

Economics 335
Game Theory Problem Set

Name: _____

1. Consider the following two-person game:

		Player 2		
		L	C	R
Player 1	U	1,2	1,1	1,0
	M	3,3	5,5	2,2
	D	5,1	6,3	1,2

(a) Player 1 does not have a *dominant* strategy in this game. Explain why:

(b) Player 2 does not have a *dominant* strategy in this game. Explain why:

i. Even though there are no dominant strategies here, there are strategies that are *dominated*. Which strategy is dominated for player 1?

ii. What strategy is it dominated by?

iii. Which strategy is dominated for player 2?

iv. What strategy is it dominated by?

(c) Now, cross out the two *dominated* strategies. Look at the game that is left behind. Each player now has only two strategies, and there are only four outcomes.

i. In this smaller game, player 1 has a dominant strategy. Which one?

ii. Which strategy is dominated for player 1?

iii. Player 2 also has a dominant strategy. Which one?

iv. Which strategy is dominated for player 2?

(d) Determine the iterated dominant strategy equilibrium of the game:

i. What is the equilibrium? (What are the strategies chosen?)

ii. What are the equilibrium *payoffs* of the two players?

2. Consider the following two-person game:

		Player 2		
		L	C	R
Player 1	U	5,1	3,3	2,2
	M	3,3	5,5	2,2
	D	1,1	1,0	1,3

(a) What is player 1's best response function?

$$r^1(L) = \quad ; r^1(C) = \quad ; r^1(R) =$$

(b) What is player 2's best response function?

$$r^2(U) = \quad ; r^2(M) = \quad ; r^2(D) =$$

(c) What is (are) the pure strategy Nash equilibrium (equilibria) of this game?

(d) Using the best response functions you just worked out, show that your answer in part c. is an equilibrium (or that these are equilibria). For example, (U, L) is not an equilibrium. To show this using best response functions, note that $r^2(U) = C \neq L$.

3. Two fellow criminals are caught by police and interrogated in separate rooms. Each has two choices: collaborate with the police to get a good plea bargain, or stay mum and remain loyal to his friend. Thus, each has the strategy set: {collaborate, stay mum}. If both stay mum, then they will each be convicted of a minor offense, and will each stay in jail for a year. If one refuses to collaborate, but the other does collaborate, then the collaborator will get off with probation, but the other will go to jail for 10 years. If both collaborate, then each will get 4 years. We can express this game in normal form as follows:

		Criminal 2	
		collaborate	stay mum
Criminal 1	collaborate	-4,-4	0,-10
	stay mum	-10,0	-1,-1

- a. Each criminal has a dominant strategy.
- i. Which strategy is dominant?
 - ii. Why is it dominant?
- b. Describe the equilibrium in dominant strategies:
- i. What is the equilibrium? (What are the strategies chosen?)
 - ii. What are the equilibrium *payoffs*?
- (e) Identify another strategy combination that makes both players better off. Do you expect this outcome? Explain.

3. Two firms share a market with demand curve

$$Q = 90 - \frac{1}{2}p$$

Each has cost function

$$C(q) = 900 + q^2$$

(a) What is the outcome in this market if these firms act as price-takers? HINT: solve this problem by (i) calculating each firm's profit maximizing supply curve, (ii) calculating the market supply curve, and (iii) calculating the market equilibrium price and quantity.

- How much does each firm produce?
- What is the market price?
- What is each firm's profit?
- What is consumer surplus?

v. What is firm 1's profit-maximizing choice of q_1 when firm 2 produces q_2 units of output? (HINT: the answer will be expressed as a function of q_2 .)

vi. What is firm 2's profit-maximizing choice of q_2 when firm 1 produces q_1 ?

vii. Draw a graph of the two firms' best response functions, and illustrate the Nash equilibrium outcome.

viii. Derive the Cournot - Nash equilibrium using algebra. What are q_1 , q_2 , and p ?

ix. How do the duopolistic quantities and price compare to the competitive outcome?
Can you explain why this is the case?

x. What is the resulting profit for each firm, and how does it compare to the competitive outcome?

xi. What is the resulting consumer surplus, and how does it compare to the competitive outcome?

xii. How does the duopolistic outcome compare to the competitive outcome in terms of efficiency?

(c) Now suppose that the two firms decide to collude in order to obtain even higher profits. Thus, they agree on a price that each will charge, and they split the market and profits equally between them, i.e., each firm supplies $1/2$ of the total quantity that is produced and collects $1/2$ of the total profits.

i. What is each each firm's output quantity?

ii. How much profit does each firm earn? Compare this profit to the Cournot outcome.

- (d) We will now see that even though collusion is beneficial to these firms, it is not strategically stable. One or the other firm will have an incentive to raise output above the cartel level.
- i. Suppose that firm 1 adheres to the cartel agreement. Calculate firm 2's marginal profit—the rate of change in profits from increasing its output beyond the cartel quantity. To do this: (i) write an expression for firm 1's profits as a function of q_1 and q_2 , (ii) partially differentiate the expression with respect to q_1 , and then (iii) evaluate the expression at the cartel quantities. What does your answer tell you about firm 2's incentive to deviate from the cartel agreement?

ii. Another way to examine the incentive to deviate is to calculate the profit gain from doing so. If firm 1 adheres to the cartel agreement, what level of output is best for firm 2, and how does it compare to their agreement?

iii. How much does firm 2 gain, in terms of increased profits, by deviating from the agreement?

iv. Is the Cartel agreement a stable one? Explain.