Notes on Network Formation

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Resource Sites (Links to Readings/Software):

Trade Network Game (TNG) Home Page
www.econ.iastate.edu/tesfatsi/tnghome.htm

Formation of Economic and Social Networks
www.econ.iastate.edu/tesfatsi/netgroup.htm

Presentation Outline

- Three approaches to the study of network effects
- Two IPD game examples comparing effects of having random vs. preferential partner matching
  - **Preparatory Stuff:** Finite state machine (FSM) representation of IPD player (i.e. strategy) types
  - **Example 1:** IPD game play among ***fixed*** player types
  - **Example 2:** IPD game play among ***evolving*** player types

Three Approaches to the Study of Network Effects

- Agents interact with other agents in a **given interaction network**. Agents do not control with whom they interact, or with what regularity (**e.g. Axelrod Tournament with round-robin PD play).**
- Agents interact with other agents through **given restricted links** but they exert some control over the strength of these interactions **(e.g. Electricity Market).**
- Agents **preferentially decide** with whom they interact and with what regularity **(e.g. Labor Market).**

Network Effects vs. Network Formation Effects…Continued

Cf. Andy Clark, *Being There: Putting Brain, Body, and World Together Again*, MIT Press, 1998

- **Strong Scaffolding**: ***Given*** interaction network, or ***given*** restricted links.
- **Weak Scaffolding**: Agents ***preferentially decide*** with whom they interact, and with what regularity.
- Scaffolding as a substitute for learning and/or thinking?

Network Effects vs. Network Formation Effects…Continued

Key Question:

What difference does it make if agents can *preferentially* form their own networks?

Random vs. Preferential Matching: Two Illustrative Examples

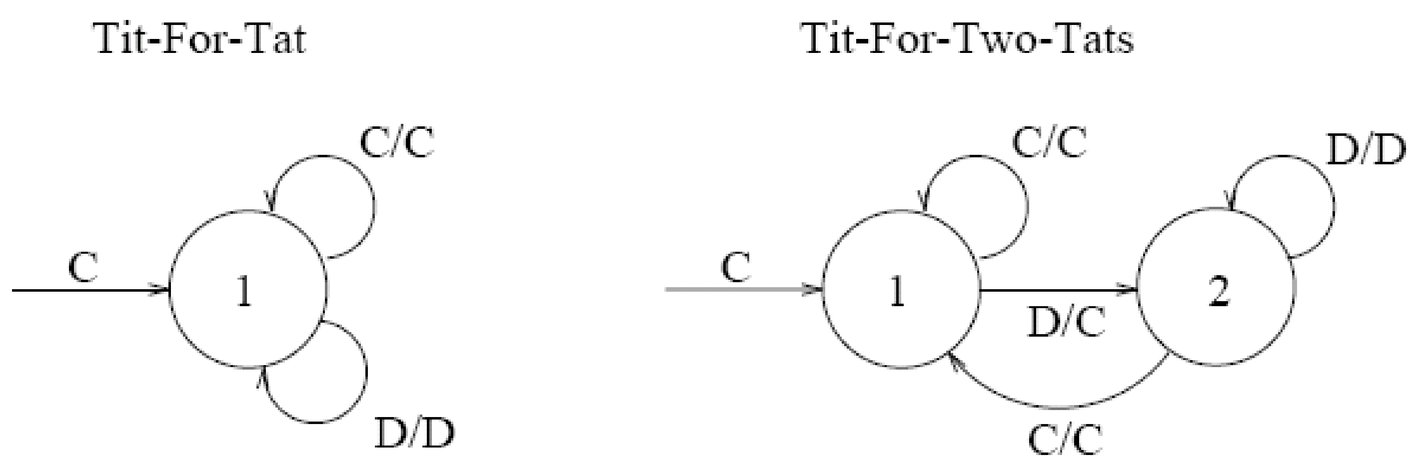
Example 1: IPD game play among fixed player types

Ref.[1]: L. Tesfatsion, "How Economists Can Get Alife," *SFI Volume*, 1997

Example 2: IPD game play among evolving player types

Ref.[2]: D. Ashlock, M. Smucker, A. Stanley, and L. Tesfatsion, *BioSystems*, 1996

