The next few problems will all make use of the same game. Suppose that a land-owner (he) leases land to a farmer (she). Each year, certain improvements can to be made to the land. One cannot specify which improvements should be made, since numerous conditions will influence which ones are cost effective and which are not. The cost of these improvements will be born by the farmer (who leases the land). More specifically:

- Suppose that the farmer currently rents the land for $10,000 per year. This is the normal rental price for land of this quality.
- The farmer expects to make a gross profit of $13,000 on the land, including all factor costs except cost of using the land. (Her expected net profit is $3,000 for the year.) The expected profit would be the same on another piece of comparable land.
- She faces the following decision: implement an improvement or don’t do so. The cost of the improvement is $2,000 in current expenses. The benefits are $1,000 this year and $2,000 next year. The farmer has enough cash on hand to finance the improvement in the first year.
- The landowner can observe whether the improvement has been made, but they cannot contract on the improvement because a third party will have a difficult time determining whether it has been done properly.
- At the end of the season, the landowner will offer the farmer a new lease. He must decide whether to raise the rent to $12,000 per year, or to keep the rent at $10,000 as it was the previous year. If no improvement is made, the landlord can rent the land to another farmer for $10,000. If an improvement is made, the landlord can rent the land to another farmer for $12,000 (because of the $2,000 benefit from the improvement). The cost of finding another tenant is $500.
- The farmer must decide whether to renew the lease or move. (He cannot make a counter-offer.) Assume that it costs the farmer $500 to move to a new location. If he goes to another piece of land, his expected profit is $3,000 in the second year.

The following is a game tree for the actions described above. The landowner’s payoff is listed first, then the farmer’s.
1. We will look for the subgame-perfect Nash equilibrium. To do this, we will look at the two biggest subgames, and find the Nash equilibrium in each one. Consider first the subgame that begins at node $C$, after the farmer decides not to make the improvement. In this subgame, the landowner moves first, and then the farmer (at nodes $F$ and $G$). In order for there to be a subgame perfect Nash equilibrium in the game as a whole, the players must play a Nash equilibrium in this subgame (even if they never get to it when they actually play).

a. We are looking only at the subgame which begins at node $C$, the right half of the game tree. In the table below, fill in the names of the strategies for each player and the payoffs. Remember that a strategy is not an action. It is a complete plan that says which actions you will take under which conditions. The farmer may respond differently, depending on whether the landowner raises the rent or leaves it alone. Thus, his strategy must include an action at $F$ and an action at $G$. ($2 \times 2 = 4$ possible combinations) Look at pages 16 and 20 of the lecture notes on game theory, and use it as a guide.

<table>
<thead>
<tr>
<th>Landowner</th>
<th>Farmer</th>
<th>Farmer</th>
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b. What is the Nash equilibrium in this game? Circle it above. Will the landowner raise the rent? In words, explain why or why not?

c. There is only one Nash equilibrium in this subgame, and it happens also to be subgame perfect. To check this (and to check that you found the equilibrium correctly), use backwards induction on this subgame to find the subgame perfect equilibrium. Do this in three steps:

i. Look at node $G$. The farmer must decide whether to stay on the land or leave. Here, she has not improved the land, and the landowner has not raised the rent. Which choice is rational for the farmer? Determine this by looking at her own payoffs at the two terminal nodes which she must choose between (nodes $N$ and $O$). Which node gives the farmer a higher payoff? (The landowner’s payoff is irrelevant here.)

Draw an arrow toward this node and write the name of this node in parentheses next to node $G$.

ii. Now look at node $F$. Again, the farmer must decide whether to stay or leave. This time, she has not improved the land, but the landowner has raised the rent. Which choice is rational for the farmer now? (either node $L$ or node $M$).

Draw an arrow toward this node and write the name of this node in parentheses next to node $F$. 
iii. Now look at node $C$, where the landowner must decide whether to raise the rent after the farmer decides not to improve the land. In making this choice, the landowner assumes that the farmer would make a rational choice at both $F$ and $G$. You just worked out what the farmer should do at $F$ and $G$. Of the two nodes you calculated in parts (i) and (ii), which gives the landowner a higher payoff?

Which choice is rational for the landowner? Should he choose node $F$ or node $G$?

Draw an arrow toward this node.

iv. What is the subgame perfect equilibrium of the subgame beginning at $C$? Which final node will the players reach if the farmer does not make the improvement (either $L$, $M$, $N$, or $O$)?

Circle this node and write its name in parentheses next to node $C$.
Check that it is the same as the Nash equilibrium you found in part (a).

