

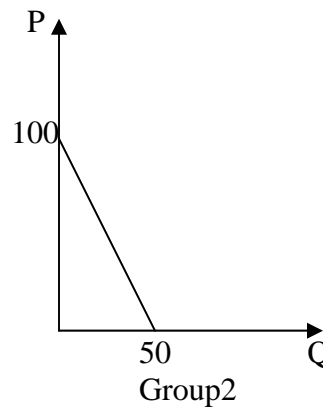
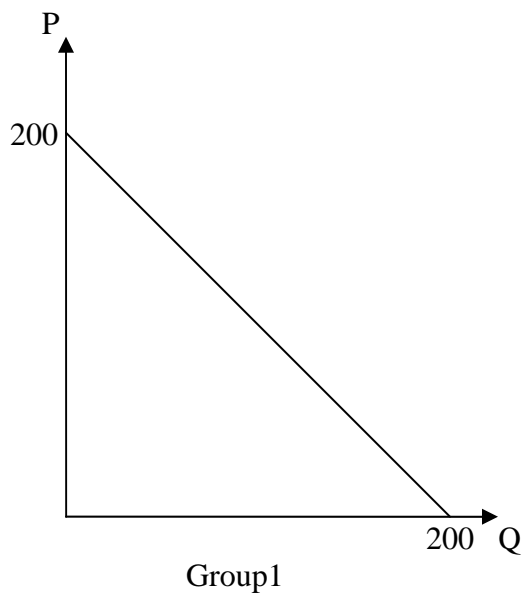
Economics 335
Price Discrimination
Answer Key

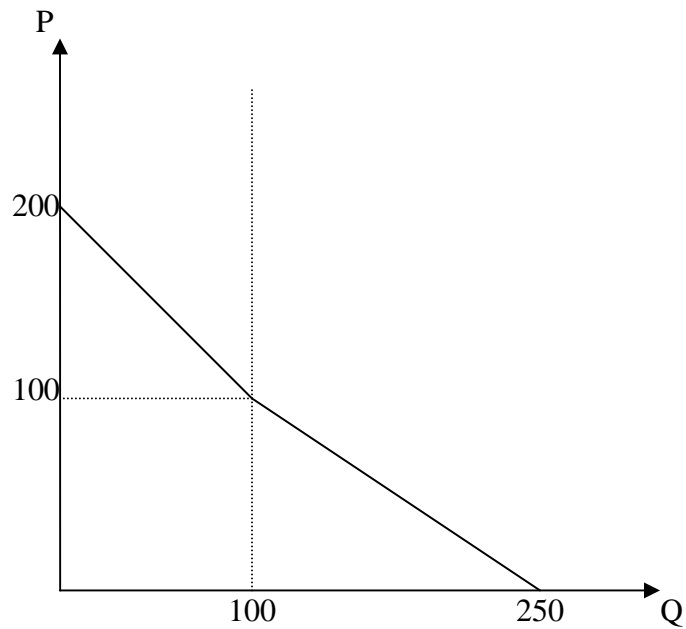
#1

(a) (page 1) You have to show with the aid of graphs that the monopolist's total demand is given by

$$Q = Q_1 + Q_2 = \begin{cases} 0 & \text{if } P \geq 200 \\ 200 - P & \text{if } 100 \leq P \leq 200 \\ 250 - 1.5P & \text{if } 0 \leq P \leq 100 \end{cases}$$

The following figures show the demand for each group and total demand.





(b) Derive the monopolist's revenue and marginal revenue as function of Q .