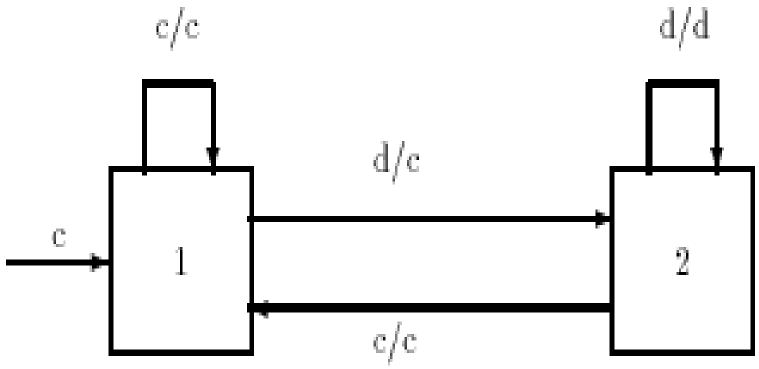
Illustrative Finite State Machine Representations for 1-State and 2-State IPD Players



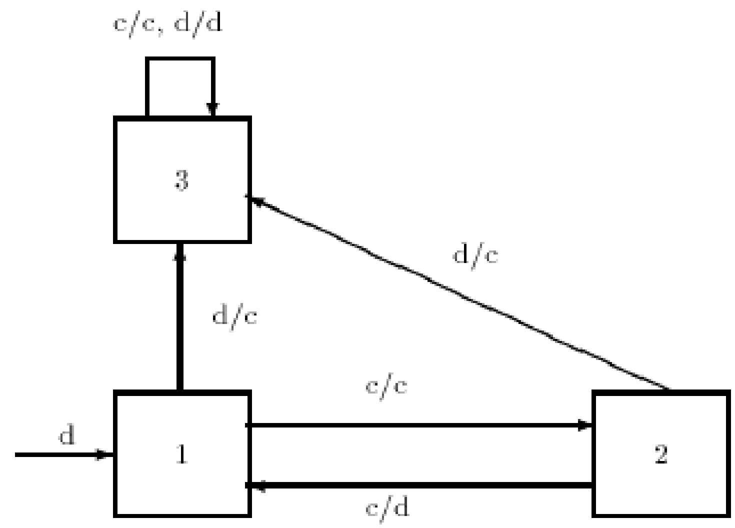
X / Y = "if rival played X last time, I play Y now."

TFTT vs. Rip-Off

QUESTIONS: What happens if TFTT is FORCED to play Rip-Off?
What happens if two Rip-Offs play each other?



(a) Tit-for-Two-Tats



(b) Rip-Off

X / Y = "if rival played X last time, I play Y now."

EXAMPLE 1: A Simple 5-Player IPD Game with Choice and Refusal of Partners

NOTE: All Example 1 results are analytically derived

- Fixed Player Population = 3 TFTTs and 2 Rip-Offs
- Players engage in 150 iterations of an Iterated Prisoner's Dilemma (IPD) Game
- The payoffs for each PD game play are centered about 0, as follows:

L (Lowest = Sucker Payoff) < **D** (Mutual Defection) < **0**

< **C** (Mutual Cooperation) < **H** (Highest=Temptation Payoff)

- In addition, PD payoffs satisfy **[L + H] / 2 < C**.

Example 1… Payoffs for Each Play of the Prisoner's Dilemma (PD) Game

Player 2

Player 1		C	D
	C	(C,C)	(L,H)
	D	(H,L)	(D,D)

L (Lowest) < **D** (Mutual D) < **0** < **C** (Mutual C) < **H** (Highest)

Example 1…
Expected Payoff Assessments

- Each player A assigns an **initial expected payoff U^o** to each other player B
- Expected payoff assessments U are continually updated based on play history (simple averaging)
- Player A finds player B **tolerable** as long as player A assigns a nonnegative expected payoff U to B
- Player A stops making play offers to (or accepting play offers from) any player B who becomes **intolerable ($U < 0$)**

Example 1…Preferential Matching

- At start of each iteration, each player A makes a play offer to a ***tolerable*** player B he judges to offer the currently highest expected payoff U.
- Player A "flips a coin" to settle ties and goes inactive if he judges every other player to be ***intolerable*** (U < 0).
- If player A has a play offer ***refused*** by a player B:
  - He suffers a **negative refusal payoff R ("shame")**
  - He then redirects his offer to a tolerable player B' he judges to have the next highest expected payoff U
  - If all other players are intolerable, he goes inactive.

