Economics 335

Answers-Duopoly/Duopsony Problem Set

#1 Basic idea of this question is the same as Duopoly model, except in this case, they are buyers instead of sellers.

a) The description of this Cournot Duopsony Game is as following:

-Players: Beef packing firm i, where $i = 1, 2$

-Action: Choosing the quantity of live cattle that each firm will purchase, $x_i \in [0, \infty)$

-The key rule: Non-coorporative, simultaneous move

-Payoff: Profit π_i, where π_i can be derived as following: Since one unit of live cattle x_i is transformed into 0.8 unit of boxed beef, by defining Q_i as the quantity of boxed beef, production function can be written as $Q_i = 0.8x_i$. The cost of processing live cattle is $c(x_i) = 0.25x_i^2$ and the cost of buying live cattle is ωx_i. Thus, profit function would be

$$\pi_i = PQ_i - c(x_i) - \omega x_i$$
$$(12.5)(0.8x_i) - (0.25x_i^2) - \omega x_i$$
$$10x_i - 0.25x_i^2 - \omega x_i$$

b) To find out firm's best response function, we have to solve profit maximization problem by plug the inverse supply function, $\omega = \frac{(x_1 + x_2)}{2} + 1$, into the above profit function

$$\pi_i = 10x_i - 0.25x_i^2 - \left[\frac{(x_i + x_j)}{2} + 1\right]x_i, \quad \text{where } i,j \in \{1,2\} \text{ but } i \neq j.$$

First order condition for this problem is

$$\frac{\partial \pi_i}{\partial x_i} = 10 - \frac{1}{2}x_i - \left[\frac{x_i + x_j}{2} + 1\right] - \frac{x_i}{2} = 0$$

and by solving for x_i, one will get the best response function

$$10 - \frac{1}{2}x_i - \left[\frac{x_i + x_j}{2} + 1\right] - \frac{x_i}{2} = 0$$
$$x_i = 6 - \frac{1}{3}x_j$$

c) Solve for the Cournot Nash equilibrium. By using the best response function derived in the previous question

$$x_1 + \frac{1}{3}x_2 = 6$$
$$\frac{1}{3}x_1 + x_2 = 6$$

the solution can be obtained, which is $x_1 = \frac{9}{2}$, $x_2 = \frac{9}{2}$, $\omega = \frac{11}{2}$.

d) To find out the dead weight loss due to this Duopsony, first you need to find the competitive equilibrium solution. Due to the competitive market, the price of cattle is considered as given, or determined by the market. Thus, first order condition is

$$\frac{\partial \pi_i}{\partial x_i} = 10 - \frac{1}{2}x_i - \omega = 0$$

and the demand for cattle by each firm would be

$$x_i = 20 - 2\omega.$$

That is, the market demand for the cattle is $X(\omega) = x_1 + x_2 = 40 - 4\omega$, or $\omega = 10 - \frac{1}{4}X$. The same result can be obtained by looking at VMP_i, which will give the demand for cattle by each firm(Note: you have to be very careful when you calculate VMP_i, because in this question, total cost consists of two parts, the cost of production processing and the cost of buying input). Once, you find the market demand, it is quite clear to have competitive equilibrium solution by solving demand and supply functions

$$X = 40 - 4\omega$$
$$\omega = \frac{X}{2} + 1$$

$$Solutionis : [X = 12, \omega = 7]$$

By using the information from the market demand function at the quantity $X = 9$, $\omega\big|_{X=9} = \frac{31}{4}$, dead weight loss will be

$$DWL = \left(\frac{31}{4} - \frac{11}{2}\right)(12 - 9)\left(\frac{1}{2}\right) = \frac{27}{8} = 3.375$$