The revenue is price times quantity, therefore,

$$R = P \cdot Q = \begin{cases} 0 & \text{if } Q \geq 250 \\ 166\left(\frac{2}{3}\right)Q - \frac{2}{3}Q^2 & \text{if } 100 \leq Q \leq 250 \\ 200Q - Q^2 & \text{if } Q \leq 100 \end{cases}$$

and the marginal revenue is derivative of revenue function with respect to Q

$$MR = \frac{\partial R}{\partial Q} = \begin{cases} 0 & \text{if } Q \geq 250 \\ 166\left(\frac{2}{3}\right) - \frac{4}{3}Q & \text{if } 100 \leq Q \leq 250 \\ 200 - 2Q & \text{if } Q \leq 100 \end{cases}$$

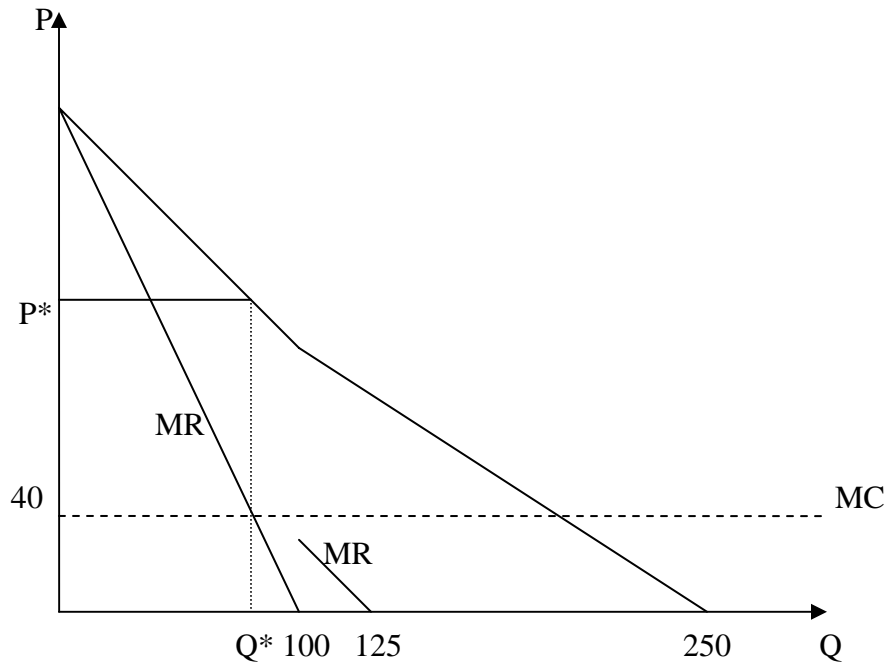


Figure 1: Market Demand with Two Consumer Groups

(a) (page 2) Draw the firm's marginal cost curve and indicate the price and quantity at which the monopolist maximization profits.

Since the firm's cost function is $C(Q) = 40Q$, thus marginal cost would be 40. By using this information, MR and Demand function from previous question, you can find that $P^* = 120$ and $Q^* = 80$ (see also above figure).

(b) Given $P^* = 120$ and $Q^* = 80$, monopolist profit is

$$\pi = (P - 40)Q = (80)(80) = 6400$$

Consumer surplus for each group? It is obvious that there is no consumer surplus for group2 because price is too high for them to enter the market. Thus, only need to care is group1.

$$CS(\text{group1}) = (200 - 120) \cdot 80 \cdot \left(\frac{1}{2}\right) = 3200$$

By adding all surplus, CS and profits, the total social surplus would be

$$\text{Total Social Surplus} = 6400 + 3200 = 9600$$

(c) In this question, we assume that the monopolist can distinguish between the two group of consumers; that is, monopolist can charge each group a different price.

