Notes on Wilhite (*CE*, 2001)

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Main Reference:

Allen Wilhite, **"Bilateral Trade and `Small World' Networks,"** *Computational Economics* 18 (2001), 49-64

www.econ.iastate.edu/tesfatsi/SmallWorldNetworksBilateralTrade.Wilhite.pdf

NOTE: **For details about the C++ Program implementation, contact Allen Wilhite at wilhitea@email.uah.edu**

Small-World Networks & Economics

A number of ACE researchers have begun to consider small-world networks in relation to economic processes.

For example, Wilhite (2001) uses an ACE model of a bilateral exchange economy to explore the consequences of restricting trade to four different types of networks, including a small-world trade network.

Small-World Network

As defined by Watts and Strogatz (1998), a ***Small-World Network*** is a network that can be represented as a simple connected graph G exhibiting two properties:

- **Global Reach:** The presence of "short cut" connections between vertices results in short characteristic path length $L(G)$.
- **Local Connectivity**: Each vertex of G is linked to a relatively well-connected set of neighboring vertices, resulting in a large value for the clustering coefficient $C(G)$.

Wilhite's Basic Approach

Examine the trade-off between market efficiency and transaction costs under four types of trade networks:

* **Completely connected trade network** (every trader can trade with every other trader);
* **Locally disconnected trade network** (disjoint trade groups);
* **Locally connected trade network** consisting of trade groups aligned around a ring with a 1-trader overlap at each meeting point;
* **Small-world trade network** constructed from the locally connected trade networks by permitting from 1 to 5 randomly specified short-cut trade links between members of non-adjacent trade groups.

Four Possible Trade Networks

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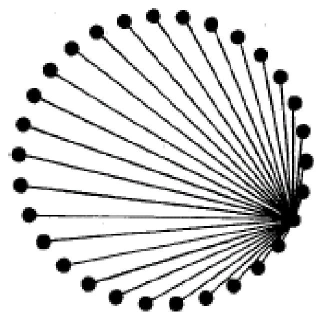
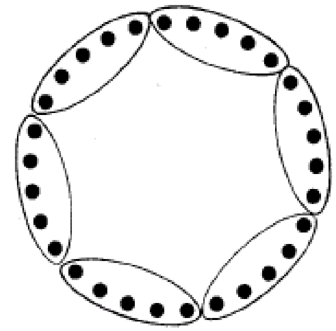
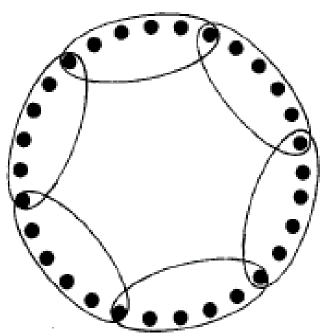


Panel a:
Global Network
Trade routes for one trader

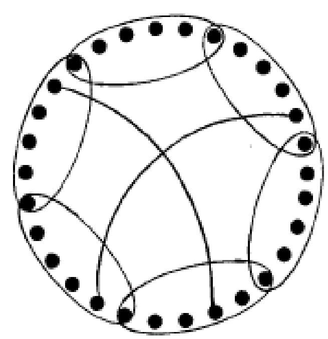


Panel b:
Local Disconnected Network
six groups, five agents per group

Note: Depicted links are for a *typical* trader in the global network



Panel c:
Local Connected Network
six groups, six agents per group



Panel d:
Small-world Network
two crossover agents

Figure 1. Sketches of four networks (population: 30 agents).

Basic Wilhite Model

- Exchange economy with 2 durable (non-perishable) goods: (1) a good g_1 that must be traded in whole units; and (2) an "infinitely divisible" good $g_{2.}$
- 500 traders initially endowed with **random** positive amounts of g_1 and g_2 (g_2 = numeraire good, i.e., the price of g_1 is expressed as units of g_2 per unit of g_1)
- Each trader i has the same form of utility function measuring preferences for the two goods:

