Answer Outline

Midterm Exam (80 Points Total) — **Leigh Tesfatsion**
DATE: Thursday, March 13, 11:00am — **Econ 308, Spring 2008**

QUESTION 1:SHORT ANSWERS [15 Points Total, 5 Points Each Part A-C (About 15 Minutes)]

Part A [5 Points] In order for a system to be referred to as *complex*, what two key properties do researchers generally agree the system should possess?

Part A Answer Outline: From "Complex Systems," Syllabus Section I.A
http://www.econ.iastate.edu/classes/econ308/tesfatsion/casdef.pdf

> A complex system is a system that: (1) is composed of interacting units (components, primitive elements, constituents,), and (2) exhibits emergent properties, i.e., properties arising from the interactions of the units that are not properties of the individual units themselves.

Part B [5 Points] What does it mean to say that a market is organized as a *bilateral trade* market? Give a simple specific example of a bilateral trade market.

Part B Answer Outline: From required Section III notes ("Modeling Behavior, Learning, and Interaction Networks in Dynamic Market Economies")
http://www.econ.iastate.edu/classes/econ308/tesfatsion/ACEMarketModelingIntro.pdf

Two basic forms of trading:

1. Bilateral trading (Seller ↔ Buyer): No intermediary between buyer and seller
2. Mediated trading (Seller ↔ Mediator ↔ Buyer)

Simple Example of Bilateral Trade: Any direct buyer-to-seller trading, such as workers seeking to "buy" a job from an employer

Part C [5 Points] Carefully define IN WORDS the meaning of a *(pure) strategy* for an ITERATED game played for TMax iterations, where $TMax \geq 2$. Give a specific example of a (pure) strategy for a player in a 2-person simultaneous-move Iterated Prisoner's Dilemma (IPD) game played for two iterations.

Part C Answer Outline: From required Section II notes: "Game Theory: Basic Concepts and Terminology"
http://www.econ.iastate.edu/classes/econ308/tesfatsion/gamedef.pdf

A *pure strategy* for a player in any particular game (including any iterated game) is a complete contingency plan, i.e., a plan dictating what move that player should take in each possible situation ("information state") in which he could find himself.

Example: For a 2-person simultaneous-move Iterated Prisoner's Dilemma (IPD) game played for two iterations, every pure strategy consists of the following:

- A specific choice X (either C or D) for a move in the first iteration of the game;
- A rule that dictates which move Y (either C or D) to make on the second iteration of the game for each possible move Z (either C or D) of the rival player in the first iteration of the game.

Since there are precisely two ways to choose X, two ways to choose Y, and two ways to choose Z, it follows that there are $2\times\ 2\times\ 2 = 16$ possible pure strategies that could be specified here to answer this portion of Part C.
A specific example of such a pure strategy is as follows: Start by defecting, then do the opposite of what your rival did on the first move (i.e., defect if he cooperated and cooperate if he defected).

QUESTION 2: [30 Points Total, 5 Points Each Part A-F (About 30 Minutes)]

Consider a market for apples consisting of two profit-seeking sellers S1 and S2 plus three profit-seeking buyers B1, B2, and B3.

Table 1, below, presents the specific apple reservation prices (per bushel) for sellers and buyers for each successive bushel of apples they sell and buy, respectively.

Table 1: True Apple Reservation Prices for Sellers and Buyers

Bushels	S1	S2	B1	B2	B3
1	\$10.00	\$10.00	\$70.00	\$70.00	\$25.00
2	\$50.00	\$50.00	\$70.00	\$50.00	0
3	\$65.00	\$70.00	\$65.00	\$25.00	0
4	\$70.00	\$70.00	\$65.00	0	0
5	∞	∞	0	0	0
7	∞	∞	0	0	0
8	∞	∞	0	0	0

IMPORTANT NOTE:

The **Blank Graphs** provided for Q2:A through Q2:D, below, should be used to provide the requested graphical answers for Q2:A through Q2:D **in clear carefully labeled form**.

If you wish, you can superimpose some or all of your graphical answers for Q2:A through Q2:D on top of each other in one graph instead of graphing each one separately *as long as the resulting graph(s) clearly depict all required graphical answers for Q2:A through Q2:D.*

Q2:Part A (5 Points) Using the information in Table 1, graphically depict below the **True Total Supply Schedule** for this apple market with apple quantity (in bushels) on the horizontal axis and the apple price per bushel on the vertical axis.

Part A Answer Outline: See graph at end of Q2.

Q2:Part B (5 Points) Using the information in Table 1, graphically depict below the **True Total Demand Schedule** for this apple market with apple quantity (in bushels) on the horizontal axis and the apple price per bushel on the vertical axis.

Part B Answer Outline: See graph at end of Q2.

Q2:Part C (5 Points) Using your findings in parts Q2:A and Q2:B, graphically depict all possible **Competitive Market Clearing (CMC) Points** for this apple market.

Part C Answer Outline: See graph at end of Q2.

Q2:Part D: (5 Points) Using your findings in parts Q2:A through Q2:C, graphically depict **Net Seller Surplus** and **Net Buyer Surplus** at any CMC point. (NOTE: You do not have to calculate the NUMERICAL value of the Net Buyer Surplus or Net Seller Surplus. Just provide a carefully labeled GRAPHICAL depiction of the Net Buyer Surplus and Net Seller Surplus at any CMC point.)

Part D Answer Outline: See graph at end of Q2.

Q2:Part E (5 Points)

E.1 Define in words what is meant by **Total Net Surplus** for this apple market.

> **Part E.1 Answer Outline:** From the required Section II notes "Market Basics for Price-Setting Agents"
> http://www.econ.iastate.edu/classes/econ308/tesfatsion/MBasics.SlidesIncluded.pdf
>
> Net Seller Surplus at any CMC Point (Q*,P*) for this apple market is the area between the CMC price P* and the true total supply curve, up to the CMC output level Q*.
>
> Net Buyer Surplus at a CMC point (Q*,P*) for this apple market is the area between the true total demand curve and the market price P*, up to the CMC output level Q*.
>
> Total Net Surplus at any CMC point (Q*,P*) for this apple market is the sum of Net Seller Surplus and Net Buyer Surplus at this CMC point.

E.2 Explain carefully WHY the Total Net Surplus extracted at any CMC point for this apple market is as large as it can possibly be. That is, explain why the Total Net Surplus extracted at any point OTHER than a CMC point CANNOT BE STRICTLY LARGER than the Total Net Surplus extracted at any CMC point.

Part E.2 Answer Outline:

By construction, to determine a CMC point for this apple market, the buyer units are plotted from left to right in *descending* reservation-price order to get the Total Demand Schedule and the seller units are plotted from left to right in *ascending* reservation-price order to get the Total Supply Schedule. The CMC point(s) are the intersection point(s) of these two schedules.

