Problem Set # 6

Key 1

1.a

$$\pi = 100(20x_1 + 15x_2 - x_1^2 - x_2^2) - 600x_1 - 100x_2$$

1.b

$$\frac{\partial \pi}{\partial x_1} = 100(20 - 2x_1) - 600 = 0$$

$$\Rightarrow 2x_1 = 20 - 6$$

$$\boxed{x_1^* = 7}$$

$$\frac{\partial \pi}{\partial x_2} = 100(15 - 2x_2) - 100 = 0$$

$$\Rightarrow 2x_2 = 15 - 1$$

$$\boxed{x_2^* = 7}$$

$$q^* = 20(7) + 15(7) - 7^2 - 7^2$$

$$= 140 + 105 - 49 - 49$$

$$\boxed{q^* = 147}$$

$$\Pi^* = 100 \cdot 147 - 600(7) - 100(7)$$

$$= 14700 - 4200 - 700$$

$$\boxed{\Pi^* = 9800}$$

1.c

$$\Pi = 100(20x_1 + 15x_2 - x_1^2 - x_2^2) - (200 + 50x_1)x_1 - 100x_2$$

1.d

$$\frac{\partial \Pi}{\partial x_1} = 100(20 - 2x_1) - 200 - 100x_1 = 0$$

$$\Rightarrow \quad 20 - 2x_1 - 2 - x_1 = 0$$

$$\boxed{x_1^* = 6}$$

$$\frac{\partial \Pi}{\partial x_2} = 100(15 - 2x_2) - 100 = 0$$

$$\boxed{x_2^* = 7}$$

$$q^* = 20(6) + 15(7) - 6^2 - 7^2$$

$$= 120 + 105 - 36 - 49$$

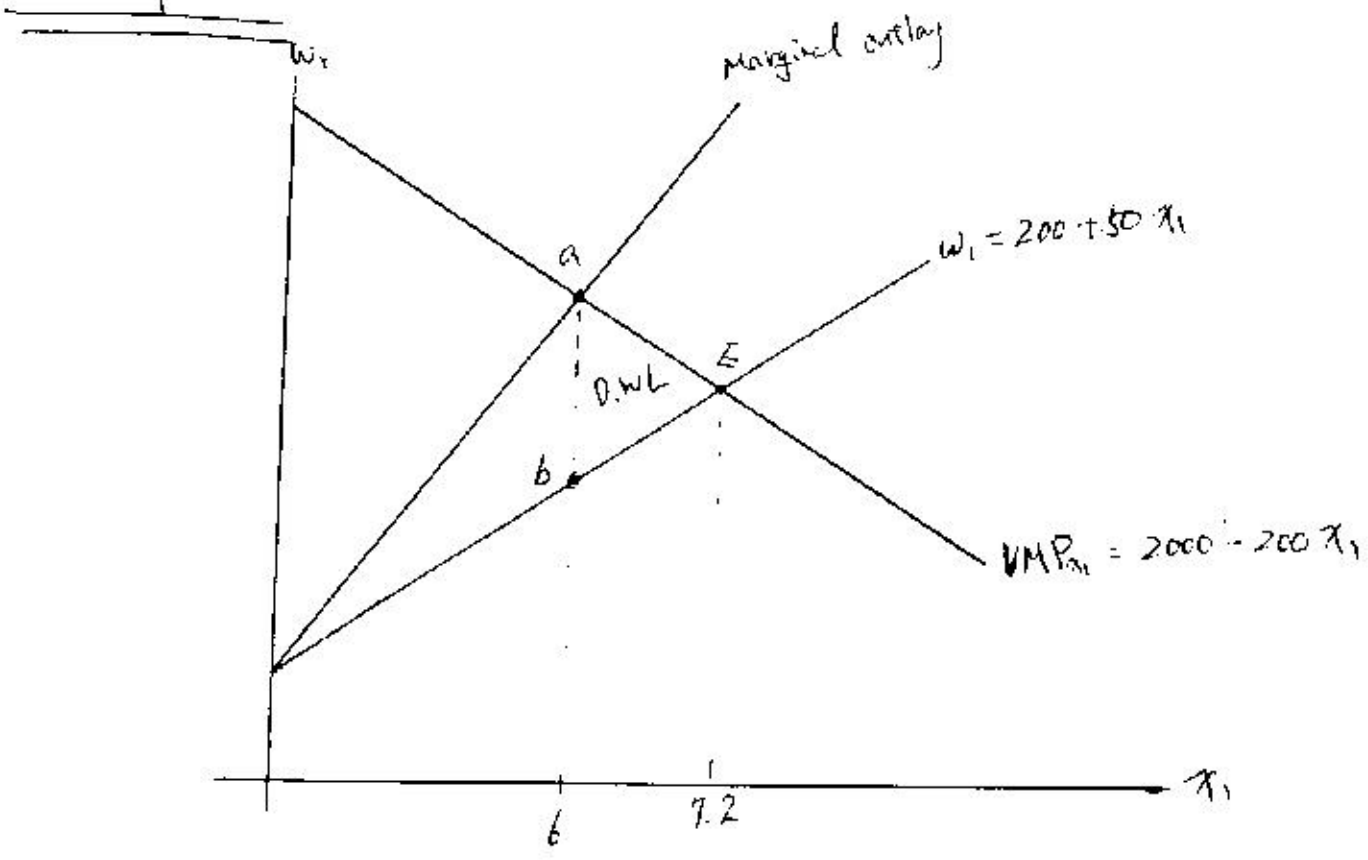
$$\boxed{q^* = 140}$$

$$\pi^* = 100(140) - (200 + 50(6))(6) - 100(7)$$

$$= 14000 - 3000 - 700$$

$$\boxed{\pi^* = 10300}$$

- Graph



1.e Competitive equilibrium will give the efficient result and to find out efficient level of x_1, first equate demand and supply of x_1

$$\Rightarrow \quad VMP_{x_1} = w_1 = 2000 - 200x_1 = 200 + 50x_1$$

$$\Rightarrow \quad 1800 = 250x_1$$

$$\boxed{x_1^* = \frac{36}{5} \text{ or } 7.2}$$