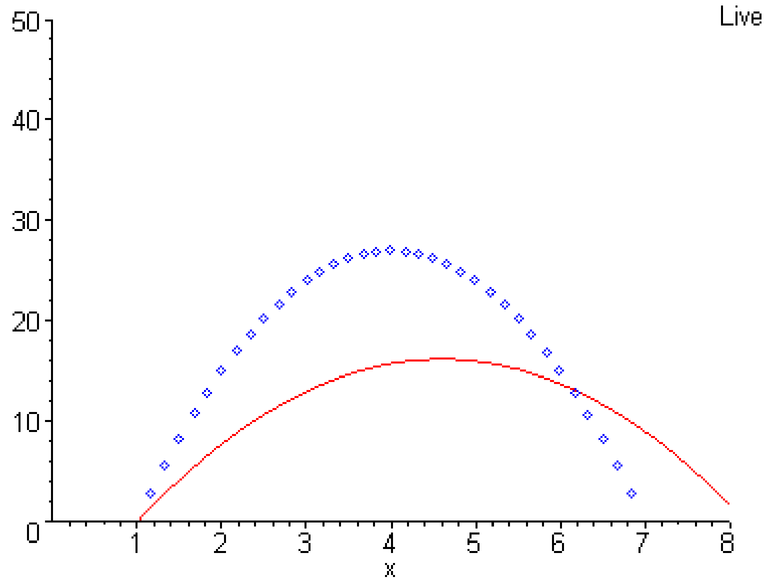
e) Now, we have to consider the Bertrand Duopsony case.
Write an expression for each packing firms live cattle supply curve

$$x_1(\omega_1, \omega_2) = \begin{cases} 0 \text{ if } \omega_2 > \omega_1 \\ X(\omega) = 2\omega - 2 \text{ if } \omega_1 > \omega_2 \\ \frac{1}{2}X(\omega) = \omega - 1 \text{ if } \omega_1 = \omega_2 \end{cases}$$

Similarly for x_2

$$x_2(\omega_1, \omega_2) = \begin{cases} 0 \text{ if } \omega_1 > \omega_2 \\ X(\omega) = 2\omega - 2 \text{ if } \omega_2 > \omega_1 \\ \frac{1}{2}X(\omega) = \omega - 1 \text{ if } \omega_1 = \omega_2 \end{cases}$$

f)(This question will not be graded) The graph blow shows firm's profit for the each case ("dot" curve shows the firm i's profit when $\omega_i > \omega_j$, $x_i = X = 2\omega - 2$, and $x_j = 0$ for i, $j \in \{1, 2\}$ but $i \neq j$)



As you can see from the graph, for most of the region, the profit is higher if only one firm is buyer of live cattle. That is, there is an incentive for each firm to raise the price of cattle until both curve intersect at the point where $\omega = 6.14$. Once, it reaches this point, there is no incentive for firms to increase the price. Therefore, equilibrium price of cattle is $\omega = 6.14$. (Note: This is a little bit different from simple Bertrand game. For simple Bertrand game, the result would be the same as competitive equilibrium result. However, in our case, because of the processing cost in addition to the cost of purchasing cattle, we could not obtain the same result as competitive equilibrium.)

g) Because of higher price of live cattle, packing firms prefer Cournot competition and ranchers prefer Bertrand competition

h) Stackelberg Game:

-Players: Packing firm i, $i \in \{1, 2\}$

-Action: Choosing the quantity of live cattle that each firm will purchase, $x_i \in [0, \infty)$

-The key rule: Firm 1 will move first
-Payoff: Profits obtained by selling boxed beef

i) To solve Stackelberg Game, first we need to look at the follower's problem (or firm 2's problem). From the result of question a), firm 2's demand for live cattle is $x_2 = 6 - \frac{1}{3}x_1$. Now, look at the leader's problem (firm 1's problem). Since firm 2's demand for live cattle depend on the firm 1's demand, thus $x_2 = 6 - \frac{1}{3}x_1$must be substituted into firm 1's profit function before it's maximization problem is solved.

$$\pi_1 = 10x_1 - 0.25x_1^2 - \left[\frac{\left(x_1 + 6 - \frac{1}{3}x_1\right)}{2} + 1 \right] x_1$$
$$= 10x_1 - 0.25x_1^2 - \left[\frac{1}{3}x_1 + 4 \right] x_1$$

The first order condition for this problem is

$$10 - \frac{1}{2}x_1 - \left[\frac{1}{3}x_1 + 4 \right] - \frac{1}{3}x_1 = 0$$

by solving for x_1, one will get $x_1 = \frac{36}{7}$. Plug this into both firm 2's demand and supply function, $x_2 = 6 - \frac{1}{3}\left(\frac{36}{7}\right) = \frac{30}{7}$, and $\omega = \frac{\left(\frac{36}{7} + \frac{30}{7}\right)}{2} + 1 = \frac{40}{7}$.

j) Since we already knew the competitive equilibrium result, so only the information we need to find out the DWL is the level of ω derived from market demand function when $X = \frac{36}{7} + \frac{30}{7} = \frac{66}{7}$.

$$\omega|_{X=66/7} = \omega = 10 - \frac{1}{4}\left(\frac{66}{7}\right) = \frac{107}{14}$$