Matching of buyer with seller units then proceeds from left to right, so each successive matching necessarily yields the largest possible net surplus extraction (given all previous surplus extractions). However, if matching proceeds beyond the CMC point(s), the seller reservation prices are higher than the buyer reservation prices and the net surplus extracted is therefore *negative*, which reduces the total net surplus extracted. Consequently, the *maximum* total net surplus is extracted if and only if the matching of buyers and sellers proceeds up to – but not beyond – the CMC point(s).

Q2:Part F (5 Points)

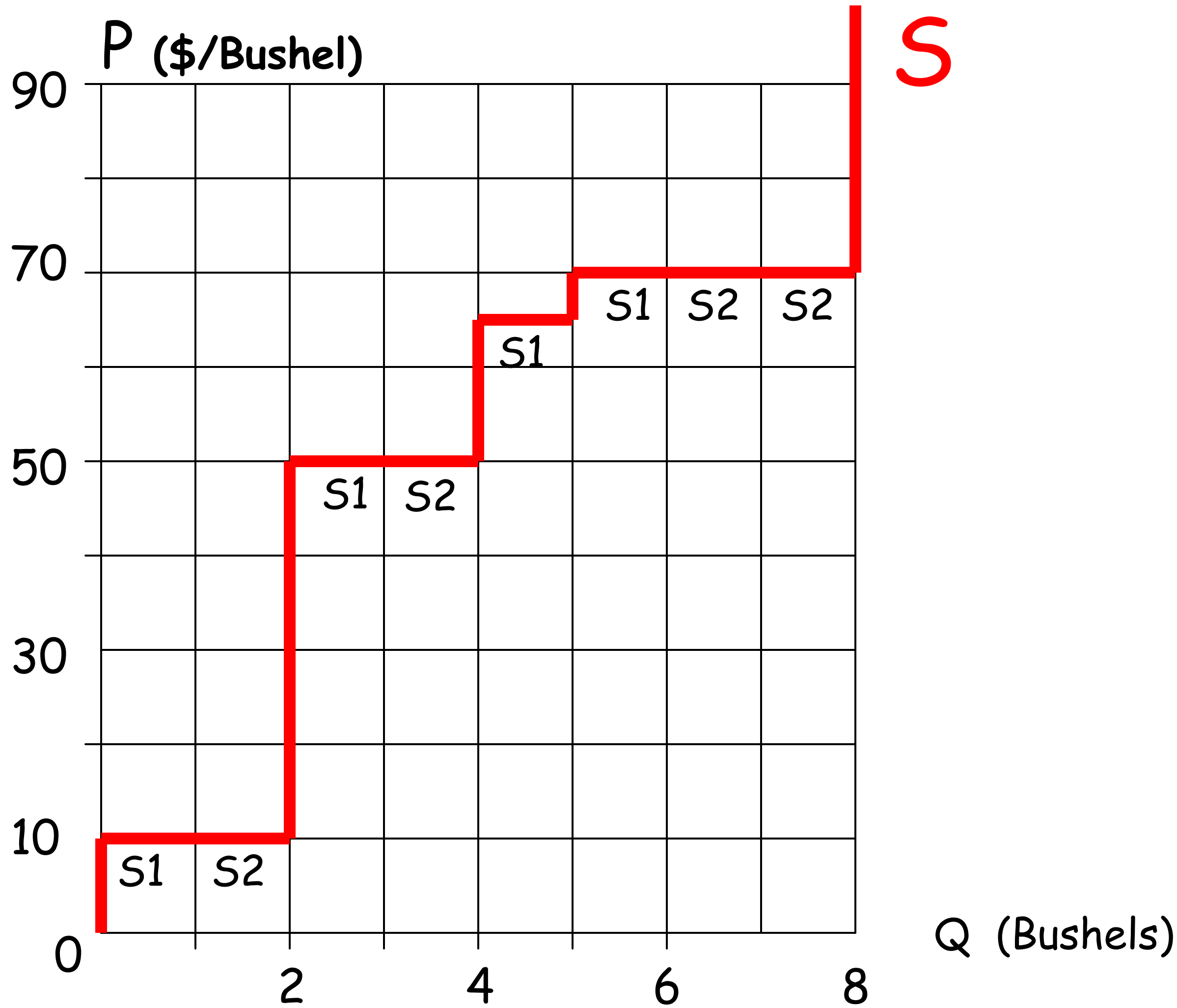
Suppose the apple market is conducted through an auctioneer, as follows: Sellers and Buyers EXPRESS (REPORT) supply and demand schedules to the auctioneer. Based on these EXPRESSED (REPORTED) supply and demand schedules – the only information he receives – the auctioneer then calculates and publicly posts what he believes to be a CMC price for the market.

Based on your findings from parts Q2:A through Q2:E, does **Buyer B1** have any incentive to report an expressed demand schedule to the auctioneer that **deviates** from his true demand schedule, given that all OTHER sellers and buyers report their true supply and demand schedules to the auctioneer?

