

Economics 335
Problem Set 3

NAME: _____

Profit-maximizing input choices

1. Consider the following production function for a firm using two inputs x_1 and x_2 ,

$$q = 20x_1 + 14x_2 - 2x_1^2 + 2x_1x_2 - x_2^2,$$

where q denotes the quantity of output that is produced.

(1.a) What is the marginal (physical) product of x_1 ?

(1.b) What is the marginal (physical) product of x_2 ?

(1.c) Does an increase in the quantity of x_2 increase or decrease the marginal product of x_1 ?
Can you provide an example of a real world production process that may exhibit this property?

(1.d) Holding x_2 constant at 20 units, graph the relationship between x_1 and output.

(1.e) Examine the shape of the curve that you have just drawn. Explain how you might use calculus to identify the quantity of x_1 that produces the **maximum** output quantity (holding x_2 at 20 units).

Assume that the price of the output is \$5 , and the prices of the inputs are $w_1 = \$10$ and $w_2 = \$20$.

(1.f) What is the value marginal product (VMP) of x_1 ?

(1.g) What is the value marginal product (VMP) of x_2 ?

(1.h) Write down an expression for the firms profits in terms of x_1 and x_2 .

(1.i) Use calculus to identify the values of x_1 and x_2 that provide the maximum amount of profit.

(1.j) Now suppose that the price of x_1 increases to $w_1 = \$30$. Re-calculate the profit maximizing quantity of x_1 and x_2 .

(1.k) How does the *optimal* x_1 change when w_1 is increased to \$30? Provide an intuitive explanation for this result.

(1.l) How does x_2 change when w_1 is increased to \$30? Provide an intuitive explanation for this result.

Profit Maximizing Output Choice

Consider the following cost function, $c(q) = 625 + 20q + q^2$.

(We will often write cost functions as functions of quantity only; costs of production also depend on input prices but since they are held constant they are suppressed)

2.a Identify the graph the following: fixed costs (FC), variable costs (VC), marginal costs (MC).

2.b Identify and graph the following: average fixed costs (AFC), average variable costs (AVC).

2.c Draw a graph of average total costs (AC).

2.d Calculate the output quantity that minimizes AFC, AVC, AC, MC.

3. Consider a firm that purchases inputs x_1, x_2, \dots, x_n and produces output q . The firm can purchase any input quantities that it likes at constant prices w_1, w_2, \dots, w_n . It can also sell any output quantity that it likes at constant price p . The objective of the firm is to select inputs and outputs to maximize its profits.

We can formulate the firm's problem in two stages. In stage one, the firm selects inputs x_1, x_2, \dots, x_n to minimize the cost of producing output q . In stage two, it selects the profit-maximizing output quantity, q .

3.a Write down a mathematical expression for the first stage cost minimization problem.

$$C(w, q) =$$

3.b Using the above cost function write down an expression for firm profits

$$\pi(p, w) =$$

3.c Holding prices constant, graph the firm revenues, costs and profits as a function of the output quantity that it produces.

3.d How might you use calculus to determine the output quantity that maximizes the firm's profits?

3.e Write down the first-order condition that determines the profit maximizing output quantity.

3.f Write down the second-order condition for the problem in part 3.e. Does the first-order condition identify a maximum or a minimum? Explain.

3.g Suppose that the cost function for the firm is $C(q) = q + .25q^2$. How much should the firm produce if the output price is $p = 9$ dollars? How much should the firm produce if the output price is $p = 11$ dollars?

3.h Now let the output price be denote simply as p . Use the first order condition for profit maximization to identify a general relation between the quantity that the firm will produce and the output price. What name do economists use to describe this relationship?