2. Now, consider the other big subgame, the one which begins at node $B$, after the farmer decides not to make an improvement.

a. Again, express this subgame in normal form in the table below:

<table>
<thead>
<tr>
<th>Farmer</th>
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b. There are two Nash equilibria in this game. Circle them in the table above. Which do you think is most plausible and why?

c. We will use the concept of subgame perfection to choose between these two equilibria. Again, we will use backwards induction.

i. Look at node $E$. Here the farmer must decide whether to stay or leave. Which choice is rational (node $J$ or $K$)?

Draw an arrow toward this node and write its name next to node $E$.

ii. Look at node $D$. Which choice should the farmer make ($H$ or $I$)?

Draw an arrow toward this node and write its name next to node $D$. 

3
iii. Now look at node B. The landlord must decide whether to raise the rent or not. He will assume that the farmer would be rational at nodes D and E. You just worked out what the farmer should do at D and E. Of the two nodes you calculated in parts (i) and (ii), which gives the landowner a higher payoff?

Assuming that the landlord knows your answers to parts i and ii, what node should the landlord choose at B (D or E)?

Draw an arrow toward this node.

iv. What is the subgame perfect equilibrium of the subgame beginning at B? Which final node is reached when the farmer does do the improvement (either H, I, J, or K)?

Circle this node and write its name next to node B.

v. Which of the two Nash equilibria is not subgame-perfect? Place a star by the other one.

3. Now suppose that the farmer knows that, whichever choice she makes at node A, a subgame perfect Nash equilibrium will follow in the ensuing subgame. In other words, she assumes that, if they start at node C, they will reach the node you wrote down as the answer to 2.c.iv above. (You wrote the name of this node next to node C.) If they start at B, then they will reach the node you wrote down as the answer to 3.c.iv above. (You wrote the name of this node next to node B.).

a. Which node is better for the farmer? The answer to 2.c.iv, or the answer to 3.c.iv?

b. So which node is better for the farmer, node B or node C?

c. Should the farmer make the improvement? Why or why not?
4. Write out fully the strategies of both players at the subgame perfect Nash equilibrium. Recall that a strategy is a full plan of action. For the landowner, a strategy says which actions to take at nodes C and D. (So his strategy has two items in it.) For the farmer, a strategy says which actions to take at nodes A, D, E, F, and G. (So her strategy has five items in it).
   a. Landlord’s strategy:

   b. Farmer’s strategy:

   c. How is this equilibrium an example of the hold-up problem?

5. Now suppose that we change the game a little. Now the farmer and the landlord can sign a two-year lease, so that the rent in the second year is predetermined. The landlord cannot raise the rent. To be precise, suppose that they agree that the rent will be $10,250 in both years. Then the game becomes a very simple one. The farmer simply chooses whether to do the improvement, and then the players have their payoffs.
   a. What are the players’ payoffs if the farmer makes the improvement?

   b. What are the players’ payoffs if the farmer does not make the improvement?

   c. Will the farmer make the improvement?

   d. What are the payoffs of the two players? How do they compare to the payoffs in the previous game?

   e. Explain why a longer term lease solves the hold-up problem in this case?
6. Consider two firms in a Cournot duopoly. Firm 1 produces $q_1$ units and has constant marginal cost $c_1 = 40$. Firm 2 produces $q_2$ units and has constant marginal cost $c_2 = 56$. The inverse demand curve is

$$P = 120 - 20Q$$

where $Q = q_1 + q_2$.

a. Calculate the Cournot Nash equilibrium.

i. What is the price as a function of $q_1$ and $q_2$?

ii. What is firm 1’s profit as a function of $q_1$ and $q_2$?

iii. Calculate firm 1’s best response function.

iv. What is firm 1’s profit as a function of $q_1$ and $q_2$?

v. What is firm 2’s best response function?

vi. Combine these to get the Nash equilibrium.

vii. What is the equilibrium price?
viii. What is each firm’s profit level?

b. Now suppose that firm 1 acts as the Stackelberg leader.
   i. Suppose that firm 1 assumes that firm 2 will play its best response level of $q_2$, taking $q_1$ as fixed. What is firm 1’s profit as a function of $q_1$?

   ii. Calculate firm 1’s optimal level of $q_1$.

   iii. What are the levels of $q_2$ and $P$ at the equilibrium?

   iv. What is each firm’s profit level?

   v. What do you notice?

   vi. Explain why this outcome is a subgame-perfect equilibrium. What is the game being played here?