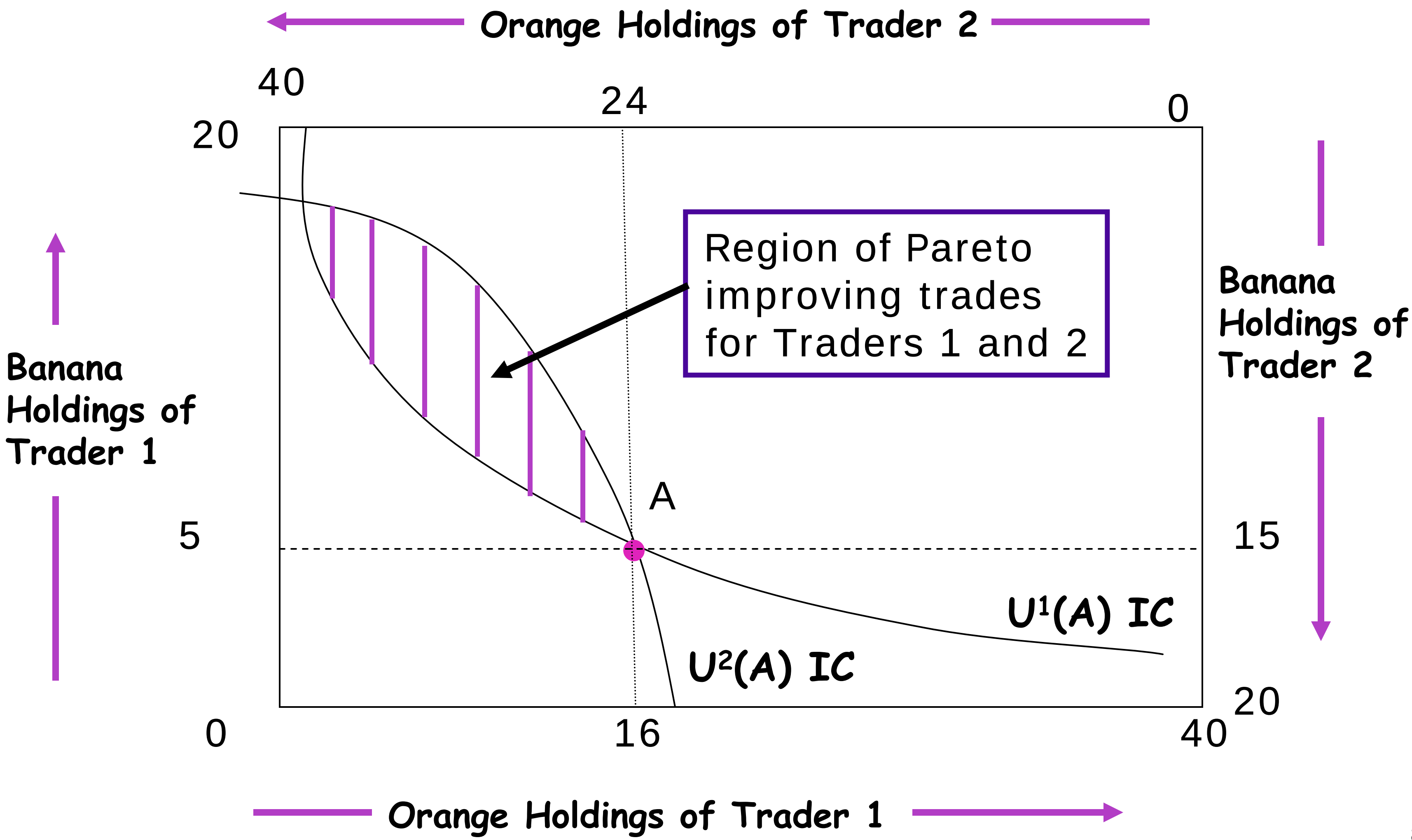
$$U^i = g^i{}_1 g^i{}_2 \quad , i = 1,\dots,500$$

- Traders are rational, non-strategic, and myopic agents who try to improve their utility in each period by voluntary feasible trades of g_1 and g_2.

Basic Model…Continued

- Opportunity for mutually beneficial trade exists for two traders if they have different reservation prices ("marginal rates of substitution" MRS) giving the rates at which they are ***just willing*** to exchange g_2 for g_1.
- To see the intuition for this, consider an "Edgeworth Box" pure exchange economy with two goods - oranges g_1 and bananas g_2 in fixed supplies 40 and 20 – that are currently allocated between Traders 1 and 2 at point *A*.
  - $MRS^i(A) = [\Delta U^i(A)/\Delta g^i_1]/\Delta U^i(A)/\Delta g^i_2] = \Delta g^i_2/\Delta g^i_1 = g^i_2/g^i_1$
  - $U^i(A)$ IC = Indifference Curve of Trader i passing through *A*
  - $MRS^i(A)$ = -1 times the slope of Trader i's indifference curve passing through *A*

Edgeworth Box Illustration



Basic Model…Continued

- Each trader is limited to trades within its own particular neighborhood as determined by the trade network.
- By random selection (without replacement), each trader in each neighborhood searches for trade partners within its neighborhood and selects a trade partner offering a "best" mutually beneficial price (if such a trade partner exists).
- Reservation prices are assumed to be truthfully revealed.

