Name: 

Consider the following production function for a firm using two inputs $x_1$ and $x_2$:

$$q = 20x_1 + 15x_2 - x_1^2 - x_2^2$$

where $q$ denotes output. Assume that the price of output is $100.00 and the prices of inputs are $w_1 = $600 and $w_2 = $100.

1.a Write down an expression for the firm’s profit as a function of $x_1$ and $x_2$.

1.b Calculate the profit maximizing quantity of $x_1$ and $x_2$, the optimal output quantity and the profit for the firm.

1.c Now assume that the firm is monopsonist in the purchase of input 1. The price of input 1 depends on the quantity of $x_1$ that is purchased. Suppose that $w_1 = 200 + 50x_1$. Write down an expression for the firm’s profit as a function of $x_1$ and $x_2$. 
1.d Calculate the profit maximizing levels of $x_1$, $x_2$, $q$, and determine the profits of the firm.

We will now compare the monopsonistic solution to the *socially efficient* market outcome. Holding $w_2$ constant a $100, draw a diagram that depicts the price, i.e., $w_1 = 200 + 50x_1$, the marginal outlay and the value marginal product of the input $x_1$.

1.e What is the socially efficient quantity of $x_1$? Explain intuitively why this is the socially efficient quantity.
1.f What output quantity and profit is achieved under the socially efficient input quantity? Verify that profit is lower than in the monopsonistic outcome.

1.g What is the dead weight loss due to monopsony?

Suppose that a honey producer and an apple orchardist own adjacent farms. The production function for honey, \( q^H \) is given by
\[
q^H = 10x_B + 2x_T - x_B^2.
\]
where \( x_B \) denotes the quantity of bee hives that are kept on the honey farm, and \( x_T \) denotes the acres of apple trees that are maintained on the apple orchard. The production function for apples, \( q^A \), is given by
\[
q^A = 7x_T + .5x_B - .5x_T^2.
\]
Suppose that price of honey is \( P^H = $2 \) per unit; the price of bees is \( w^B = $4 \); the price of apples is \( P^A = $10 \); the price per acre of apples trees is \( w^T = $5 \). Suppose that the bee-keeper and the apple orchardist operate independently. Each chooses the private profit maximizing input choice.

2.a Write down an expression for the profits of the bee-keeper, and determine the privately optimal quantity of hives that are maintained.
2.b Write down an expression for the profits of the apple orchardist, and determine the acres of trees on the orchard.

2.c Now calculate the socially optimal number of bee hives and acres of apple trees.
2.d Explain intuitively why the privately optimal and the socially optimal outcomes differ in this problem.

Consider an industry that produces a valued good along with pollution. The inverse demand for the good is \( P(Q) = 5 - .5Q \). The private cost of producing the good (e.g., labor, energy and capital costs) is \( c^P(Q) = 2Q \). The total social cost is the private cost plus the environmental cost and is equal to \( c^{P+E}(Q) = 3Q \).

3.a Draw a graph depicting the demand, the private marginal cost and the social marginal cost of this good.
3.b Calculate the privately optimal market outcome (quantity supplied and market price), the socially efficient market outcome and the dead weight loss due to the externality.

3.c Suppose that this good is produced by a monopolist. Draw a diagram depicting the market outcome under monopoly.
3.d Calculate the monopoly market outcome (price and quantity), and the dead weight loss in this market.

3.e Is society better off under a monopoly or perfect competition in this particular market? Carefully explain your answer.