Example 1…More on Preferential Matching

- Each player A updates his expected payoff U for another player B whenever he receives any payoff from interaction with B (either a refusal payoff or a game payoff)

  EXAMPLE: If player A has played B twice in the past and received payoffs p1 and p2, his current expected payoff U' for player B is

$$U' = [U^o + p1 + p2]/3$$

- If U falls below 0, player B is deemed **intolerable**

  ➔ player A will not direct any more play offers to B and player A will refuse any play offers received from B in the future.

Example 1…Key Issues

- Fixed population consisting of two agent types: 3 TFTTs & 2 Rip-Offs
- With **RANDOM** matching, Rip-Offs will chew TFTTs to pieces
- How does the introduction of **PREFERENTIAL** matching affect the relative long-run **fitness (accumulated points)** of TFTTs vs. RipOffs?
- How does the initial expected payoff level U^o affect long-run fitness outcomes?

Example 1…Visualization of Case Findings

Network Visualization:

- **Boxes = Players**
- **Box size = Long-run fitness level**
- **Lines = Persistent interactions**

Treatment Factor:

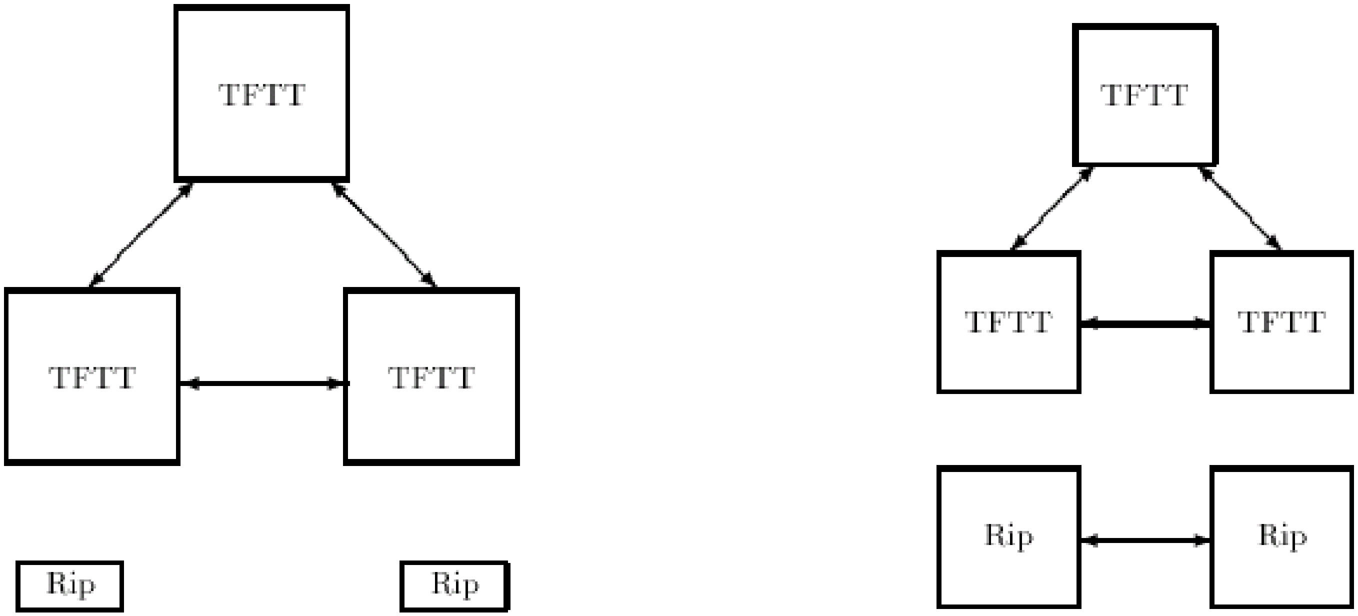
Initial expected payoff assessment U^o

stance towards strangers, same for each player

Four Cases for U^o:

very low; low; high; very high

TFTT vs. Rip-Off with Relatively Low U^o Values: ($L < D < 0 < C < H$, and $R < 0$)



(a) Case (CP.1)

