1. Suppose that there are two beef packing firms operating in a market. There is no further threat of entry. The price for the firm’s output, boxed beef, is $12.5 and is constant, i.e., the firms are price takers in the boxed beef market. The cost of processing live cattle into boxed beef is \( c(x_i) = 0.25x_i^2 \), where \( x_i \) is the quantity of live cattle processed by firm \( i = 1, 2 \). The rate of recovery from a live animal is constant at 80% (in other words, one unit of live cattle \( x_i \) is transformed into 0.8 units of boxed beef). The beef packers face an upward-sloping supply curve from ranchers given by \( X(w) = -2 + 2w \), where \( w \) is the live cattle price.

(a) Suppose that processing firms compete non-cooperatively by choosing the quantity of live cattle to process. Set up a Cournot Duopsony game to analyze the competitive process. Identify the players, their actions, the key rules of the game, and the payoffs?

(b) Solve for each firm’s best response function.
(c) Solve for the Cournot-Nash equilibrium.

(d) Calculate the dead weight loss due to duopsony.

Now consider the beef-packing market outcome that is predicted by the Bertrand duopsony model. That is, suppose that ranchers are active price seekers that deliver their entire supply of beef to the packing plant that offers the highest price for live cattle. The packing firms compete by simultaneously choosing live cattle prices that they offer to ranchers. Denote these prices as $w_1$ for firm 1 and $w_2$ for firm 2.

(e) Write an expression for each packing firms live cattle supply curve.
(f) What is the Nash equilibrium of the cattle purchasing game? What quantity of live cattle would be purchased by each packing firm?

(g) Do packing firms prefer Cournot or Bertrand competition? Do ranchers prefer Cournot or Bertrand competition? Explain your answers.

Suppose now that packing firm 1 has signed a contract with the ranchers. The contract prohibits any rancher from selling cattle to packing firm 2 until firm 1 has satisfied its live cattle demands. In return, packer 1 has agreed to make a substantial payment to the ranchers’ producer organization. Notice that the contract guarantees that packing firm 1 has a first mover advantage.

(h) Set up a Stackelberg game to analyze the competitive process, i.e., who are the players, what are their actions, what are the rules and what are the payoffs?
(i) Solve for the Nash equilibrium outcome of the game.

(j) What is the dead weight loss due to Stackelberg competition?
2. Suppose that a fertilizer distributor in Northwestern Iowa is currently operating as a monopolist. Your company, Cyclone Fertilizer and Seed (CF&S) is contemplating opening a production facility in the region to compete with this firm. The fixed cost, which includes a fair market return on the initial investment cost of building the plant is $900. Your firm has hired a marketing agency that estimates the demand for fertilizer in the region at \( Q(p) = 100 - \frac{1}{2}p \). In the past year the monopolist sold 45 units of fertilizer in the region. The cost function for the monopolist is unknown, since the monopolist is unwilling to disclose information about its business practices. The information that you do have is that marginal costs are constant since the technology used to produce fertilizer is similar to yours. Your marginal cost is $30, but because the monopolist has been in business for some time it is possible that its unit costs are lower.

a. Based on the information that you have, what is your best estimate of the monopolist’s marginal cost?

b. Suppose that the monopolist has threatened to continue to produce 45 units if you decide to build the fertilizer plant. What is the residual demand for CF&S fertilizer if the monopolist carries out this threat? Can CF&S make a profit if the monopolist carries out its threat?
c. Should CF&S build the fertilizer plant? If so, how much should they produce and how much profits will be earned? HINT: To answer this question develop a two-stage game of entry and subsequent competition (you will have to make an assumption about how competition will proceed once you have entered the market), and solve for the subgame-perfect Nash equilibrium of the game.