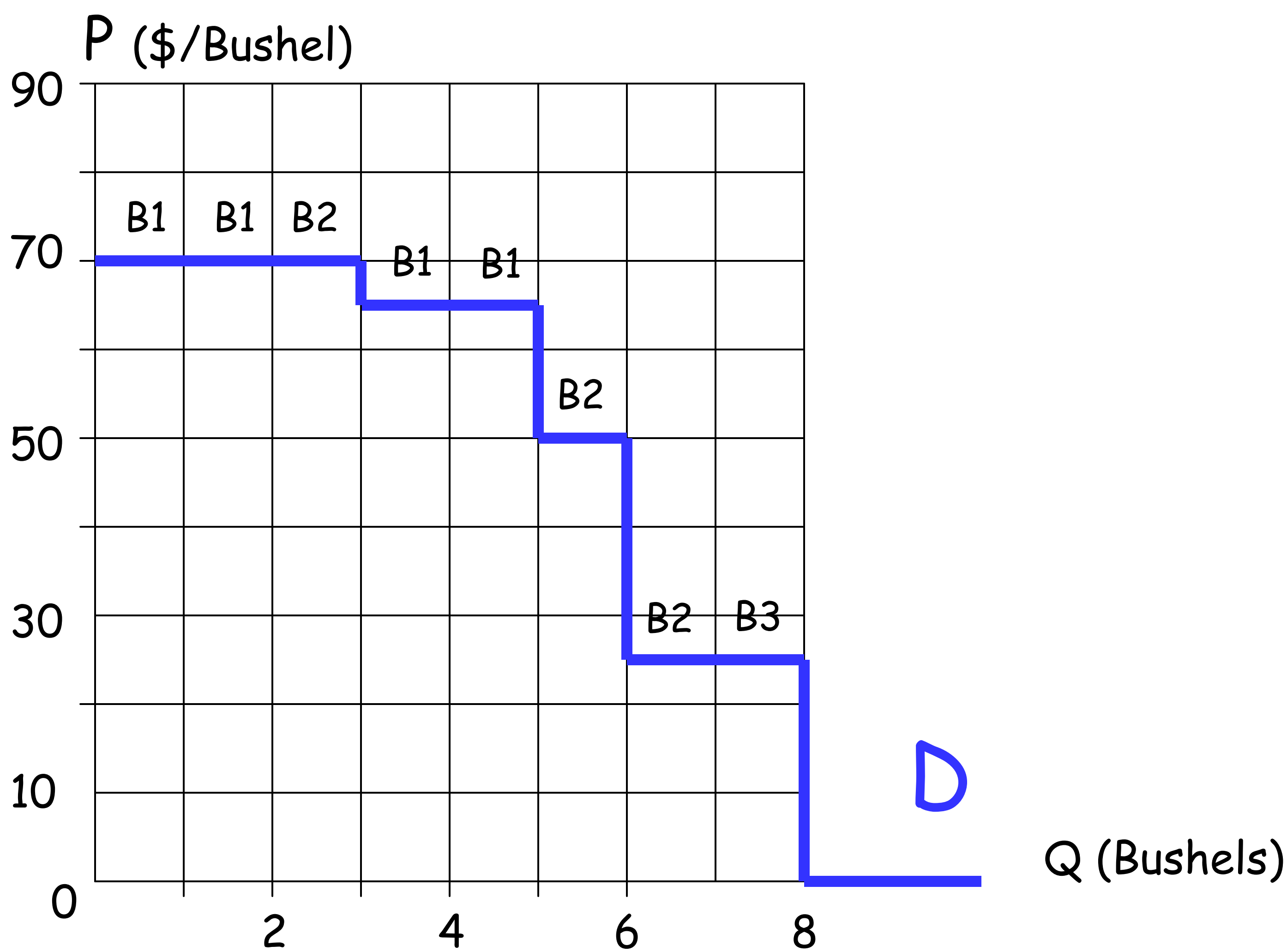
If your answer is yes, carefully explain the exact nature of the desirable deviation for Buyer B1, and justify carefully WHY this deviation is desirable for Buyer B1. If your answer is no, carefully explain why no such desirable deviation exists for Buyer B1.

Part F Answer Outline: B1 *does* have an incentive to report an expressed demand schedule that deviates from his true demand schedule. Indeed, he can increase his profits from $10.00 to approximately $55.00 if he withdraws his fourth bid unit altogether and reduces the REPORTED reservation value for his third bid unit from $65.00 to $50.01. See the attached graphical argument at the end of Q2.

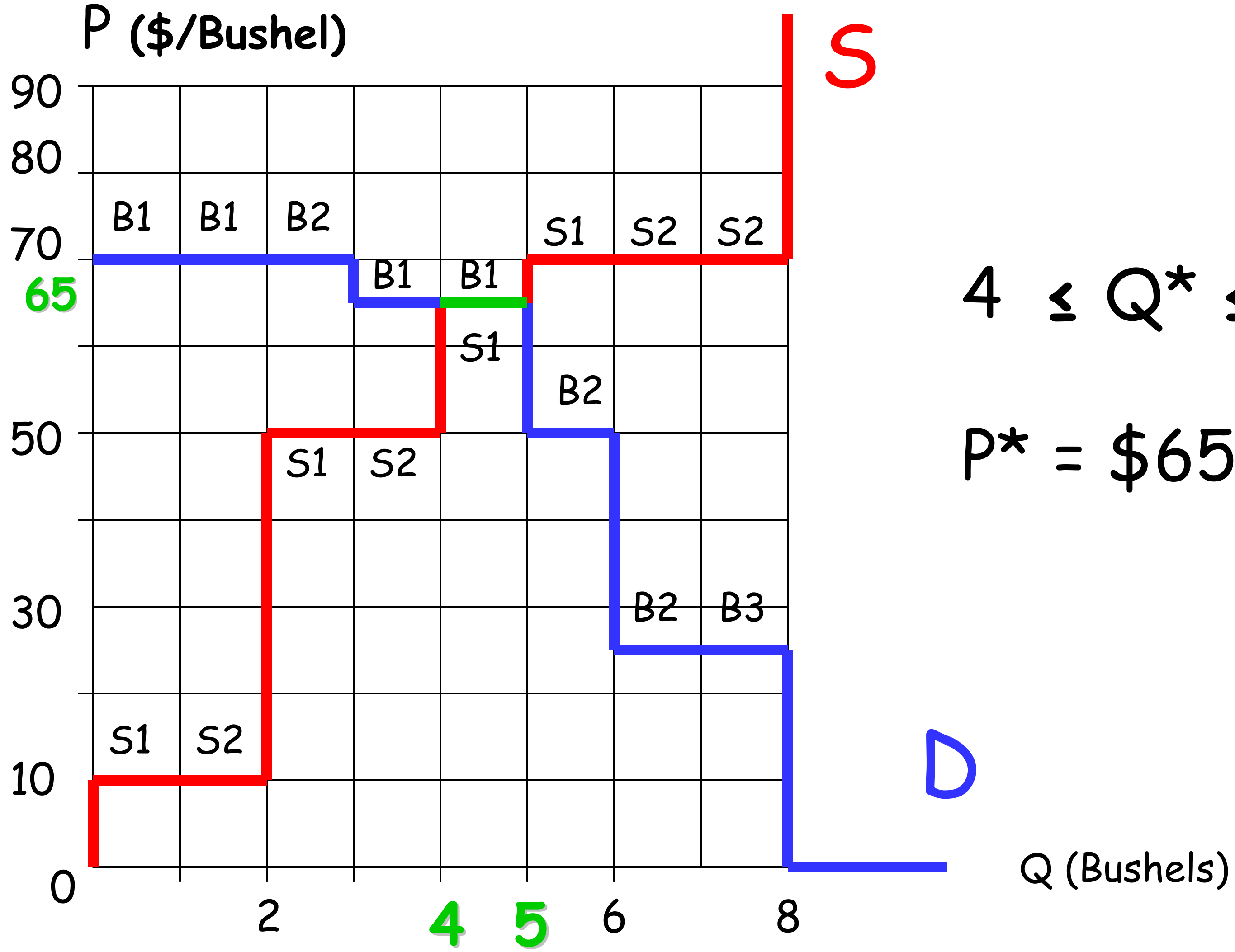
Q2 (PART A): Determine and graph the True Total Supply Schedule S