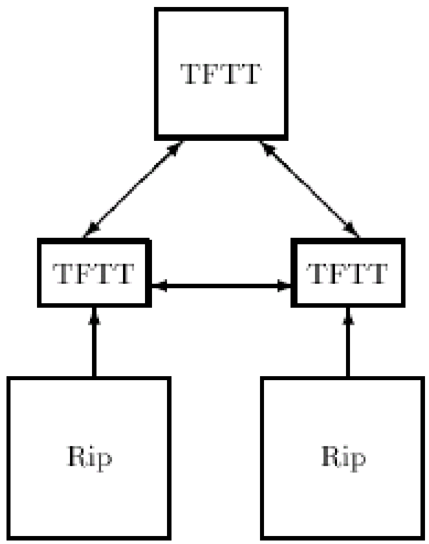
$0 < U^o < -D$

(b) Case (CP.2) or (CP.3)

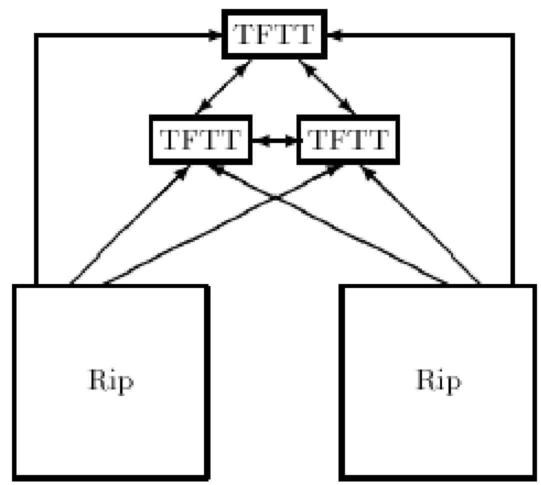
$(-D \leq U^o < -L)$ or $(C < -L \leq U^o)$

Note: **A larger box indicates a relatively higher long-run fitness.**

TFTT vs. Rip-Off with Relatively High U^o Values: ($L < D < 0 < C < H$, and $R < 0$)



(c) Case (CP.4): $-L \leq C$ with $-L \leq U^o < (H + C)/2$



(d) Case (CP.4): $-L \leq C$ with $(H + C)/2 \leq U^o$

Figure 2: Long-Run Trade Networks Under Assumption (CP) for the Illustrative 5-Tradebot TNG. A relatively larger box indicates a definitely higher fitness score for a sufficiently long trade cycle loop. In case (d), the Rip-TFTT interactions are stochastic if $(H + C)/2 = U^0$ and deterministic if $(H + C)/2 < U^0$.

EXAMPLE 2: Evolutionary IPD Game Play with Preferential Partner Matching

Reference [2]:

Dan Ashlock, Mark Smucker, Anne Stanley, and Leigh Tesfatsion, *BioSystems*, 1996

Key Issue Studied:

What happens in an evolutionary IPD game if players ***preferentially choose and refuse their partners*** instead of having their interactions exogenously determined by a random matching device?

Base-Line IPD Game Parameter Specifications

PD Payoffs: Sucker=0, MutD=1, MutC=3, Tempt.=5

Initial Expected Payoff: $U^o = 3$ = Mutual C

Intolerable: U falls below 1.6 (between MutD & MutC)

Refusal Payoff: R = 1.0

Number of Players	$N = 30$
Number of Generations (Tournaments)	$G \geq 50$
Number of Iterations per Tournament	$I = 150$
Initial Expected Payoff:	$\pi_0 = 3.0$
Minimum Tolerance Level:	$\tau = 1.6$
Refusal Payoff:	$R = 1.0$
Wallflower Payoff:	$W = 1.6$
Memory Weight:	$\omega = 0.7$
Number of Elite	$X = 20$
Mutation Probability	$\mu = 5/1000$

Table 3: Parameter Settings for the Standard IPD/CR Scenario

Evolutionary IPD Game with Random Partner Matching

- **Initial Strategies:** Each player in an initial population of 30 players starts with a ***randomly*** specified IPD strategy
- **Random Matching:** Each player is ***randomly*** matched in each iteration with another player to play a PD game
  - ➔ No choice or refusal of partners permitted, no refusal payoffs sustained, & no tolerability assessments are made.