Then, DWL would be

$$DWL = \left(\frac{107}{14} - \frac{40}{7}\right)\left(12 - \frac{66}{7}\right)\left(\frac{1}{2}\right) = \frac{243}{98}$$

#2 In this market, only one fertilizer distributor is currently operating as a monopolist, and CF&S is contemplating opening a production facility. The fixed cost is $900 and your marginal cost is $30. From the marketing agency's estimations, you know the demand for fertilizer in the region at $Q(p) = 100 - \frac{1}{2}p$. Even though your firm knows that the monopolist sold 45 units of fertilizer in the past year, firm doesn't know the cost function for the monopolist, except you know the marginal costs are constant.

a) Because you know the demand function, only you have to do is to find the marginal revenue at $Q = 45$. The inverse demand function is $p = 200 - 2Q$. Therefore, total revenue of monopolist is $TR = PQ = (200 - 2Q)Q$ and marginal revenue at $Q = 45$ is, $MR|_{Q=45} = 200 - 4(45) = 20$. Thus, estimated marginal cost would be $MC = MR = 20$

b) The inverse market demand for two firms is $p(q_1, q_2) = 200 - 2Q = 200 - 2(q_1 + q_2)$, where q_1 is the quantity level of CF&S and q_2 is that of monopolist. Because of $q_2 = 45$, the residual demand for CF&S is $p(q_1, q_2) = 200 - 2(q_1 + 45) = 110 - 2q_1$. If profit maximization problem with this residual demand shows the positive profit, of course, CF&S can make profit even though there is threat from monopolist. The profit function for CF&S with the cost function, $c(q_1) = 900 + 30q_1$, is

$$\pi_1 = (110 - 2q_1)q_1 - 900 - 30q_1$$

and

$$\frac{\partial \pi_1}{\partial q_1} = 110 - 4q_1 - 30 = 0$$

Thus, the profit maximizing quantity level is $q_1 = 20$. To check whether profit is positive, just plug this result into the profit function

$$\pi_1(q_1 = 20) = (110 - 2(20))(20) - 900 - 30(20) = -100$$

This implies that your firm can not make positive profit under this residual demand. (Note: you can solve this question by looking at price, $p(q_1, q_2)$, and CF&S average cost function, and you will get the answer $\pi_1 > 0$ if $q_1 \in (-\infty, 0]$. That is, as long as $q_1 > 0$, the it's profit will always be negative.)

c) In the previous question, the key assumption is that monopolist threatened that it will continue to produce 45 units of output and CF&S also believes that, too. However, there is no reason why monopolist will not change the quantity level if actually CF&S entered into the market. Thus, in this question, we have to consider this possibility. If CF&S decides not to enter the market, then the result will be monopoly outcome, and CF&S make no profit. On the other hand, if CF&S decides to enter the market, the result would be Cournot competition. Now, let's consider the Cournot game. CF&S's profit function can be written as

$$\pi_1 = (200 - 2(q_1 + q_2))q_1 - 900 - 30q_1$$

and first order condition for this profit maximization problem is

$$\frac{\partial \pi_1}{\partial q_1} = 200 - 2(q_1 + q_2) - 2q_1 - 30 = 0$$

By solving for q_1, one will get the best response function

$$q_1 = \frac{85}{2} - \frac{1}{2}q_2$$

For the incumbent firm, it's profit function based on the estimated marginal cost would be,

$$\pi_2 = (200 - 2(q_1 + q_2))q_2 - 20q_2$$

By taking derivative w.r.t. q_2 and solve for q_2, the best response function for incumbent firm can be obtained

$$q_2 = 45 - \frac{1}{2}q_1$$

To find out the Cournot solution, you have to solve these two best response functions, and the solution for this game is

$$q_1 = \frac{80}{3}, q_2 = \frac{95}{3}, p = \frac{250}{3}$$

Given this result, CF&S's profit will be

$$\pi_1 = \left(\frac{250}{3}\right)\left(\frac{80}{3}\right) - 900 - 30\left(\frac{80}{3}\right) = \frac{4700}{9} = 522.22$$

and incumbent's profit is

$$\pi_2 = \left(\frac{250}{3}\right)\left(\frac{95}{3}\right) - 20\left(\frac{95}{3}\right) = \frac{18050}{9} = 2005.6$$

As you can see, CF&S can make positive profit by entering the market. Therefore, this Cournot outcome should be subgame perfect NE.