Q2 (Part B): Determine and Graph the True Total Demand Schedule D



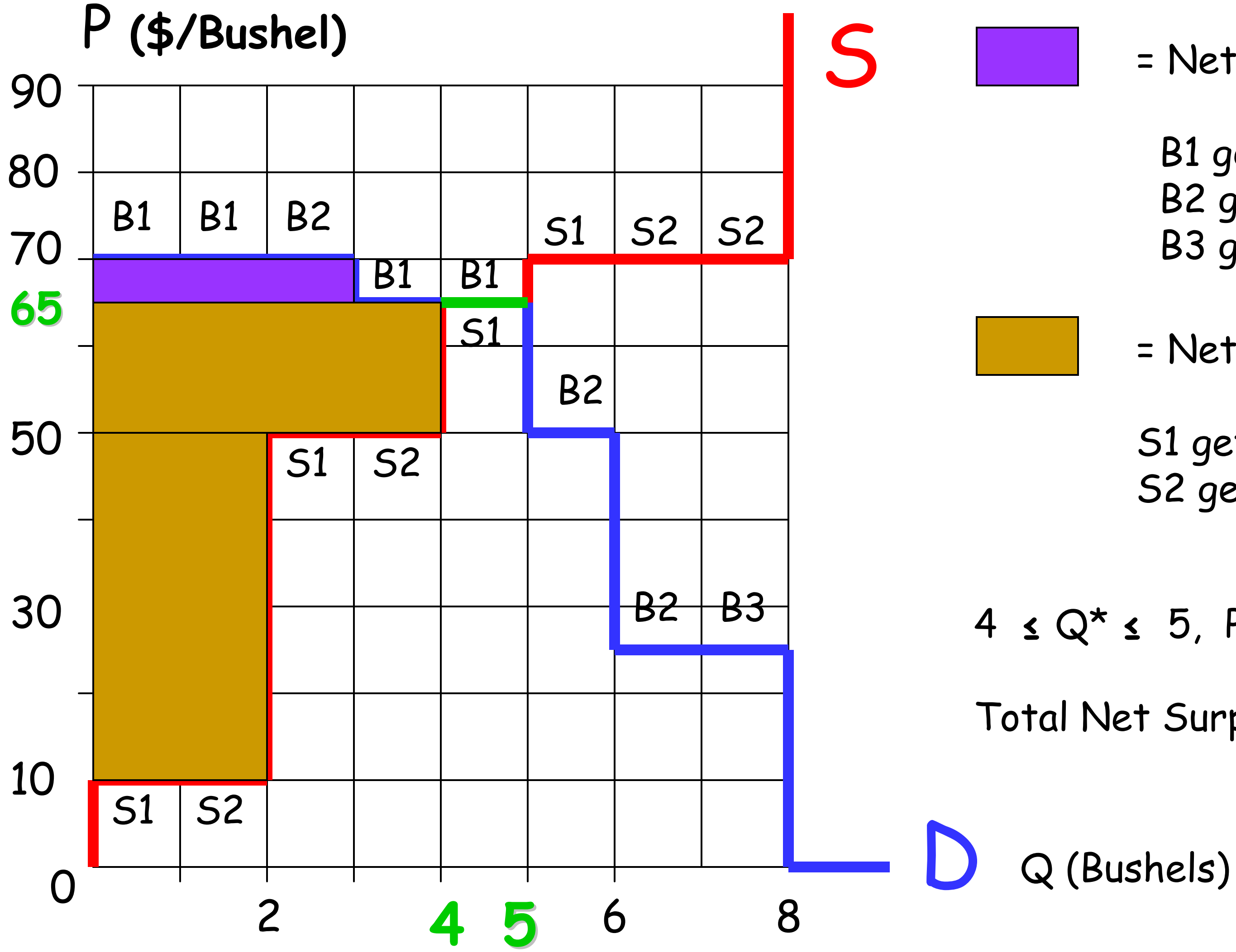
Q2 (PART C): Graph the Competitive Market Clearing (CMC) Points for This Market