Evolutionary IPD Game with Random Partner Matching…

- After **150 iterations (= one generation),** a Genetic Algorithm (GA) is used to construct a ***new*** set of 30 IPD strategies from players' current IPD strategy set.
- Players then enter another 150 iterations of PD game play with ***random*** matching
- This continues for **500 generations**
- **Forty runs (500 generations each)** were conducted

IPD with Random Partner Choice

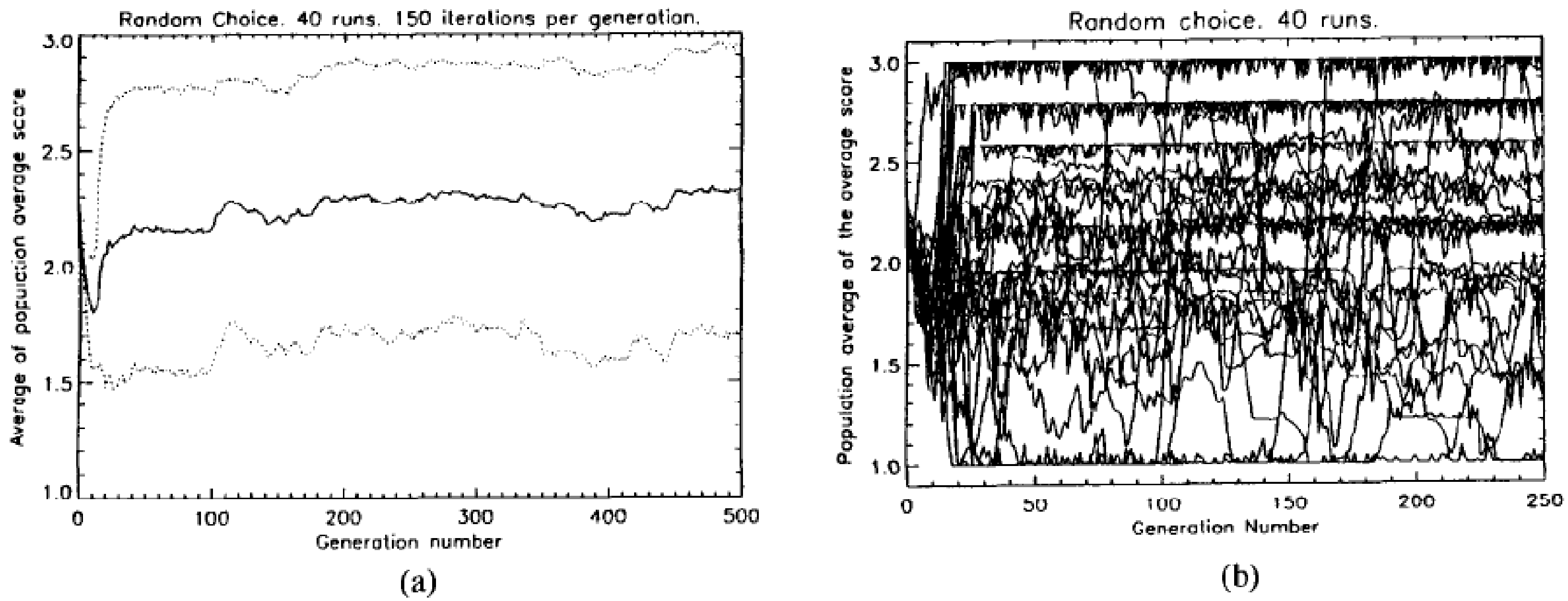


Fig. 3. Random choice evolved for 500 generations. Each player chooses exactly one partner at random on each of the 150 iterations comprising an IPD tournament. (a) The overall average fitness achieved by successive generations across 40 runs. The dashed lines (error bounds) show this overall average fitness plus or minus one standard deviation. (b) Each line shows the average fitness achieved by successive generations during one of the 40 runs. Note the wide spread and the horizontal bands. The bands tend to occur because populations become genetically homogeneous and mutants tend to do poorly.

Evolutionary IPD Game with Preferential Partner Matching

- Each player in initial 30-player population has a ***randomly*** specified IPD strategy