Obviously at this quantity level, wage is equal to VMP_{x_1}; that is, wage reflects correct value of labor productivity. On the other hand, under the monopsony case, market power creates distortion and $w_1 (\neq VMP_{x_1})$ is set at too low. This results in (causing) DWL

1.f Efficient quantity of x_2^* is 7 Thus,

$$q^* = (20(7.2) + 15(7) - (7.2)^2 - 7^2)$$

$$\boxed{q^* = 148.16}$$

$w_1 = 200 + 50(7.2) = 560$

$\pi^* = 100(148.16) - 560(7.2) - 100(7)$

$= 10084 < 10300$

Thus the profit is lower than in monopsony case.

1.g Obviously, DWL is from only input (x_1) market.

From the efficient MKT outcome,

$x_1^c = 7.2$, $w_1 = 560$

* See graph on page 3

Now consider Monopsony case.

$x_1^m = 6$. $VMP_{x_1} = 800$, $w_1 = 500$

$\Rightarrow \Delta x_1 = x_1^c - x_1^m = 1.2$

$\Rightarrow$ distance b/w a and b $= VMP_{x_1} - w_1$

$= 300$

$\Rightarrow$ **D.W.L $= 300(1.2)\left(\frac{1}{2}\right) = 180$**

2.a

$$\pi^B = p^B q^B - w^B x_B$$

$$= 2(10x_B + 2x_T - x_B^2) - 4x_B$$

$$\frac{\partial \pi^B}{\partial x_B} = 2(10 - 2x_B) - 4 = 0$$

$$2x_B = 10 - 2$$

$$\boxed{x_B^* = 4}$$

2.b

$$\pi^A = p^A q^A - w^T x_T$$

$$= 10\left(7x_T + \frac{1}{2}x_B - \frac{1}{2}x_T^2\right) - 5x_T$$

$$\frac{\partial \pi^A}{\partial q^A} = 10(7 - x_T) - 5 = 0$$

$$\boxed{x_T^* = 6.5}$$

2. c

To find out the socially optimal number, you have ~~to~~ consider both profit functions simultaneously