$4 \le Q^* \le 5$

$P^* = \$65/\text{bushel}$

Q2 (PART D): Graph Net Seller Surplus and Net Buyer Surplus at any CMC Point



= Net Buyer Surplus

B1 gets [$5+$5]=$10
B2 gets $5
B3 gets $0

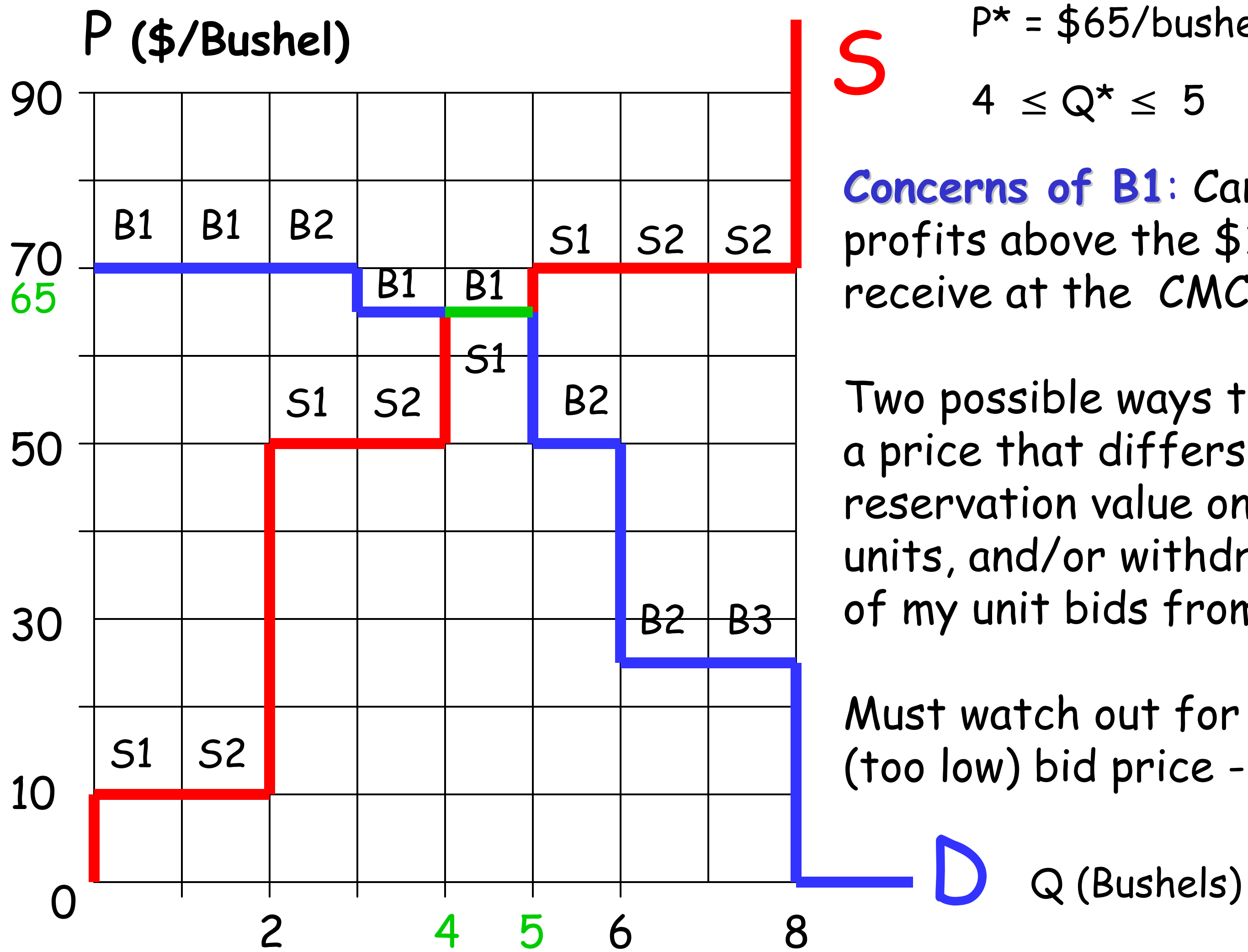
= Net Seller Surplus

S1 gets [$55+$15]=$70
S2 gets [$55+$15]=$70

4 ≤ Q* ≤ 5, P* = $65/bushel

Total Net Surplus = $155

Q2 (PART F): All else equal, does B1 have any incentive to report a DIFFERENT demand schedule?



P* = $65/bushel

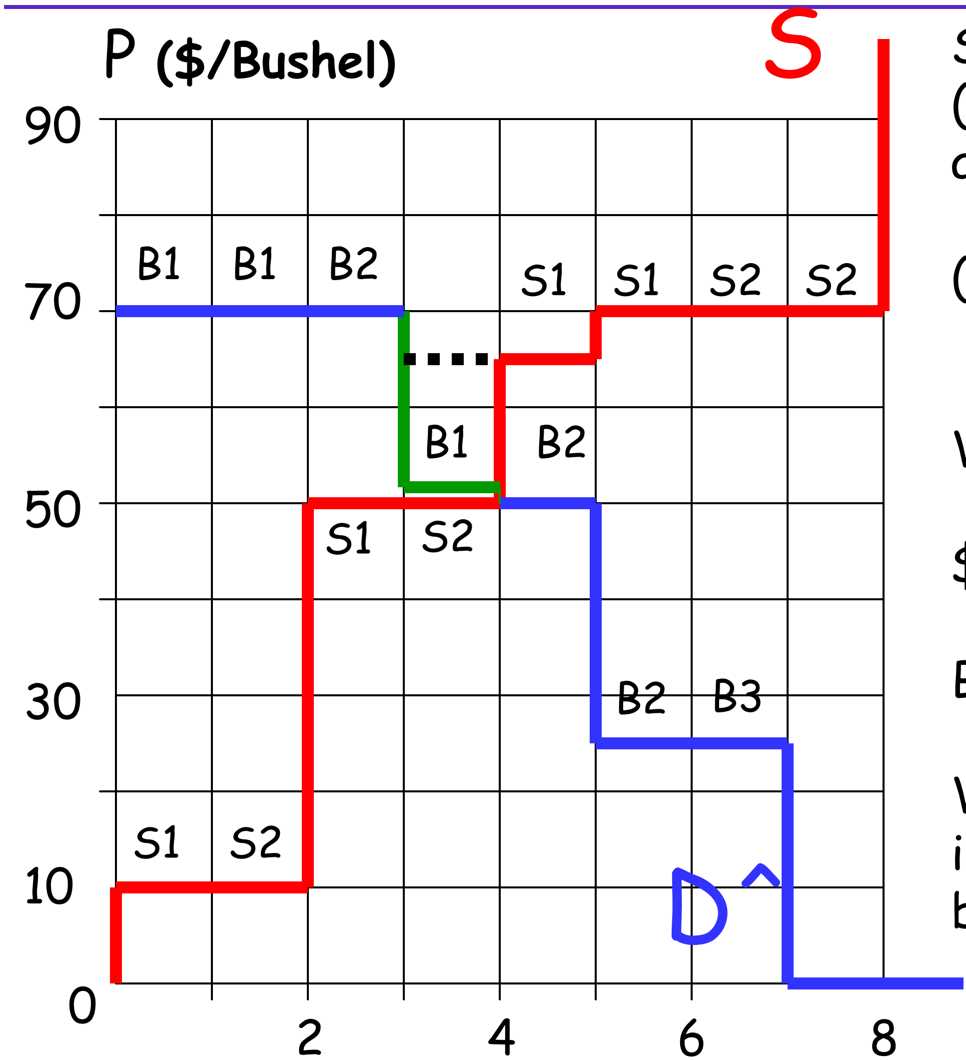
$4 \leq Q^* \leq 5$

Concerns of B1: Can I increase my profits above the $10 profits I receive at the CMC points ?

Two possible ways to do this: Bid a price that differs from my true reservation value on one or more units, and/or withdraw one or more of my unit bids from the market.

Must watch out for overly aggressive (too low) bid price --- why?

Q2 (PART F): A profitable way for B1 to make additional profits by deviating



Suppose B1 withdraws his fourth ("$65/bushel") unit from the market altogether and reports the price for his THIRD unit at $50.01/bushel (thus still beating out B2's second unit at $50/bushel) instead of $65.

