PARTIAL DERIVATIVES

In economics, we often want to know how changes in one economic variable (say x,) may lead to changes in another (say y), all else equal. Given a general or specific function expressing the relationship between the variables, these relative changes are expressed by the partial derivative.

Example: \[ Y = f(x_1, x_2, \ldots, x_n) \]

This equation says that, in general, y is related to a list of x’s.

The partial derivative \( \frac{\partial y}{\partial x_1} \) (also noted as \( y' \) or \( \frac{df}{dx_1} \)) measures the change in y due to an infinitesimal increase in \( x_1 \), while all other x’s stay the same. It is the limit of \( \Delta y/\Delta x \) as \( \Delta x \) approaches 0. This is the definition of various “marginal” concepts in economics. In a graph with \( x_1 \) on the horizontal and y on the vertical axis, \( \frac{\partial y}{\partial x_1} \) is the slope of the curve at a specific point.

TOTAL DERIVATIVE

On some occasions in economics, all else is not equal or constant and we want to know how small changes at the same time in several economic variables (e.g. \( x_1, x_2, \ldots, x_n \)) may effect another (say y). The sum total effect on y due to small changes in all of the x’s (i.e. \( dx' \)'s) is known as the total differential (dy) where

\[
\frac{dy}{dx} = \frac{\partial f}{\partial x_1} \cdot dx_1 + \frac{\partial f}{\partial x_2} \cdot dx_2 + \ldots \frac{\partial f}{\partial x_n} \cdot dx_n
\]

Calculating the effect on y due to a change in \( x_1 \), in this case requires calculating how changes in \( x_1 \) effects y directly and how it effects y indirectly by impacting the other x variables. Mathematically, this is determined by dividing the total differential by \( dx_1 \). The result is also known as the total derivative.

\[
\frac{dy}{dx_1} = \frac{\partial f}{\partial x_1} \cdot \frac{dx_1}{dx_1} + \frac{\partial f}{\partial x_2} \cdot \frac{dx_2}{dx_1} + \ldots \frac{\partial f}{\partial x_n} \cdot \frac{dx_n}{dx_1}
\]