$$\Rightarrow \max \pi^T + \pi^A \quad (= \pi^S)$$

or

$$2(10x_B + 2x_T - x_B^2) - 4x_B + 10(7x_T + \tfrac{1}{2}x_B - \tfrac{1}{2}x_T^2) - 5x_T$$

$$\frac{\partial \pi^S}{\partial x_B} = 2(10 - 2x_B) - 4 + 10\left(\tfrac{1}{2}\right) = 0$$

$$\Rightarrow 20 - 4x_B - 4 + 5 = 0$$

$$4x_B = 21$$

$$\boxed{x_B^* = \frac{21}{4} \text{ or } 5.25}$$

$$\frac{\partial \pi^S}{\partial x_T} = 4 + 10(7 - x_T) - 5 = 0$$

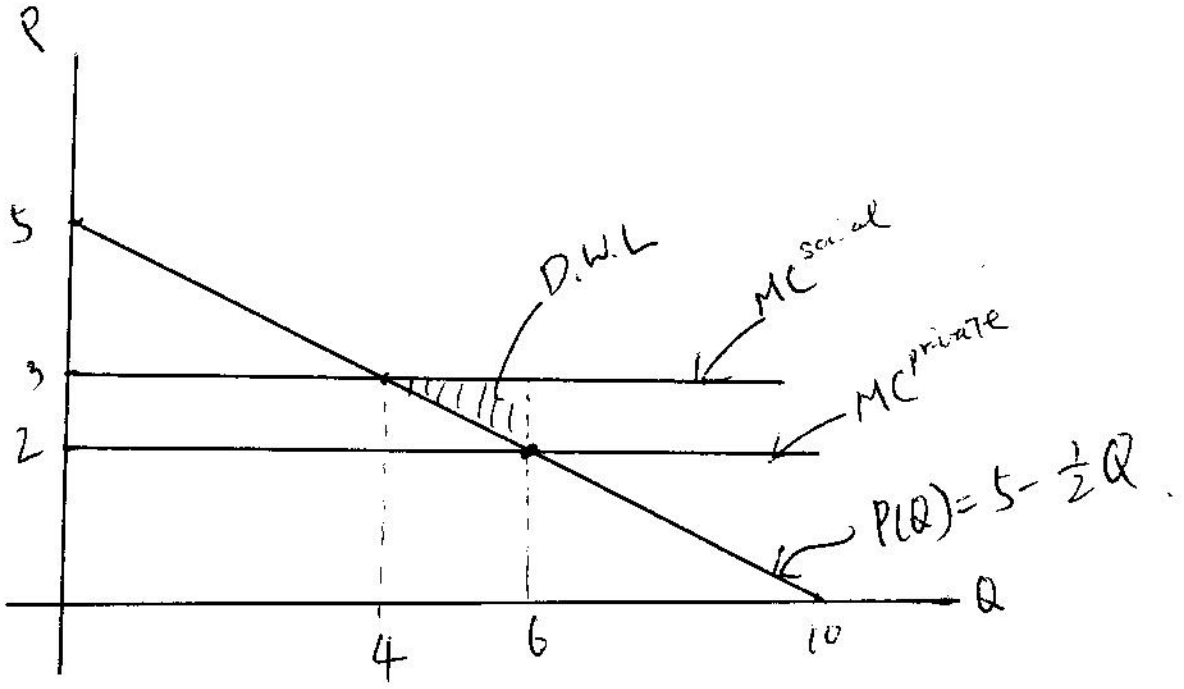
$$\Rightarrow 10x_T = 69$$

$$\boxed{x_T^* = 6.9}$$

2.d as you can see from production functions, there are positive externalities.

