1. A monopolist faces two types of customers, which will be referred to as group 1 and group 2 consumers. The inverse demand of group 1 consumers is given as $P_1(Q_1) = 200 - Q_1$, where $P_1$ is the price that they pay and $Q_1$ is the quantity consumed. For group 2, the inverse demand is $P_2(Q_2) = 100 - 2Q_2$. The monopolist’s cost function is $C(Q) = 40Q$.

(a) Suppose that the monopolist is unable to distinguish between the two groups of consumers, and must therefore charge them all the same price $P = P_1 = P_2$. Show with the aid of graphs that the monopolist’s total demand is given by

$$Q = Q_1 + Q_2 = \begin{cases} 0 & \text{if } P \geq 200 \\ 200 - P & \text{if } 100 \leq P \leq 200 \\ 250 - 1.5P & \text{if } 0 \leq P \leq 100 \end{cases}$$

(b) We can rewrite this demand function to obtain

$$P(Q) = \begin{cases} 0 & \text{if } Q \geq 250 \\ 166\frac{2}{3} - \frac{2}{3}Q & \text{if } 100 \leq Q \leq 250 \\ 200 - Q & \text{if } Q \leq 100 \end{cases}$$

Using this, derive the monopolist’s revenue and marginal revenue as functions of $Q$. 


Figure 1: Market Demand with Two Consumer Groups

Figure 1 shows the firm’s demand and marginal revenue curves. Note that the marginal revenue curve is discontinuous. When the quantity increases beyond 100, the second group of consumers enters the market, so marginal revenue jumps up from 0 to $33\frac{1}{3}$.

(a) Draw in the firm’s marginal cost curve and indicate the price and quantity at which the monopolist maximizes profits.

(b) At the price $P = $120, what is the monopolist’s profit? What is the consumer surplus of each group? Add these to obtain the total social surplus that is created.
(c) Now suppose that the monopolist can distinguish between the two groups of consumers, and can charge each group a different price. Derive the monopolist’s profit maximizing price for each group.

(d) At these prices \( P_1 \) and \( P_2 \), what is the monopolist’s total profit from the two markets? What is the consumer surplus in each market? Add the monopolist’s profit and the two consumer surpluses to get the total social surplus.

(e) How does your result compare to the result when the monopolist could not price discriminate?
   - Who gains?
   - Who loses?
   - How do they compare in terms of efficiency? (To compare efficiency, you need to look at the total surplus.)
2. The manager of a local movie theater suspects that demand for movies depends on when the movie is shown. Early moviegoers, who go to films before 5 p.m., are not willing to pay as much as those who go to movies in the evening. The manager does some market research and discovers that the demand curves for daytime (D), and evening (E) moviegoers are given by

\[ Q_D = 6 - P_D \]
\[ Q_E = 8 - P_E \]

The marginal cost of showing a movie is constant and equal to $4 per customer. This includes the costs of ticketing and of cleaning the seats after each movie.

(a) Suppose that the manager charges separate prices for afternoon and evening tickets.

i. What is the profit maximizing price and quantity for daytime and evening movies?

ii. What is the price elasticity of demand \((\varepsilon = \frac{dQ}{dP} \frac{P}{Q})\) in each market at these profit maximizing prices?
iii. Recall that

\[ MC = MR = P \left( 1 + \frac{1}{\varepsilon} \right) \]

Therefore, we expect that

\[ P_D \left( 1 + \frac{1}{\varepsilon_D} \right) = P_E \left( 1 + \frac{1}{\varepsilon_E} \right) \Rightarrow \frac{P_D}{P_E} = \frac{1 + \frac{1}{\varepsilon_E}}{1 + \frac{1}{\varepsilon_D}} \]

- Verify that \( \frac{P_D}{P_E} = \frac{1 + \frac{1}{\varepsilon_E}}{1 + \frac{1}{\varepsilon_D}} \) holds in this case.
- Given the elasticities you have just calculated, how would you interpret the fact that evening prices are higher and daytime prices are lower?

iv. How much profit does the theater manager earn from each type of consumer?

v. What is the consumer surplus of each type of consumer?
vi. Add the theater’s profit to the consumer surpluses to get the total social surplus.

(b) Now suppose that the manager decides to adopt a two-part pricing scheme, or two-part tariff. During the day, customers must pay a cover charge for entering the theater complex, and then a small additional ticket charge for every movie seen. The manager adopts a similar scheme for the evening shows.

i. What is the optimal charge per movie seen?

ii. What is the optimal cover charge for consumers who come during the day?

iii. What is the optimal cover charge for consumers who come in the evening?

iv. How much profit does the theater earn from each type of consumer?
v. What is the consumer surplus of each type of consumer? (There is very little math to do here. Think logically and the answer will be obvious.)

vi. Add the theater’s profit to the consumer surpluses to get the total social surplus.

(c) How do the two pricing schemes compare?

• Which is best for the theater?
• Which is best for consumers?
• Which is best for efficiency?
3. Suppose that the Agribusiness firm you work for has developed a new pesticide that is far superior to other pesticides on the market. Your firm holds a patent, which means that for the next several years you have an exclusive right to manufacture and sell the product. In other words, you are a monopolist. Marketing research indicates that your customer base consists of large corporate farmers and family farmers. A marketing research consultant has estimated the following demand curves:

\[ Q = 10 - 0.5P \]  
Demand from corporate farmers

\[ Q = 13 - P \]  
Demand from family farmers

The cost of producing the pesticide is \( C(Q) = 8Q \) where \( Q \) is quantity expressed in kilograms. There is no way to distinguish corporate farmers from family farmers. You must decide on the size of the container and the price per container in which the pesticide will be sold. Containers hold between 1-10 kilograms of pesticide. What container sizes and price per container will maximize your profits?