Problem Set #1: Mathematics and Calculus Primer

Name: ____

We will model economic phenomenon using functional relationships. Single variable calculus, algebra and basic geometry skill will be important. I assume that you have successfully completed the class prerequisites and can work simple mathematical problems. The following is a brief review of some important concepts.

Greek letters will often be used to denote a parameter. Common Greek letters include:

α = alpha	θ= theta	λ= lambda
β = beta	σ= sigma	π= pi

EXPONENT AND OTHER RULES

Rule	Examples
$x^n = x$ multiplied by itself n times.	$x^3 = x{\cdot}x{\cdot}x$, $x^1 = x$
$x^{-a} = 1/x^a$	$x^{-2} = 1/x^2$
$x^a x^b = x^{a+b}$	$x^2x^3 = x^5$, $x^2x^{-1} = x$
$x^a/x^b = x^{a-b}$	$x^2/x^3 = x^{-1} = 1/x$
$x^{1/a}$ = the $a'th$ root of x	$x^{1/2} = \sqrt{x}$
$x^a y^a = (xy)^a$	$x^2y^2 = (xy)^2$
$(xa)b = xab$	$2(3x) = 6x$
$b(x-y) = bx - by$	$-2(6-2x) = -12 + 4x$

FUNCTIONS

The equation $y = 3x - 5$ associates a number y with a number x. This relation is a function, which may also be thought of as a mapping from the domain of the function, x, to the range of the function, y. We can write a general functional relation $f(x)$, where f is some mathematical formula that tells us what value of y corresponds to a value x.

1a. Calculate the value of the function $f(x) = x^2 - 3x + 1$ when $x = 0$.

1b. Calculate the value of the function $f(x) = x^2 - 3x + 1$ when $x = -1$.

1c. Calculate the value of the function $f(x) = x^2 - 3x + 1$ when $x = \sqrt{3}$.

2a. Solve for the value of x for which $f(x) = 0.5x - 12 = 20$.

2b. Suppose $f(x) = 0.5x - 2$. Solve for the values of x for which $f(x) = f(x)^2$. HINT: You will need to use the quadratic formula to solve this problem; if $ax^2 + bx + c = 0$, then $x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

3. Suppose $f(x)/x = 4$ where $f(x) = ax - b$. Solve for x.

We will often work with functions of several variables; $y = f(x_1, x_2, ..., x_n)$, where x_i is the $i'th$ function argument and n is an integer. This function can also be thought of as a mapping; if we provide the ingredients $(x_1, x_2, ..., x_n)$ then f determines a value for y.

Consider the following function: $f(x_1, x_2) = x_1 + 2x_2 - (x_1 x_2)^{1/2}$.

4a. Evaluate this function at $x_1 = x_2 = 5$.

4b. Evaluate this function at $x_1 = 4$, and $x_2 = 0$.

4c. Evaluate this function at $x_1 = 3^a$, and $x_2 = 5^a$.

PARTIAL DERIVATIVES

In economics, we often want to know how changes in one economic variable lead to changes in another. For example, how much additional corn will be produced per acre (all else equal, including weather) when one more hour of labor services are applied to the acre? If the relationship between inputs and outputs can be represented with a function then calculus can be used through the use of a partial derivative. For example, the partial derivative $\partial y/\partial x_1$ (oftentimes denoted as f_1 or f_{x_1}) measures the change in $y = f(x_1, x_2, ..., x_n)$ due to an infinitesimal increase in x_1, holding $(x_2, ..., x_n)$ fixed. This is the foundation of various marginal concepts in economics.

CALCULATIONS OF DERIVATIVES

Function	Derivative	Rule
$f(x) = a$	$\partial f(x)/\partial x = 0$	constant rule
$f(x) = ax^n$	$\partial f(x)/\partial x = nax^{n-1}$	power rule
$f(x) = x$	$\partial f(x)/\partial x = 1$	
$f(x) = u(x)v(x)$	$\partial f(x)/\partial x = \partial u(x)/\partial x \cdot v(x) + \partial v(x)/\partial x \cdot u(x)$	product rule
$f(x) = u(x) + v(x)$	$\partial f(x)/\partial x = \partial u(x)/\partial x + \partial v(x)/\partial x$	
$f(x) = u(v(x))$	$\partial f(x)/\partial x = \partial u(x)/\partial v \cdot \partial v(x)/\partial x$	chain rule

5. Suppose $f(x) = x^2 - 3x + 1$. Calculate $\partial f(x)/\partial x$ and then evaluate this derivative at $x = 0$.

6. Suppose $f(x) = x^3 - 2x^2$. Calculate $\partial^2 f(x)/\partial x^2$ and then evaluate this second derivative at $x = 1$.

7. Suppose $f(x_1, x_2) = x_1 + 2x_2 - (x_1x_2)^{1/2}$. Calculate the first partial derivatives of this function: $\partial f(x_1, x_2)/\partial x_1$ and $\partial f(x_1, x_2)/\partial x_2$.

8. For the same function, $f(x_1, x_2) = x_1 + 2x_2 - (x_1 x_2)^{1/2}$, calculate the second partial derivatives: $\partial^2 f(x_1, x_2)/\partial x_1^2$, $\partial^2 f(x_1, x_2)/\partial x_1 \partial x_2$, and $\partial^2 f(x_1, x_2)/\partial x_2^2$.

9. Draw a graph of the function $f(x) = 3 + 3x - \frac{1}{3}x^2$ for $x \in [0, 10]$.

10. Calculate the first derivative of the function, $f(x) = 3 + 3x - \frac{1}{3}x^2$, set it equal to zero and solve for the corresponding value of x. Carefully explain how you have used calculus to find the value of x that maximizes $f(x)$.

11. Consider the function $y = 3z^{b-1}$. Calculate the first partial derivatives of this function with respect to z. Assuming $z > 0$, for what values of b is the first partial derivative strictly positive?