The MR of group1 is

$$MR_1 = 200 - 2Q_1$$

by equating $MC = MR_1$, you will find

$$40 = 200 - 2Q_1$$

$$Q_1 = 80$$

$$P_1 = 120$$

Similarly, do the same thing for group2,

$$MR_2 = 100 - 4Q_2$$

$$40 = 100 - 4Q_2$$

$$Q_2 = 15$$

$$P_2 = 70$$

(d) Given prices calculated above, find out profit from each market, total profit, each group's CS, and total social surplus.

$$\pi_1 = (120 - 40) \cdot 80 = 6400$$

$$\pi_2 = (70 - 40) \cdot 15 = 450$$

$$\pi = \pi_1 + \pi_2 = 6400 + 450 = 6850$$

$$CS_1 = (200 - 120) \cdot 80 \cdot \left(\frac{1}{2}\right) = 3200$$

$$CS_2 = (100 - 70) \cdot 15 \cdot \left(\frac{1}{2}\right) = 225$$

$$\text{Total Social Surplus} = 6850 + 3200 + 225 = 10275$$

(e)

- Who gains? Group2 and Monopolist (but not group1): Both CS_2 and monopolist's profits increase. But, since CS_1 does not change, so there is no gain or loss for group1.

- Who lose? No one

- How do they compare in terms of efficiency? Price discrimination because it creates higher social surplus.

#2

$$\text{Daytime } Q_D = 6 - P_D$$

$$\text{Evening } Q_E = 8 - P_E$$

$$MC = 4$$

(a)

i) What is the profit maximizing price and quantity for daytime and evening movies?

For daytime, the MR is

$$MR_D = 6 - 2Q_D$$

and

$$MC = MR_D$$

$$4 = 6 - 2Q_D$$

$$Q_D = 1$$

$$P_D = 5$$

Similarly, for evening,

$$MR_E = 8 - 2Q_E$$

$$MC = MR_E$$

$$4 = 8 - 2Q_E$$

$$Q_E = 2$$

$$P_E = 6$$

ii) What is the price elasticity of demand in each market at these profit maximizing prices?

$$\frac{\partial Q_D}{\partial P_D} = -1 = \frac{\partial Q_E}{\partial P_E}$$
$$\Rightarrow \varepsilon_D = (-1) \left(\frac{P_D}{Q_D} \right) = -5$$
$$\varepsilon_E = (-1) \left(\frac{P_E}{Q_E} \right) = -3$$

iii)

-Verify that $\frac{P_D}{P_E} = \frac{1 + \frac{1}{\varepsilon_E}}{1 + \frac{1}{\varepsilon_D}}$ hold in this case

Right hand side is

$$\frac{P_D}{P_E} = \frac{5}{6}$$

and left hand side is

$$\frac{1 + \frac{1}{\varepsilon_E}}{1 + \frac{1}{\varepsilon_D}} = \frac{1 - \frac{1}{3}}{1 - \frac{1}{5}} = \frac{10}{12} = \frac{5}{6}$$

- Given the elasticities you have just calculated, how would you interpret the fact that evening prices are higher and daytime prices are lower?

Since $|\varepsilon_D| > |\varepsilon_E|$ implies that given price changes, the change in quantity is relatively larger in market for daytime than market for evening. That is, even if monopolist raise the price for evening, since there is only few change in quantity, one's profit will rise. Therefore, evening prices are higher than daytime prices. Overall, $|\varepsilon_D| > |\varepsilon_E|$ implies that if smaller market has the more elastic demand, it must have the lower price.

iv) How much profit does the theater manager earn from each type of consumer?

$$\pi_D = (5 - 4)(1) = 1$$

$$\pi_E = (6 - 4)(2) = 4$$

v) What is the consumer surplus of each type of consumer?

$$CS_D = (6 - 5)(1) \left(\frac{1}{2}\right) = \frac{1}{2}$$

$$CS_E = (8 - 6)(2) \left(\frac{1}{2}\right) = 2$$

vi) add the theater's profit to the consumer surplus to get the total social surplus

$$total\ social\ surplus = 1 + 4 + \frac{1}{2} + 2 = 7.5$$

(b) In this question, we are assuming that manager adopt a two-part tariff

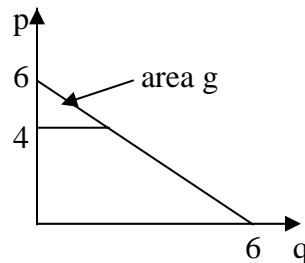
i) what is the optimal charge per movie seen?

