first, than labor. This is correct since firms do choose capital first in the short-run. Let’s put the isoquant picture on the side for comparison.

Suppose a firm can choose three capital levels: low, median, and large plant sizes. We can draw three short-run AC curves (Figure 7.9). Note that each point on a SRAC corresponds to a level of labor input. [Putting three horizontal lines in the isoquant picture, we can see the corresponding short-run product and then we know the short-run cost by calculating the money amount.]

Now suppose the firm wants to produce q1, it can choose among three levels of K. One of them gives the lowest cost and the other two are the “wrong plant size.” What will firm do if it has the wrong plant size? In the short-run, it can do nothing. But it can adjust given sufficient time.
In the long-run, the firm chooses the plant size that minimizes its cost of production. So the firm picks the plant size that has the lowest average cost for each possible output level. If there are many possible plant sizes, LRAC is smooth and U-shaped.

A firm chooses the lowest SRAC for a given level of q.

\[ \Rightarrow \text{The union of the lowest SRACs constitutes the LRAC.} \]

When the firm can change capital input continuously, the LRAC is the “envelope” of all the SRACs.

We can see the LRAC is always less than or equal to the SRAC. This is because, for any projected q, if the firm is at the right plant size, the current level of capital is the optimal choice and the cost cannot be lower. So, SRAC = LRAC. If the firm is at a wrong plant size, then it can adjust the level of capital (to the LR optimal level) and reduce cost. In this case, LRAC < SRAC.

**Long-run Expansion Path VS Short-run Expansion Path**

In the short-run, capital is fixed. Firms can only change the inputs of labor. So the short-run expansion path is a horizontal one. When firm increases labor, output changes. But since firm is unable to vary both inputs, the cost of producing 200 will be 4.616, rather than 4kr. While in the long run, the optimal inputs will be point z, and the cost will be 4, which is lower than the cost for short-run.
Figure 7.10