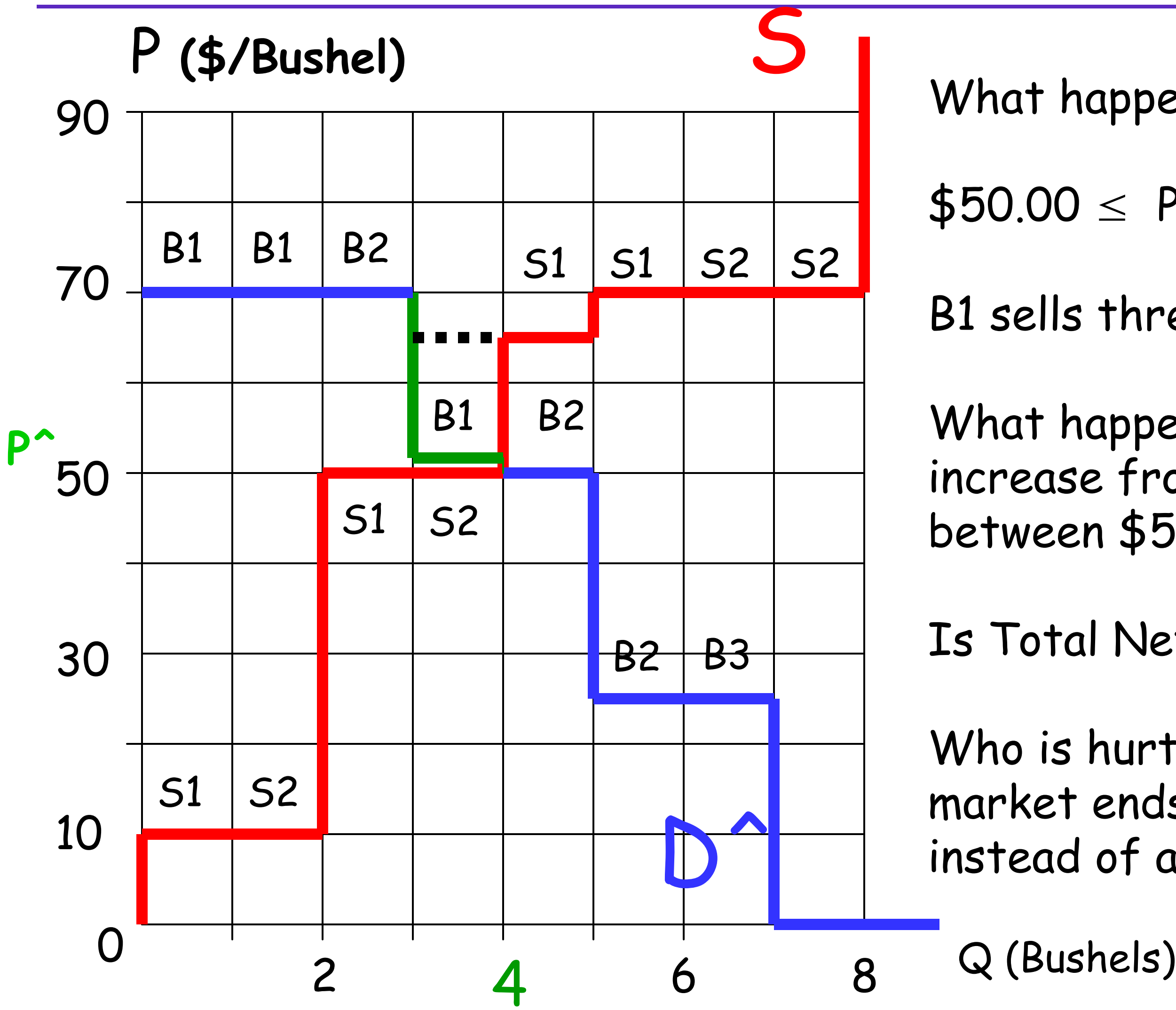
What happens to the "CMC" point(s)?

$\$50.00 \leq P\hat{} \leq \$50.01$, $Q\hat{} = 4$

B1 sells three units.

What happens to B1's profits? They increase from $10 to an amount between $54.97 and $55!!

Q2 (PART F): A profitable way for B1 to make additional profits by deviating



What happens to the "CMC" point(s)?

$\$50.00 \leq P\hat{} \leq \$50.01$, $Q\hat{} = 4$

B1 sells three units.

What happens to B1's profits? They increase from $10 to an amount between $54.97 and $55!!

Is Total Net Surplus reduced?

Who is hurt, who is helped, if the market ends up at (Q^,P^)=(4,$50) instead of at (Q*,P*) = (4,$65) ?

QUESTION 3: CREATIVE MODELING [35 Points Total (About 35 Minutes)]

GAS WAR!!

This question asks you to consider how the strategic pricing and location decisions of two rival gas stations might be modeled using game theory and agent-based modeling tools under alternative demand scenarios.

As depicted in the accompanying 8×8 checkerboard graphic, suppose two rival profit-seeking gas stations, G1 and G2, are located along the boundaries of an urban downtown area (inner four squares) surrounded by a suburban residential area (surrounding 60 squares). Due to restrictive zoning laws, G1 and G2 are the ONLY gas stations permitted to locate within the indicated urban/suburban areas.

At the beginning of Day 1, G1 and G2 must post a gas price ($/gallon) in an attempt to attract customers.

Suppose for simplicity that this price posted by G1 and G2 is restricted to two possible values:

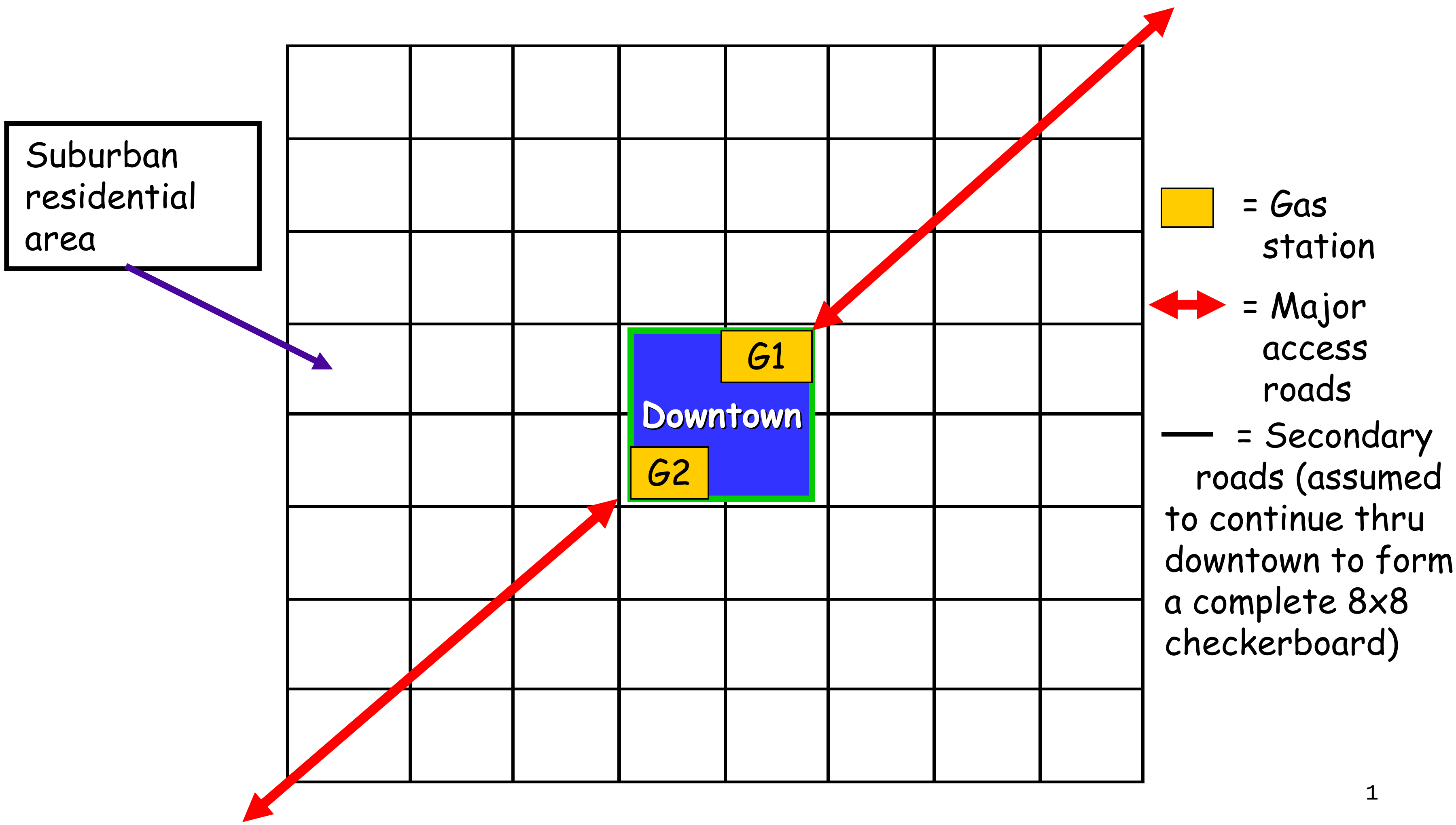
- H=High Price ($4.00/gallon) or
- L=Low Price ($3.00/gallon),

and that whatever prices G1 and G2 post on Day 1 must be maintained forever afterwards.

