Classroom Exercise, February 18

Name: ____________________________

1. Consider a monopolist with the following cost and inverse demand functions.
   \[ C(y) = 10000 + \frac{1}{20}y^2 \]
   \[ D(p) = 3800 - 20p \]
   
   a. What is this firm’s revenue as a function of \( y \)?

   b. What is this firm’s profit as a function of \( y \)?

   c. What is this firm’s marginal revenue as a function of \( y \)?

   d. What is this firm’s marginal cost function?

   e. Calculate the monopolistic (profit maximizing) price and output level. You can do this by setting \( MR = MC \), or by setting \( \frac{d\pi}{dy} = 0 \).

   f. Calculate the monopolist’s profit level.
g. Now compare this to the efficient (competitive) price and quantity. Set price equal to marginal cost and solve for \( y \). Verify that the competitive output level is higher than the monopoly output level and that the price is lower than the monopoly level.

h. What is the profit level of the firm at the efficient output level you just calculated? Verify that it is lower than the monopoly level.

i. What is the price elasticity of demand at the monopolistic output level? Verify that demand is elastic \(|\varepsilon_d| > 1\).

j. What is the marginal cost at the monopolistic output level?

k. What is the marginal revenue at this output level?

l. Verify that

\[
MC = MR = p_m \left( 1 + \frac{1}{\varepsilon_d} \right)
\]
1. Consider a monopolist with the following cost and inverse demand functions.

\[ C(y) = 10000 + \frac{1}{20}y^2 \]
\[ D(p) = 3800 - 20p \]

a. What is this firm’s revenue as a function of \( y \)?

\[ p = \frac{3800 - y}{20} \]
\[ R = py = \frac{3800 - y}{20} \cdot y = 190y - \frac{1}{10}y^2 \]

b. What is this firm’s profit as a function of \( y \)?

\[ \Pi = 190y - \frac{1}{10}y^2 - \left(10000 + \frac{1}{20}y^2\right) \]
\[ = 190y - \frac{1}{10}y^2 - 10000 \]

c. What is this firm’s marginal revenue as a function of \( y \)?

\[ MR = \frac{d}{dy} \left(190y - \frac{1}{10}y^2\right) = 190 - \frac{1}{10}y \]

d. What is this firm’s marginal cost function?

\[ MC = \frac{1}{10}y \]

e. Calculate the monopolistic (profit maximizing) price and output level.

You can do this by setting \( MR = MC \), or by setting \( \frac{d\Pi}{dq} = 0 \).

\[ MR = 190 - \frac{1}{10}y \quad \text{and} \quad MC = \frac{1}{10}y \]
\[ 190 - \frac{1}{10}y_m = \frac{1}{10}y_m \quad \Rightarrow \quad 190 = \frac{1}{5}y_m \quad \Rightarrow \quad y_m = 950 \]
\[ p_m = \frac{3800 - y_m}{20} = \frac{3800 - 950}{20} = 142.5 \]

f. Calculate the monopolist’s profit level.

\[ \Pi_m = 190y - \frac{1}{10}y^2 - 10000 = 190(950) - \frac{1}{10}(950)^2 - 10000 \]
\[ = 80250 \]

g. Now compare this to the efficient (competitive) price and quantity. Set price equal to marginal cost and solve for \( y \). Verify that the competitive output level is higher than the monopoly output level and that the price is lower than the monopoly level.

\[ p = \frac{3800 - y_c}{20} = \frac{1}{10}y_c = MC \]
\[ \Rightarrow y_c = 1266.7 > 950 = y_m \]
\[ p_c = \frac{3800 - y_c}{20} = \frac{3800 - 1266.7}{20} = 126.67 < 142.5 = p_m \]
h. What is the profit level of the firm at the efficient output level you just calculated? Verify that it is lower than the monopoly level.

\[
\Pi_e = 190y_e - \frac{1}{10}y_e^2 - 10000 = 190\left(1266\frac{2}{3}\right) - \frac{1}{10}\left(1266\frac{2}{3}\right)^2 - 10000
\]

\[
= 70222\frac{2}{3} < \Pi_m
\]

i. What is the price elasticity of demand at the monopolistic output level? Verify that demand is elastic \(|\varepsilon_d| > 1\).

\[
\varepsilon_d = \frac{dQ}{dp} \frac{P}{Q} = \frac{d}{dp}(3800 - 20p) \frac{P}{Q} = -20 \frac{P_m}{y_m} = -20 \frac{142.5}{950} = -3
\]

\[
|\varepsilon_d| = |-3| = 3 > 1
\]

j. What is the marginal cost at the monopolistic output level?

\[
MC = \frac{1}{10}y_m = \frac{950}{10} = 95
\]

k. What is the marginal revenue at this output level?

\[
MR = 190 - \frac{1}{10}y_m = 190 - 95 = 95
\]

l. Verify that

\[
MC = MR = p_m\left(1 + \frac{1}{\varepsilon_d}\right)
\]

\[
MC = MR = 95
\]

\[
p_m\left(1 + \frac{1}{\varepsilon_d}\right) = 142.5\left(1 + \frac{1}{-3}\right) = 95
\]

2. Now we are going to calculate who gains and who loses from monopolistic pricing. The following figure illustrates the result in this market.
a. What is the consumer surplus with competition?
This is the area of the triangle \( ACp_c \).
\[
\frac{1}{2} (\text{base} \times \text{height}) = \frac{1}{2} y_c (190 - p_c)
\]
\[
= \frac{1}{2} \left( 1266 \frac{2}{3} \right) \left( 190 - 126 \frac{2}{3} \right) = 40111
\]

b. What is the consumer surplus with monopoly?
This is the area of the triangle \( ABp_m \).
\[
\frac{1}{2} (\text{base} \times \text{height}) = \frac{1}{2} y_m (190 - p_m)
\]
\[
= \frac{1}{2} (950)(190 - 142.5) = 22563
\]

c. Subtract these two to get the loss to consumers from monopoly.
\[
40111 - 22563 = 17548
\]
This is the same as the area of the region \( p_mBCp_c \).

d. Now compare the profit levels in competition and monopoly \( (\Pi_m \text{ and } \Pi_c) \) to get the gain to producers from monopoly.
\[
\Pi_m - \Pi_c = 80250 - 70222 \frac{2}{9} = 10027 \frac{7}{9}
\]

e. Subtract the gains to producers from the loss to consumers to get the dead-weight loss
\[
17548 - 10027 \frac{7}{9} = 7520.2
\]

f. We can also calculate the deadweight loss graphically as the area of the triangle \( BCD \). To calculate the height of this triangle, we need the value of \( p \) at \( D \). This is \( MR \) when \( y = y_m \).
\[
MR = 190 - \frac{1}{10} y = 190 - \frac{1}{10} (950) = 95
\]
\[
\frac{1}{2} (\text{height} \times \text{width}) = \frac{1}{2} (p_m - 95)(y_c - y_m)
\]
\[
= \frac{1}{2} (142.5 - 95) \left( 1266 \frac{2}{3} - 950 \right) = 7520.8 \approx 7520.2
\]