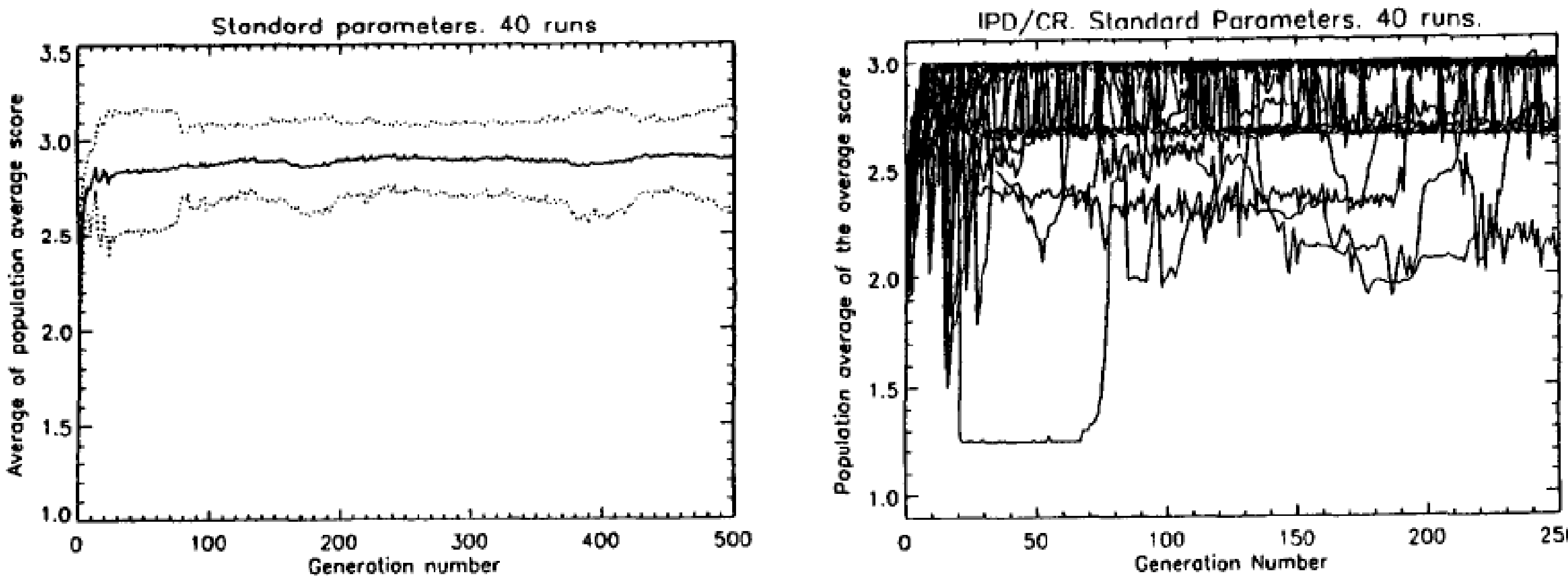
- **Preferential Matching:** Essentially the same as in the 5-Player IPD game

  - Players ***choose and refuse*** game partners on the basis of repeatedly updated expected payoff assessments (using WEIGHTED payoff averages), refuse to play with intolerable players, & receive refusal payoffs when their play offers are refused.

Evolutionary IPD Game with Preferential Partner Matching…

- After **150 iterations (= one generation),** a GA is used to construct a ***new*** set of 30 IPD strategies from players' existing IPD strategy set.
- Players then enter another 150 iterations of PD game play with ***preferential*** matching
- This continues for **500 generations**
- **Forty runs (500 generations each)** were conducted

IPD with Preferential Partner Choice

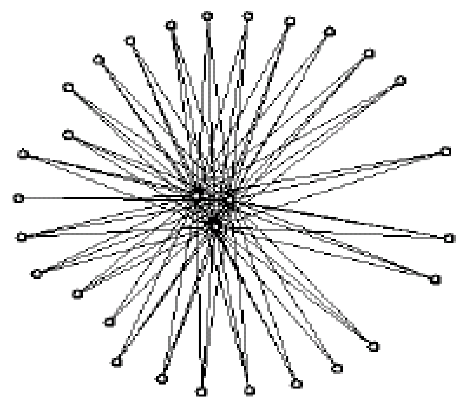


(a) (b)

Fig. 4. IPD/CR evolved for 500 generations with all parameters at their standard scenario levels. As in Fig. 3, each player chooses at most one partner in each iteration. (a) Overall average fitness across 40 runs and error bounds. (b) Average fitness achieved by successive generations for 40 individual runs. Note how few fitness levels are achieved in comparison to Fig. 3. The jumps in average fitness from the fitness region near 2.69 to a level above the mutual cooperation fitness region at 3.0 are observed frequently, and indicate the Raquel-and-the-Bobs phenomenon discussed in the text.