Suppose, also, that gasoline can be stored indefinitely, and that G1 and G2 have identical variable production costs: namely, each gallon of gas they supply to a customer costs them $1.00.

Finally, assume the daily flow of traffic into the urban downtown area along EACH major access road is 100 vehicles, with 10% in need of gas, and that each gas fill-up is for 10 gallons.

City Grid for Gas War



Part A:[**10 Points Total**] Consider the following demand scenario:

> **Demand Scenario 1:** The ONLY vehicle traffic flowing into and out of the urban downtown area is along the two major access roads, and ALL vehicle owners in need of gas search for the LOWEST price of gas.

Part A.1] Given Demand Scenario 1, develop and justify a plausible daily payoff matrix for G1 and G2 that captures their strategic pricing problem.

Part A.1 Answer Outline:

		G2	
		L	**H**
G1	**L**	(**\$200**, **\$200**)	(**\$400**, **\$0**)
	H	(**\$0**, **\$400**)	(**\$300**, **\$300**)

Daily Profit Payoff Matrix for Gas War Problem
under Demand Scenario 1 (Search for Lowest Price)

Cost per Customer: \$1.00

L = Price Low (\$3.00/**gallon**) → **Profit** = \$**2.00/gallon**
H = Price High (\$4.00/**gallon**) → **Profit** = \$**3.00/gallon**

Total Daily Customer Gasoline Demand: 200 gallons
(2 major access roads × 10 customers × 10 gallons per Day)

Additional Behavioral Assumption: Customers divide equally between G1 and G2 when prices equal.

Part A.2: Carefully analyze and describe the strategic structure of the resulting game. In particular, which price choices for G1 and G2 (if any) are Nash equilibria? Pareto efficient? Pareto dominated? Represent coordination failure? Also, does either G1 or G2 have a dominant price choice?

Part A.2 Answer Outline:

- The price-choice pair (L,L) is the ONLY Nash equilibrium – that is, the only price-choice pair such that neither player has an incentive to change its price choice given the price choice of the other player.

- The price-choice pairs (L,H), (H,L), and (H,H) are EACH Pareto efficient – that is, starting from any one of these three pairs, there is no OTHER price-choice pair the players could move to that would result in all players being at least as well off and at least one player strictly better off. (Another way to phrase this is that these three price-choice pairs are not Pareto dominated.)

- The price-choice pair (L,L) is Pareto-dominated by (H,H) in the sense that (H,H) gives each player at least as much payoff and at least one player strictly more. None of the other three price-choice pairs is Pareto-dominated (i.e., each of these other pairs is Pareto efficient).

- The price-choice pair (L,L) is a Pareto-dominated Nash equilibrium, hence by definition it represents a situation of coordination failure *for the two gas stations* – not for the customers!

- The price choice L is a dominant price choice for each player, in the sense it gives the highest payoff regardless of the other player's price choice.

Remark: Recall that a Prisoner's Dilemma Game is defined by four payoffs (CC=MutualCoop, CD=Sucker, DC=Temptation, and DD=Mutual Defection) satisfying the additional condition CC > [CD + DC]/2. The above payoff matrix for Demand Scenario 1 has this Prisoner's Dilemma Configuration with L playing the role of "Defect," H playing the role of "Cooperate," and \$200=DD, \$300=CC, \$400=CD, and \$0=DC.

Part A.3 Carefully discuss the ECONOMIC implications of your findings.

Part A.3 Answer Outline:

In Parts A.1 and A.2 it is assumed that every gallon of gasoline is like every other, and that gasoline customers are fully informed about the location of all gasoline stations, have no search costs ("transactions costs"), and are only concerned with finding the lowest price for gasoline.

In this case, the only way either G1 or G2 can attract any customers at all is if they both offer the *same* price. This is an example of the famous (within economics!) dictate called the "law of one price." Moreover, either gas station can profitably undercut the other (taking all the customers) unless they both offer the same *low* price, L. Thus, absent collusion between G1 and G2, one would anticipate an outcome (L,L).

In the real-world, driving through cities, you see gas stations in any one locale typically offering very similar prices. However, every once in a while a "price war" breaks out and you see plunging prices.

On the other hand, it is also typical to see prices offered "on a declining gradient" depending on how far away a gas station is located from major access roads. In this case the gas stations are betting that customers coming off major access roads are more interested in getting gas quickly than at the cheapest possible price. The implications of this change in customer search strategy are explored in Part B, below.

Part B:[**10 Points Total**] Consider, instead, the following alternative demand scenario:

> **Demand Scenario 2:** The ONLY vehicle traffic flowing into and out of the urban downtown area is along the two major access roads, and ALL vehicle owners in need of gas search for the NEAREST gas station.

Part B.1: Given Demand Scenario 2, develop and justify a plausible daily payoff matrix for G1 and G2 that captures their strategic pricing problem.

Part B.1 Answer Outline:

		G2: L	G2: H
G1	**L**	($200, $200)	($200, $300)
	H	($300, $200)	($300, $300)

Daily Profit Payoff Matrix for Gas War Problem under Demand Scenario 2 (Search for Nearest Gas Station)

Cost per Customer: $1.00

L = Price Low ($3.00/gallon) → Profit = $2.00/gallon
H = Price High ($4.00/gallon) → Profit = $3.00/gallon

Total Daily Customer Gasoline Demand: 200 gallons
(2 Major Access Roads × 10 customers × 10 gallons per Day)

Part B.2 Carefully analyze the strategic structure of the resulting game. In particular, which price choices for G1 and G2 (if any) are Nash equilibria? Pareto efficient? Pareto dominated? Represent coordination failure? Also, does either G1 or G2 have a dominant price strategy?