Under the individual firm's profit maximization, firm will make any decision with ignoring this positive externality, which would result in "under-employment" of input. On the other hand, in Social optimal problem, all externalities are taken into account. Thus, there is no under-(or over-) employment. Therefore, the outcomes are different b/w those two cases.

3.a



3.b

Competitive equilibrium

$P = MC_{private}$

$2 = 5 - \frac{1}{2}Q$

$Q^C = 6$

$P^C = 2$

Social efficient outcome

$P = MC_{social}$

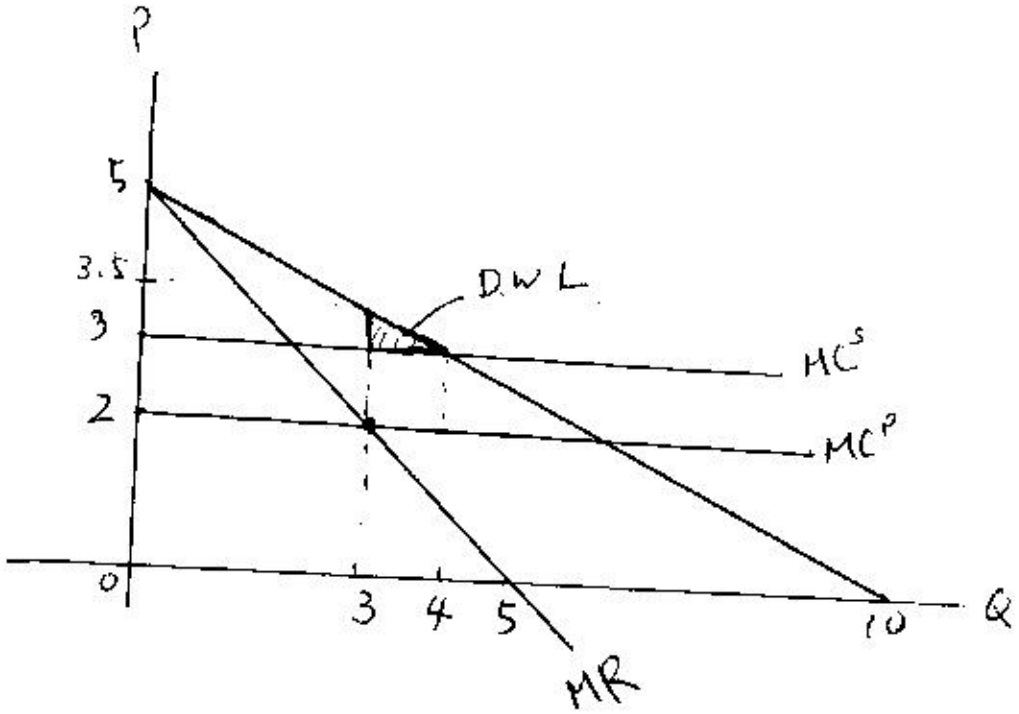
$3 = 5 - \frac{1}{2}Q$

$Q^S = 4$

$P^S = 3$

$DWL = \frac{1}{2}(1)(2) = 1$

3.c



3.d $MR = MC$

$5 - Q = 2$

$Q^m = 3$

$P^m = 5 - \frac{3}{2} = \frac{7}{2}$ or 3.5

$D.W.L = \left(\frac{1}{2}\right)(1)\left(\frac{1}{2}\right) = \frac{1}{4}$ or 0.25

3.e

Monopoly: The dead weight loss to Society is less

Because firms don't take into account the cost of pollution, the perfectly competitive market produces "too much" relative to the efficient solution.

Since monopoly restricts output, the output is closer to the socially efficient solution and socially it would be beneficial