Basic Model…Continued

- Whenever a suitable pair of trade partners i and j is determined, the two traders trade at the following "split the difference" price as long as trades are feasible and mutually beneficial:

$$p_{i,j} = \frac{g_2^i + g_2^j}{g_1^i + g_1^j} \quad i, j \in \{1, \ldots, 500\}.$$

NOTE: $p_{i,j} = MRS^i \bullet [G^i_1] + MRS^j \bullet [1 - G^i_1]$,
where $G^i_1 = g^i_1/[g^i_1+g^j_1]$

Experimental Design: Four Distinct Trade Networks

Global network:

(i)	number of groups:	1
(ii)	agents in each group:	500
(iii)	end-agents overlap?	no
(iv)	number of crossovers:	0

Local disconnected network

(i)	number of groups:	50
(ii)	agents in each group:	10
(iii)	end-agents overlap?	no
(iv)	number of crossovers:	0

Local connected network:

(i)	number of groups:	50
(ii)	agents in each group:	11
(iii)	end-agents overlap?	yes
(iv)	number of crossovers:	0

Small-world network

(i)	number of groups:	50
(ii)	agents in each group:	11
(iii)	end-agents overlap?	yes
(iv)	number of crossovers:	5

50 runs with the same 500-trader populations were conducted for each of the four tested trade networks

Illustrative depiction of the four tested trade networks for a 30-trader economy:

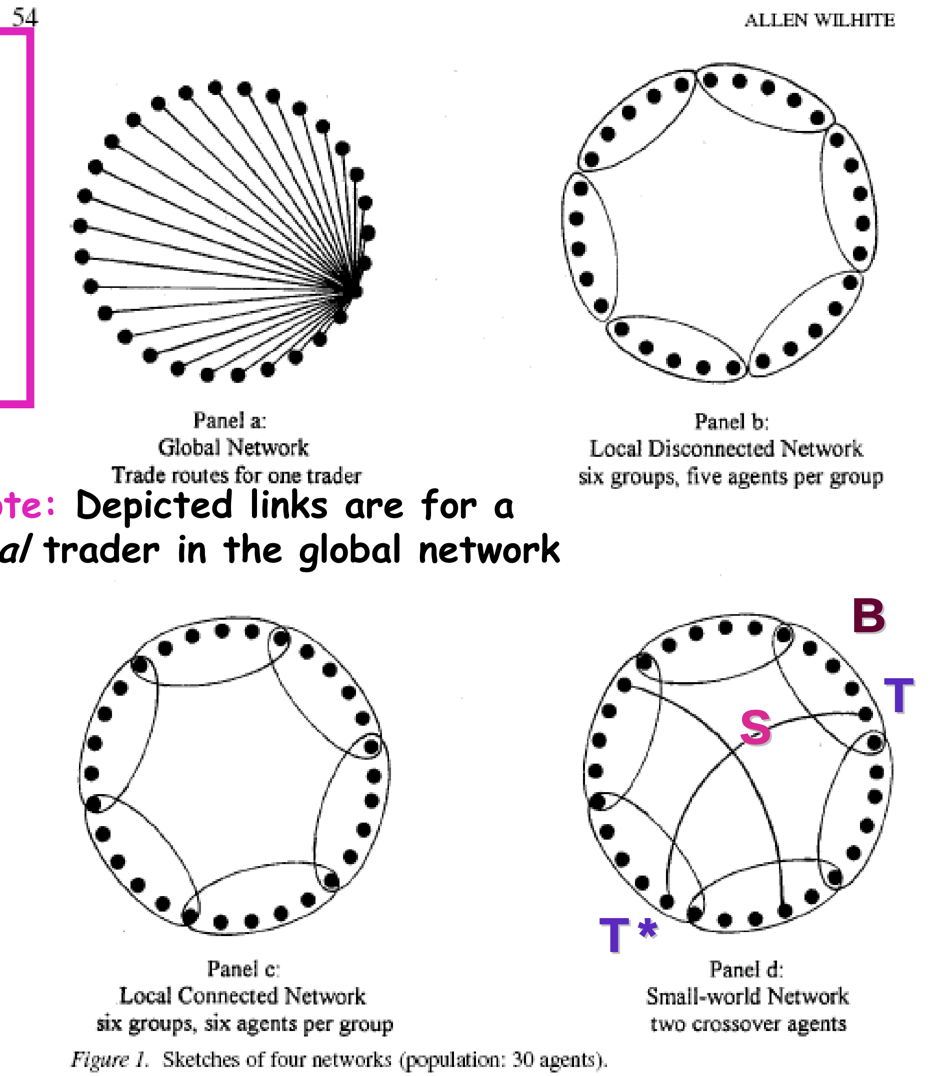


Figure 1. Sketches of four networks (population: 30 agents).

