1. An industry consists of two firms, each of which has constant marginal cost $50$ per unit produced. The market demand is given by

$$Q = 3000 - P$$

a. Describe what we mean by a firm’s “strategy” in a Bertrand model.

b. What is the Bertrand duopoly equilibrium in this market? Include in your answer prices and quantities produced by both firms.

c. Explain why this is an equilibrium
2. In this problem, you will see what happens in a duopoly model when the firms have different cost functions. Suppose that two firms exist in an industry with inverse demand curve

\[ P = 300 - \frac{2}{3}Q \]

Firm 1 has constant marginal cost 50 per unit.  
Firm 2 has constant marginal cost 80 per unit.  

a. What is the efficient outcome in this market?  
How much does each firm produce and what is the market price?  
(Think of what would happen if the firms were price-takers.)

b. Now we will calculate the Cournot Nash equilibrium.  
i. What is firm 1’s best response function?  

ii. What is firm 2’s best response function?  
(Careful! This problem is not symmetric.)
iii. Calculate the Cournot Nash equilibrium.

iv. How much profit does each firm earn?

v. How does the Cournot outcome compare to the efficient outcome? What are two inefficiencies that you see, and how can you explain them?
c. Now suppose that the two firms decide to collude and form a cartel.
   i. What is their joint profit as a function of $q_1$ and $q_2$?

ii. The firms agree on a plan that maximizes this joint profit.
    One firm will not produce at all at their optimal plan? Which firm is this?

iii. What production plan maximizes their joint profits?

iv. What profit does each firm earn?
v. How much does firm 1 have to pay firm 2 to agree not to produce? Verify that firm 1 is willing to pay this.

d. What would you predict would actually happen in an industry like this?

e. Now suppose that these two firms play a Bertrand game (rather than a Cournot game). What is the outcome in this market? *Explain your answer.*
3. John Richards owns the only poultry processing plant in the region of Roosterville.
   - For simplicity, suppose that Roosterville consists of a single road, 100 miles in length. We can express each location in town by an address $x$ which is the distance from the West end. Thus, the very West end of town is address $x = 0$. The very East end of town is address $x = 100$.
   - The plant is at the West end of town, at position $x = 0$.

   ![Plant](100 mi)

   - There are 100 chicken growers in Roosterville, each of which grows 1000 chickens per month. They are located along the road, distributed evenly at every address except $x = 0$ (where the plant is).
   - The cost of growing a chicken is $1. All growers are identical.
   - Transporting chickens is costly. Among other things, the farther you truck them, the more of them will die en route. For simplicity, assume that this cost of transportation is $.05 per chicken per mile.

a. Consider the grower who lives at address $x = 15$. If he gets $2 per chicken sold, what is his profit per chicken, taking into account transportation costs?

b. Consider the grower who lives at address $x$.
   i. If he gets $2 per chicken sold, what is his profit per chicken, taking into account transportation costs?

   ii. What is the level of $x$ at which the grower earns zero profits?

iii. Assuming that growers will grow chickens only if they won’t lose money, how many growers will supply chickens when the price offered is $2? Assume that if a grower will break even, then he will supply.
c. Suppose that John Richards offers a price of $p$ per chicken. What profit does a grower at location $x$ obtain from supplying chickens?

d. How many growers will supply chickens when the price is $p$, assuming that growers will supply only if they won't lose money?

e. Now suppose that John Richards can get $2.50 per chicken sold to supermarkets. What is his profit when he offers a price $p$ to growers?

f. Derive the profit maximizing price level.

g. At the profit maximizing price, which growers actually produce chickens and which don’t? Draw a picture giving these two sets of growers.
4. We will now vary the problem by supposing that Mr. Richards operates two plants, one at each end of town.

\[
\begin{array}{c|c}
\text{Plant 1} & \text{Plant 2} \\
\hline
\text{100 mi} & \\
\end{array}
\]

a. Assuming that there is no overlap between the growers who contract with each plant, what is his profit maxizing price strategy at plant 1? (There is no math to do.)

b. Now consider plant 2. Suppose that it offers a price \( p \).
   What profit does a grower at position \( x \) obtain by selling chickens to plant 2?

c. Calculate the critical level of \( x \) at which a grower just breaks even by supplying chickens to plant 2.

d. Now, using this level of \( x \) as a function of \( p \), calculate the profit John Rogers earns at plant 2 when he offers a price \( p \). Again assume that processed chickens can be sold for $2.50 a piece.

e. What price maximizes profit at plant 2?
f. At the equilibrium, there are three types of growers: those who sell to plant 1, those who sell to plant 2, and those who do not sell at all. Identify the levels of $x$ that describe these three types of growers. Draw a picture to illustrate your answer.

5. Continue to assume that there are two plants, but now change the assumptions, so that the cost of transportation is only $.01 per chicken per mile.
   
   a. Suppose that there is no overlap between the growers supplying chickens to the two plants. Using the same approach you used above, calculate the profit maximizing price for plant 1.
      
      i. What is the payoff of a grower at position $x$ when he sells for price $p$?

      ii. How many growers supply chickens when the price is $p$?

      iii. What is the profit of plant 1 when its price is $p$?

      iv. What is the profit maximizing price?
v. Which growers actually supply chickens? Draw a picture.

b. Given the results you just got for plant 1, what do you expect to be the result for plant 2 if their markets do not overlap? Do you notice any problem here? Explain.

c. What does the price have to be for each store for the market to be split evenly between them with no overlap (so that 50 growers sell to each store)?

d. Given that the monopolist does have to worry about markets overlapping, what is the optimal (profit maximizing) price for the two stores? Which consumers buy from each store? Explain. Draw a picture.