Since marginal cost is 4 dollar thus, optimal charge should be 4.

ii) what is the optimal cover charge for consumer who come during daytime?

Basically, what manager want to do is to take all consumer surplus so that his profit increased by that amount. Thus, area g would be the cover charge

$$Cover\ Charge = 2 \cdot 2 \cdot \frac{1}{2} = 2$$



iii) Similar for evening consumers

$$\text{Cover Charge} = 4 \cdot 4 \cdot \frac{1}{2} = 8$$

iv) How much profit does the theater earn from each type of consumer?

It is obvious those cover charges would be the profit for the manager. Thus,

$$\pi_D = 2$$

$$\pi_E = 8$$

v) What is the consumer surplus of each type of consumer?

Again, manager took all the consumer surplus, thus there is no consumer surplus.

vi) Total social surplus would be $2+8=10$

(c) How do the two pricing scheme compare?

(Note: the price scheme in #1 and #2. a) is called *third degree price discrimination*)

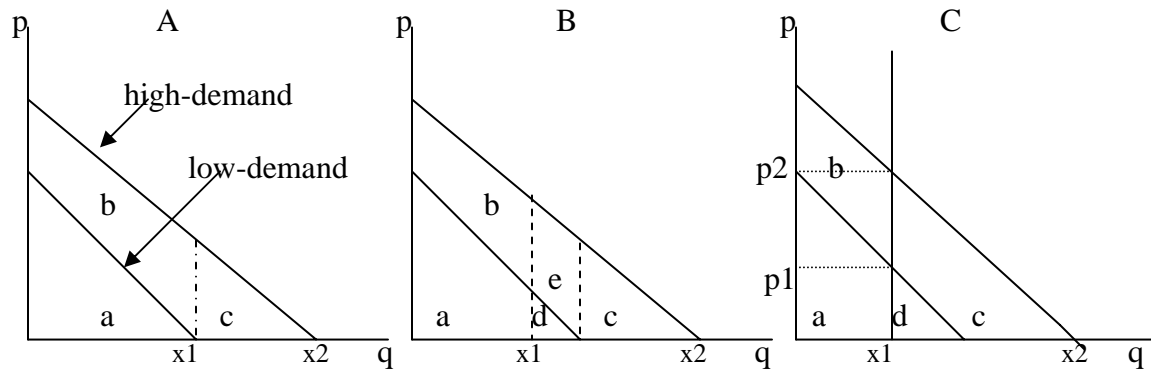
- Which is best for the theater?
Since two-part-tariff gives more profit, that price scheme is best for the theater
- Which is best for consumer?
Since there is no consumer surplus under two-part-tariff, third degree price discrimination is best.
- Which is best for efficiency?
Under the two-part-tariff, more goods are consumed (even though $CS=0$) and total social surplus is higher compared with third degree price discrimination. Thus, two-part-tariff is best for the efficiency.

#3

The best way to maximize profit is perfect price discrimination so that monopolist can take all surplus from the consumers. However, even though monopolist knows the demand for each type, one can not tell which person would be which type of consumer.

One way to solve this problem is to offer two different price-quantity packages in the market and by constructing these packages, they will give the consumers an incentive to **self select**. (This strategy is called *nonlinear pricing*)

To give the idea, let's consider the example (or see Varian, "Intermediate Microeconomics"). Suppose there are two types of consumers, A and B. Type A is high-demand consumer and type B is low-demand consumer. The figure below illustrates this situation (for simplicity, marginal cost is zero at this moment)