Note: Depicted links are for a *typical* trader in the global network

NOTE: The short-cut **S** adds one more trader **T*** to group **B**, but only trader **T** in **B** can directly trade with **T***.

Key Questions Examined

H1: Price convergence: Is there a significant difference in the dispersion of prices across each trade network?

H2: Speed of convergence: Do the different trade networks require a significantly different number of rounds of trading to reach their steady state?

H3: Number of trades: Is there a difference in the number of trades it takes for each network to reach its steady state?

H4: Search: Is there a difference in the amount of search and negotiation in each trade network?

Key Experimental Findings

Note: *Round* = One pass through all traders as initiators of trades
***Equilibrium* = No more mutually beneficial trade opportunities**

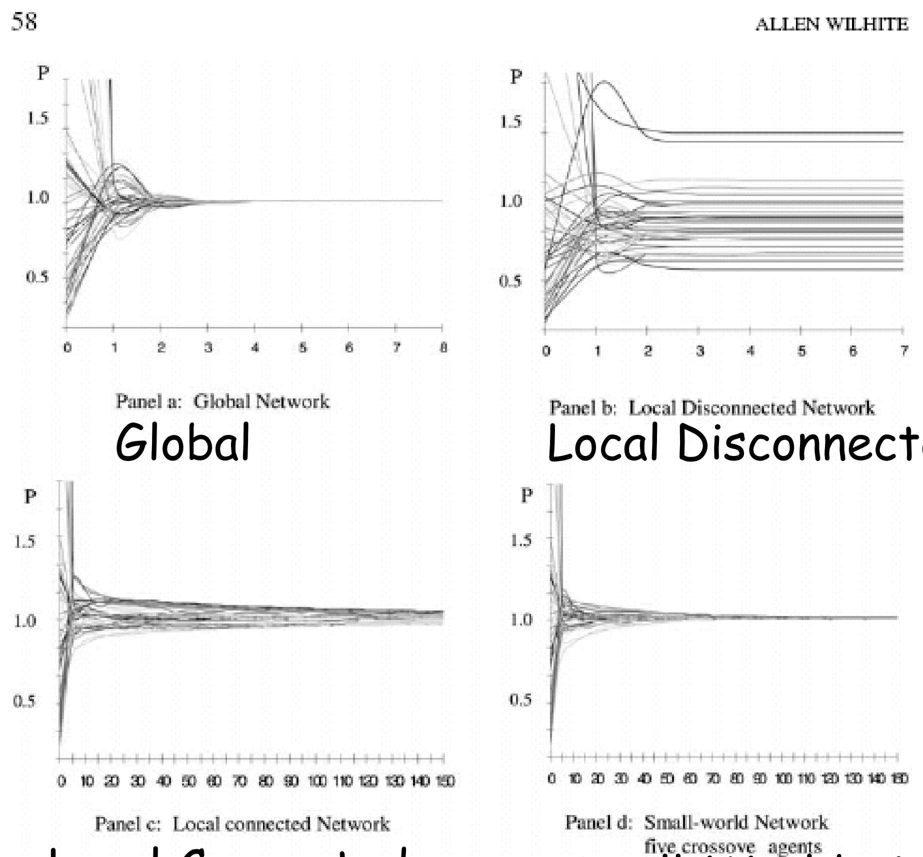
Table I. Average equilibrium characteristics.[a]

	Prices (standard deviation)	Rounds	Total trades	Total searches
Global network	1.0046 (0.00168)	8.08	1953.38	2,015,960*
Local disconnected network	1.0396 (0.2771)	7.02	1727.7	31,590
Local connected network	1.0048 (0.0146)	497.14	93,975.72	2,734,270
Small-world network	1.0045 (0.00724)	242.54	45,944.56	1,236,954

[a] Averages calculated from 50 simulations of each network configuration.