Part B.2 Answer Outline:

- The price-choice pair (H,H) is the ONLY Nash equilibrium – that is, the only price-choice pair such that neither player has an incentive to change its price choice given the price choice of the other player.
- The price-choice pair (H,H) is Pareto efficient – that is, starting at (H,H), there is no OTHER price-choice pair the players could move to that would make all players at least as well off and at least one player strictly better off. (Another way to phrase this is that (H,H) is not Pareto dominated.)
- Each of the price-choice pairs (L,L), (L,H), and (H,L) is Pareto-dominated by (H,H) in the sense that (H,H) gives each player at least as much payoff and at least one player strictly more.
- There is no Pareto-dominated Nash equilibrium, hence by definition there is no situation that represents coordination failure *for the two gas stations*.
- Move H is a dominant price choice for each player, in the sense it gives the highest payoff regardless of the other player's price choice.

Part B.3 Carefully discuss the ECONOMIC implications of your findings.

Part B.3 Answer Outline:

In economics a "good" is a bundle of attributes that have positive value for potential purchasers. These attributes can include physical form, place of availability, time of availability, and possibly additional aspects (e.g., risk considerations). Goods are economically distinct if they differ in their attributes in a way that causes people to assign different value to them.

Under Demand Scenario 2, even though the two gas stations are selling the same physical product (gasoline), customers are also interested in a second attribute of the product — the location of its sale. Indeed, in Demand Scenario 2 this second attribute is the ONLY factor determining demand for each product.

In effect, then, the two gas stations G1 and G2 are selling two different goods to two different customer bases (the vehicle traffic entering downtown on the major access road nearest to each station), and each gas station is effectively a "monopolist" for its own good. In this case, as seen above, the most profitable pricing strategy for each gas station is simply to charge the high price H. There is no price competition for customers.

Part C:[15 Points] Finally, consider instead the following alternative demand scenario:

> **Demand Scenario 3:** Vehicle traffic flows into and out of the urban downtown area along the SECONDARY roads as well as along the two major access roads. Vehicle owners in need of gas that enter downtown on SECONDARY roads ("the locals") always search for a gas station with the LOWEST price, whereas vehicle owners in need of gas that enter downtown on a MAJOR ACCESS road ("the tourists") always search for a NEAREST gas station.

Given Demand Scenario 3, G1 wants to choose a profitable pricing strategy – indeed, G1 would even be willing to relocate his gas station elsewhere in the urban downtown area or in the suburban residential area if this would ensure a suitably large increase in his profits.

However, G1 is unsure regarding relative traffic flows on secondary versus major access roads, relative demands for gas among locals versus tourists, and the intended pricing/location choices of his rival gas station G2.

To help with his pricing and location decisions, G1 decides to hire a consultant (YOU). He asks you to develop an "agent-based test bed" that he can use to explore the profitability of different possible combinations of pricing and location choices for his gas station under different plausible scenarios regarding gas customer search strategies, relative traffic flows, relative demands for gas, AND the pricing/location choices of G2.

Using simple diagrams and verbal descriptions, outline a possible way that such an agent-based test bed might be developed for G1, keeping in mind the limited time you have to devote to Q3:Part C (15 minutes).

Part C Answer Outline:

Obviously many different types of answers could be submitted for Part C. For a fifteen-minute question, only the outline of a possible approach can be expected.

These outlines will be evaluated on the basis of the following four characteristics: (1) extent to which the proposed approach is consistent with the objective of developing an "agent-based test bed" for the consulting assignment at hand; (2) originality of thought; (3) plausibility, correctness, and interest of assertions; and (4) clarity of exposition.

However, it is of interest to consider what kinds of considerations would presumably have to go into the development of such an agent-based test bed were this an actual real-world consultant project. Key development steps are indicated below.

Clearly, however, each development step would have to be guided and supported by appropriate empirical data in order to achieve a test bed with sufficient operational validity that it can generate useful scenario-conditioned predictions.

Key Development Steps:

- **Agent Hierarchy:** Identify the types of agents who will populate the test bed and their logical relationships (inheritance, composition, association,..).

  Given the stated purpose for which the agent-based test bed is desired, presumably these agents should include: vehicle drivers who will have demands for gas (differentiated into tourists from out of town and locals from residential areas); gas station managers in charge of pricing and location policies; and the city landscape, e.g., roads (differentiated into major acess and secondary), gas stations, and residential housing.

  The modeling of the city landscape might be accomplished by modeling the city as a 2-dimensional grid of "cells." However, much more is needed than a traditional 2D-cellular automaton in which each "cell" has the same rule governing its actions.

  For example, to capture a continuous roadway traversing the city, the city cells would need to have correlated but still differentiated attributes, such as different sizes, spatial coordinates, physical type (road pavement vs. building vs. open land), and so forth.

- **Flow Diagram:** Identify the basic types of dynamic agent activities and interactions to be modeled.

  For example, different types of vehicle drivers will potentially have different types of driving patterns, different demands for gas both at a point in time (across vehicle owners) and at different times of day, and different search strategies for determining from whom to purchase gas. Gas stations will presumably have different locations and different strategies for determining their gas prices.

  The interactions of these demand and supply forces could lead to traffic congestion patterns that affect the amount of gas that can feasibly be purchased in any given time period. These congestion patterns could in turn affect the future activities (e.g., gas demands and supplies) of vehicle owners and gas station managers.

- **Implementation of Agent Methods:**

  Agents that represent city landscape features (e.g., city "cells") are passive (non-cognitive) agents. Their attributes will presumably be either fixed or changing for external reasons (e.g., formation of pot-holes in proportion to intensity of use that slow down traffic flow, or a change from being an empty location to a location with a gas station). And their methods would presumably take only simple forms (e.g., a "get" method for a road location to determine its intensity of use).

  However, for the cognitive agents (vehicle owners and gas station managers), at least some of their attributes are going to have to be "state variables" that change over time due to their own actions (e.g., money holdings, amount of gas in gas tank, etc.). Moreover, behavioral methods will be needed to determine demands and supplies of gas under alternative conditions, perhaps with some kind of learning capabilities permitting the agents to change what actions they choose to take in any given situation based on past experiences.

  What makes this consulting assignment seemingly *require* some kind of agent-based modeling approach is the complex interactions (feedbacks) that would realistically be expected among the following three types of behaviors of the cognitive agents over time: the vehicle owners' plans regarding the frequency and pattern of their driving, which collectively results in potential congestive conditions and affects actual gas usage and gas demands; the particular search methods that vehicle owners use in an attempt to find "suitable" gas stations when they need gas; the placement of the gas stations; and the pricing strategies of the gas stations.

  Conjecture: In any real-world city, the *micro-level* determination of gas demands and supplies is exceedingly complex. However, *over time* there will be emergent city-wide traffic pattern flows that exhibit at least some degree of *homeostasis*, i.e., persistence over time despite perturbations going on at the micro level (see Syllabus Section I.A, Batten Chapter 1, Glossary of Terms.)