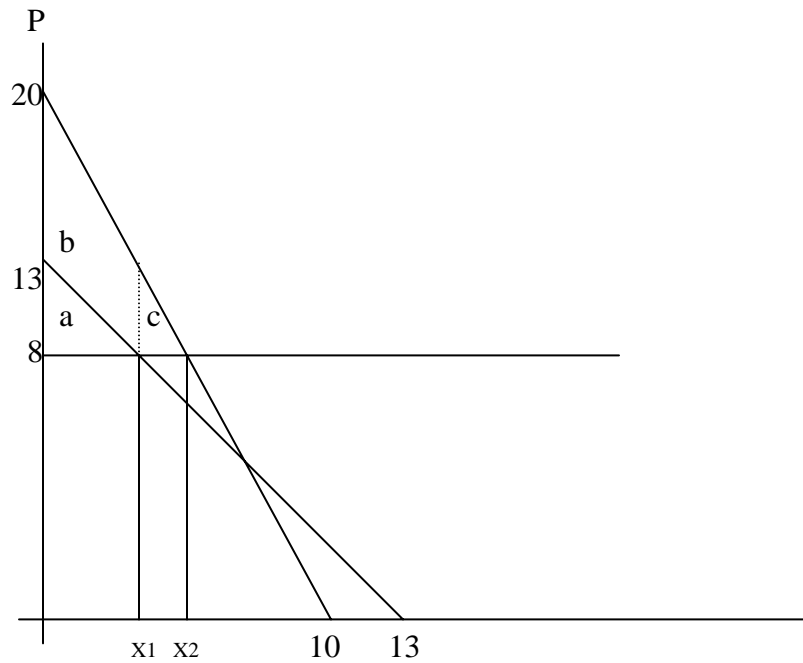
Look at the panel A. Under the perfect price discrimination, monopolist will offer x_1 at price “a” and x_2 at price “a + b + c”. However, this would not be the case and these price-quantity combinations are not compatible with self-selection. This is because high-demand consumer would find it optimal to choose the quantity x_1 and pay price “a” so that he can enjoy the consumer surplus equal to area “b”.

One thing the monopolist can do is to offer x_2 at the price “a + c”. In this case, the high-demand finds it optimal to choose x_2 and receive a surplus of “b”.

However, this is not the end of story. There is a further thing the monopolist can do to increase profits. Suppose that instead of offering x_1 at the price “a” to the low-demand consumer (type B), the monopolist offers a bit less than that at a price slightly less than “a”. This reduces the profits on type B by area “d” on panel B. But since type B’s package is now less attractive to type A, monopolist can now charge more to type A for x_2 . By reducing x_1 , the monopolist makes area “a” a little smaller but makes area “c” bigger. Then, the net result is that the monopolist’s profits increase.

Above process continues until the profit lost on type B equals the profit gained on type A (or marginal decrease in profits collected from type B equals the marginal increase in profits collected from the type A). The panel C illustrates the profit maximization solution, and in this solution, the quantity x_1 is determined where marginal decrease in profits, p_1 , must be equal to marginal increase in profits ($p_2 - p_1$).

Now, go back to our question. Since our marginal cost is 8, marginal decrease in profits would be $p_B - 8$, where subscript indicates the type of consumer. That is we have to find out the quantity which satisfy $p_B - 8 = p_A - p_B$ and $p_A, p_B \geq 8$. However, it is obvious that there is no quantity level satisfying those conditions. That is, only the monopolist can do is offer x_1 at the price “a + 8*x1” and x_2 at price “a+c+8*x2” (see figure below).



Based on each demand function $x_1=5$ and $X_2=6$ and the price (area “ $a + 8 \cdot x_1$ ”) for x_1 is $(25/2)+40=52.5$. Because area “ c ” is $(10-8) \cdot (6-5) \cdot (1/2)=1$, the price (area “ $a + c + 8 \cdot x_2$ ”) for X_2 would be $(25/2)+1+48=61.5$. Therefore, first price-quantity package would be 6kg at price 61.5 and the other is 5kg at price 52.5.

Notice under this nonlinear pricing, there is no possibility that type A pretends to be type B and purchase x_1 instead of x_2 . This is because for type A consumer, both price-quantity package will give exactly the same consumer surplus. Therefore, there is no incentive for type A consumer to behave type B.