* 2,015,960 = [500 x 499 x 8.08] = 500 traders seeking best price from 499 other traders for 8.08 rounds

Price Convergence in the Four Networks



rounds

Global

Local Disconnected

rounds

Local Connected

Small-World with
5 Crossover Traders

Price Convergence for Different Numbers of Crossover Traders

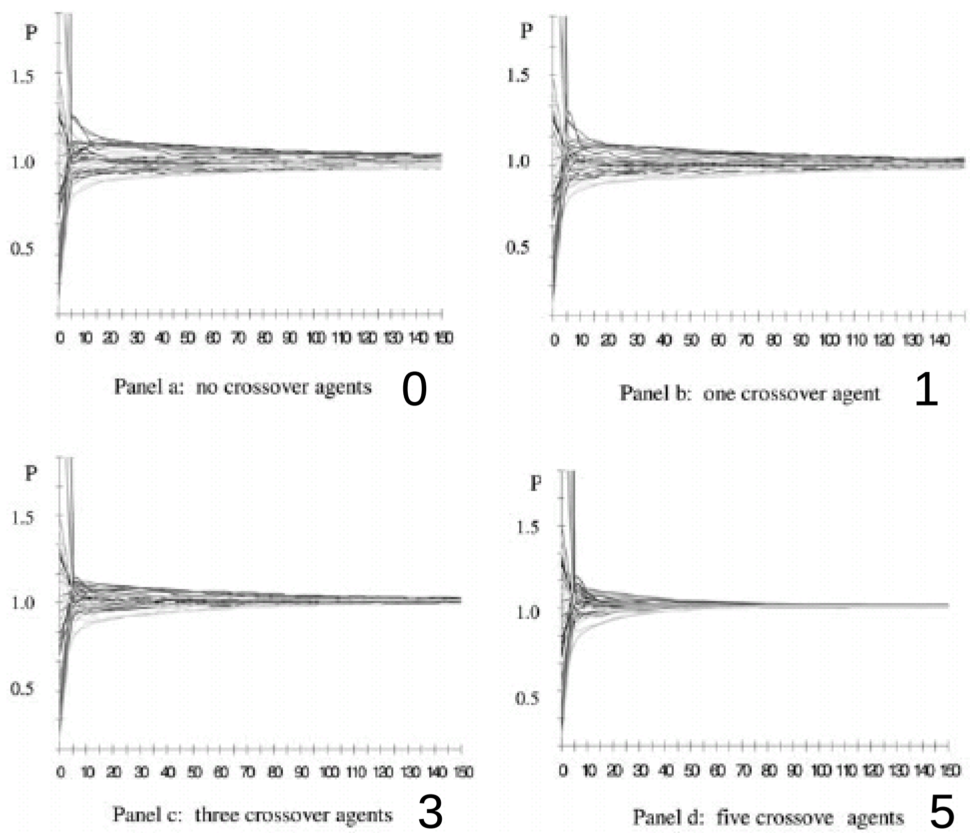


Figure 3. Dynamic price adjustment: adding crossover agents.

Relative Wealth of Crossover Traders

Table III. Wealth of crossover agents.[a]

	Agent *A*	Agent *B*	Agent *C*	Agent *D*	Agent *E*
0 crossovers	1929.61	887.73	1267.45	670.246	2059.95
1 crossover	**1997.17**	874.167	1269.23	675.557	2059.83
2 crossovers	1990.07	**1385.97**	1267.61	677.555	2058
3 crossovers	1984.94	1388.68	**1318.54**	677.591	2062.74
4 crossovers	1991.28	1389.3	1308.88	**985.451**	2066.52
5 crossovers	1981.82	1376.47	1307.04	981.252	**2105.27**

[a] The boldfaced number is the point at which that particular agent established a bridge to another trade group. Wealth from population #23, seed number 18847.

Summary of Key Findings

- The small-world trade network with 5 crossover traders provides most of the market-efficiency advantages of the completely connected trade network while retaining most of the transaction cost economies of the locally connected trade network.
- Wilhite's findings also suggest that there ***might*** exist private micro-level incentives for the ***formation*** of small-world trade networks.
- Specifically, the traders in the locally connected network who become crossover traders in the small-world trade network tend to amass greater wealth.

Extensions?

- For economic-social networks it is not satisfactory to focus solely on the implications of a fixed or parametrically varied network structure.
- Feedback mechanisms at work in economic-social networks can result in endogenous changes in the network structure over time.
- **EXAMPLE:** Network structure (whom you trade with) can affect how you behave in trading relationships, which in turn can affect whom you trade with in the future (***e.g. choice and refusal of trade partners***).