Raquel-and-the-Bobs Pattern

Actual Slice-in-Time Picture: **Inner grouping of 3 "Raquels" playing ≈ c:c with outer grouping of 27 latched "Bobs" playing ≈ d:c**



Generation 16 Iterations 120-124

Homogenous population of 30 Bobs → Rise of mutant Raquels until fitness of Bobs > fitness of Raquels → Decimation of Raquels → Back to homogeneous population of 30 Bobs → cycle repeats

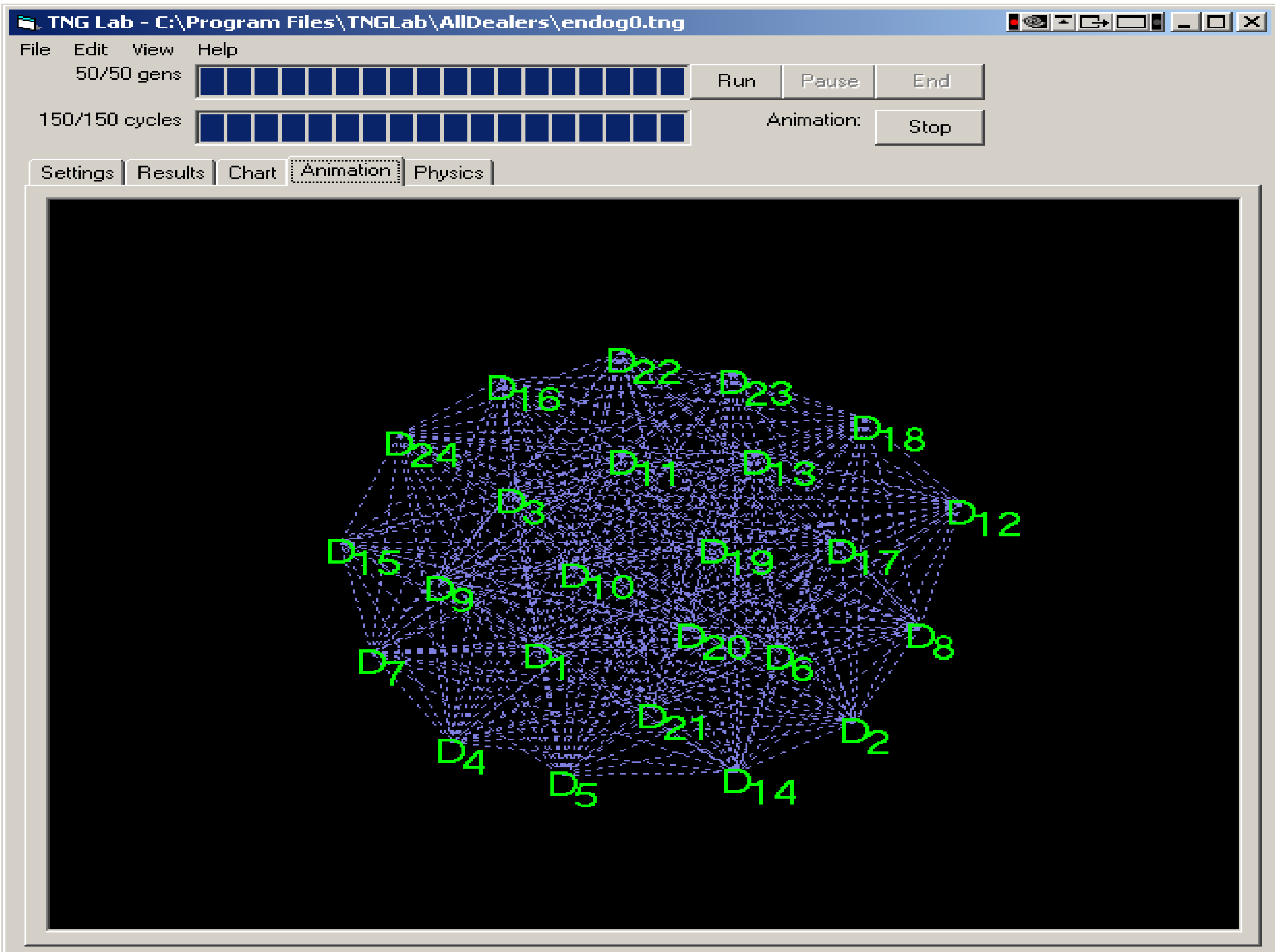
Summary of Findings for Evolutionary IPD Game Play with Preferential Partner Matching

Main Conclusions:

