Economics 335
Monopoly

Name:

1. Consider a monopolist with the following cost and demand functions:

\[ C(Q) = 900 + 0.5Q^2 \]
\[ Q^D(P) = 120 - P \]

a. What is the marginal cost function, and at what quantity of output is marginal cost minimized?

\[ MC = \frac{dC(Q)}{dQ} = Q \]
\[ \frac{dMC}{dQ} = 1 \neq 0 \rightarrow \text{This implies that } MC = 0 \text{ at } Q = 0 \]
and as long as \( Q > 0, \) \( MC > 0 \)

b. What is the and average cost function, and at what output quantity is average cost minimized?

\[ AC = \frac{C(Q)}{Q} = \frac{900}{Q} + \frac{1}{2}Q \]
\[ \frac{dAC}{dQ} = -\frac{900}{Q^2} + \frac{1}{2} = 0 \Rightarrow Q^2 = 1800 \]
\[ Q = 30\sqrt{2} \approx 42.43 \]

c. What is the inverse demand function?

\[ P = 120 - Q^D \]
d. What is firm revenue as a function of output?

\[ \text{Revenue} = P \cdot Q = (120 - Q)Q = 120Q - Q^2 \]

e. What is marginal revenue as a function of output?

\[ MR = \frac{dR}{dQ} = 120 - 2Q \]

f. Find the socially efficient output quantity, \( Q_e \) and explain why this quantity maximizes the total surplus in the market.

Under the competitive market, a price would be socially efficient. Thus, you have to find out competitive equilibrium output (and there is no deadweight losses under competitive market).

\[ MC = P_e \]

\[ \Rightarrow Q_e = P_e = 120 - Q \Rightarrow Q_e = 60 \]

g. What is the price \( P_e \) at this level of output?

\[ P_e = 120 - Q = 120 - 60 \]

\[ P_e = 60 \]
h. What is the firm's cost at this level of output?

\[ c(60) = 900 + \frac{1}{2}(60)^2 \]

\[ = 2700 \].

i. What is the firm's profit at this level of output?

\[ \pi = R - C = 60(60) - 2700 \]

\[ = 900 \].

j. Now find the level of output \( Q_m \), that this firm will choose as a monopolist.

\[ MR = MC \]

\[ 120 - 2Q = Q \]

\[ 3Q = 120 \Rightarrow Q_m = 40 \].

k. What is the price \( p_m \) at this level of output?

\[ p_m = 120 - 40 = 80 \].
1. What is marginal cost at this level of output?

\[ MC = Q_m = 40 \]

m. What is marginal revenue at this level of output?

\[ MR = 120 - 2(40) = 40 \]

n. What is the firm's profit at this level of output?

\[ \Pi_m = (120 - (40))(40) - (900 + \frac{1}{2}(40)^2) \\
= 3200 - 1700 = 1500 \]

o. What is the firm's gain from monopoly?

From the results \( g(i) \) and \( (n) \)

\[ 1500 - 900 = 600 \]
p. Calculate the elasticity of demand $\epsilon_D = \frac{\delta Q}{\delta P} \cdot \frac{P}{Q}$ at the monopolistic output level $Q_m$.

$$\frac{\delta Q}{\delta P} = -1 \implies \epsilon_D = (-1) \left( \frac{\delta Q}{\delta P} \right) = -2.$$ 

q. Verify that $MR = p \left( 1 + \frac{1}{\epsilon_D} \right)$.

$$\delta Q \left( 1 + \frac{1}{-2} \right) = \delta Q \left( \frac{1}{2} \right) = 40 = MR.$$ 

r. Use the above expression for $MR$ and the fact that the monopolist maximizes profit by choosing the quantity at which $MR = MC$ to show that the *price-cost margin*, i.e., the wedge between the price in the market and the marginal cost of production will tend to zero as the demand curve becomes perfectly elastic.

If demand is perfectly elastic, $\epsilon_D = -\infty$.

$$MR = \lim_{\epsilon_D \to -\infty} p \left( 1 + \frac{1}{\epsilon_D} \right) = p$$

Since $MR = MC$, $MC = p$.

Thus $p - MC = 0$. 

5
2. The following figure is a graphical representation of problem 1 above:

![Graph of Monopoly Market Outcome]

**Figure 1: Monopoly Market Outcome**

a. Identify and calculate the area representing the consumer surplus under the efficient output $Q_c$.

\[ CSS_c = A + B + C + D + E. \]

\[ CSS = \frac{1}{2}(60)(60) = 1800. \]

b. Identify and calculate the area representing the consumer surplus under monopoly.

\[ CSS_m = A + B. \]

\[ = \frac{1}{2}(40)(40) = 800. \]

c. What is the loss to consumers due to the monopoly?

\[ \text{Loss of Consumers is} \quad C + D + E. \]

\[ = 7 \times 1800 - 800 = 1000. \]
d. Now consider the profits of the firm. The firm gains the area C+D due to the higher price, and it loses the area H due to lower quantity sold in the market. Calculate the area C+D.

\[(20)(40) = 800\]

Calculate the area H.

\[\frac{1}{2}(20)(20) = 200\]

What is the firm's gain from monopoly?

\[800 - 200 = 600\]

e. The dead-weight loss in a market can be calculated as the foregone total surplus (lost consumer and producer surplus) from not producing the socially inefficient output quantity. Fill in the following table and calculate the foregone surplus due to monopoly?

<table>
<thead>
<tr>
<th></th>
<th>Monopoly Market</th>
<th>Efficient Market</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Surplus</td>
<td>800</td>
<td>1800</td>
<td>-1000</td>
</tr>
<tr>
<td>Producer Surplus</td>
<td>2400</td>
<td>3800</td>
<td>+600</td>
</tr>
<tr>
<td>Total Surplus</td>
<td>3200</td>
<td>3600</td>
<td>-400</td>
</tr>
</tbody>
</table>

\[DWL = 400\]

f. One can also calculate this deadweight loss directly from the graph. Identify the area that represents the dead-weight loss, and calculate the area.

\[DWL = E + H\]

\[= \frac{1}{2}(40)(20) = 400\]