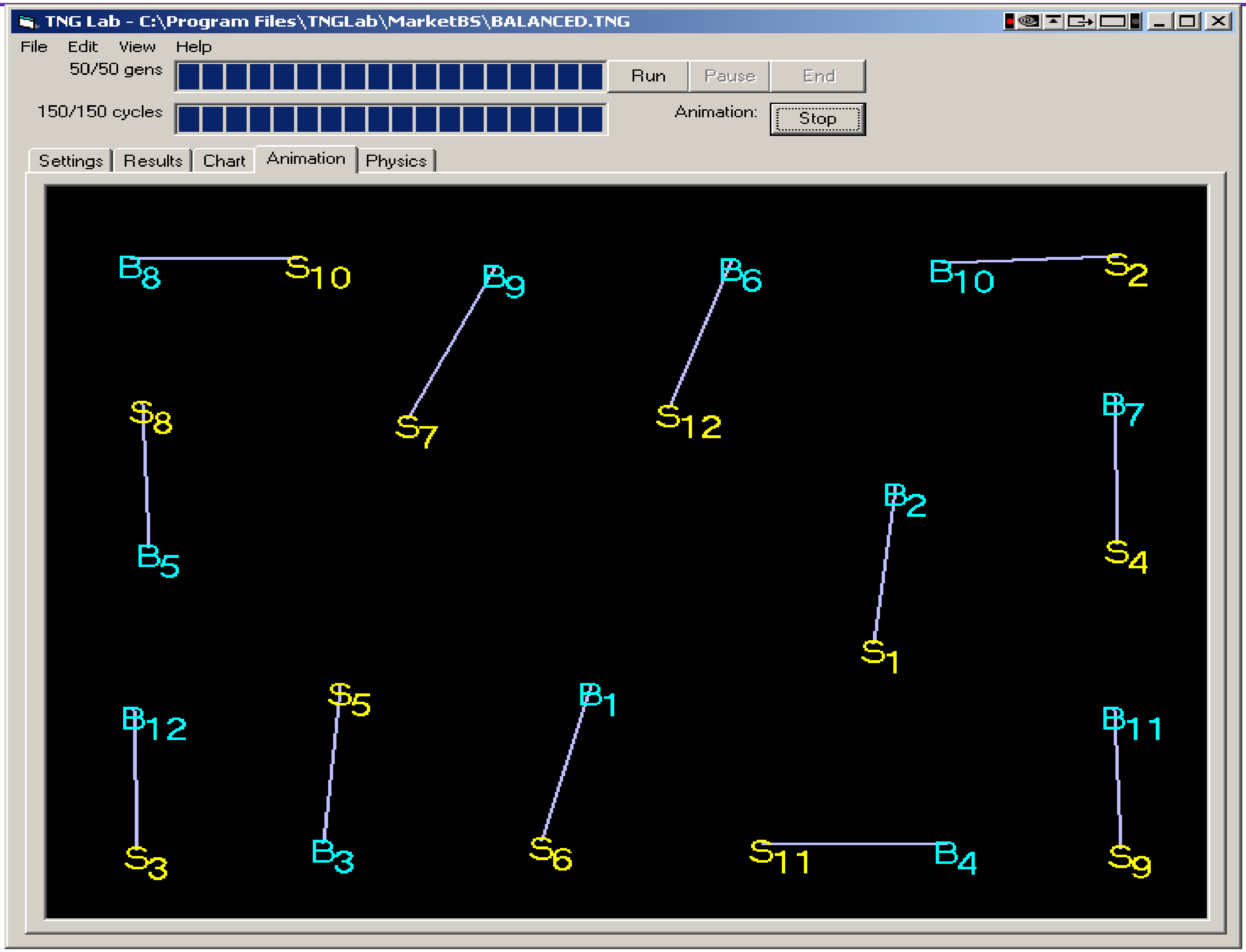
Introduction of choice and refusal of partners (in place of random matching) ***accelerates emergence of mutual cooperation*** in the IPD.

But this mutual cooperation can be supported by a ***wide variety of underlying network formations*** (latched, recurrent, star, disconnected, etc.)

An Evolved Stochastic Network (TNG Lab)



An Evolved Bi-Lateral Trade Network (Buyers-Sellers, TNG Lab)



An Evolved Tri-Lateral Trade Network (Buyers-Dealers-Sellers, TNG Lab)