Economics 335, Spring 1999  
Problem Set #13 (last one)

Due Thursday April 29, in class. This problem set is worth 22 points.

1. Consider the following payment stream.
   Let the flow at the end of period -1 be -200 with a further return at the end of period 0 of -100. Let the returns at the end of periods 1 through 4 be 20, 100, 200 and 500. The number line is as follows:

   ![Number Line]

Assume an interest rate of 5%.

a. Describe in practical terms what such an income stream means. Assume that point 0 indicates the present. Points -1, -2, etc. indicate the past, and points 1, 2, etc. indicate the future.

b. At point 0, what is the discounted value of $-100 received at point 0?

c. At point 0, what is the discounted value of $-200 received at point -1?

d. At point 0, what is the discounted value of $20 received at point 1?

e. At point 0, what is the discounted value of $100 received at point 2?

f. At point 0, what is the discounted value of $200 received at point 3?

g. At point 0, what is the discounted value of $500 received at point 4?

h. At point 0, what is the value of the entire income stream?

i. Describe in words what such a "value" means. What practical relevance can it have?
2. A biotechnology firm is considering sending ten of its staff to a training program which will train them in the use of a new technology. The cost of the training program is $3000 per employee. The expected increase in productivity for each worker is $1000 per year. The first benefits will be enjoyed at the end of the first year after the training takes place.

   a. i. Assume that the firm expects its workers to remain with the firm for three years. What is the present discounted benefit of the training program (in terms of increased productivity), if the interest rate is 10%. (You need to add the value for each of the five years, using the appropriate discount factor.)

   ii. Comparing the cost with the benefit, is the training program a good investment? Explain this result intuitively. Why would you expect to arrive at this answer, even before doing any calculations?

   b. i. Now suppose that workers can be expected to remain with the firm for four years. Now what is the present discounted benefit of the training program when the interest rate is 10%?

   ii. Is the training program worthwhile? Why or why not? What is the profit per employee (in present value terms) from doing the training?
3. It is January 15, 1999. Susan Myerson is expecting a tax refund of $2000. Her taxes are very simple, and she can easily fill out the form herself. (This will take her about two hours.) She sees an advertisement from a firm that promises to file her taxes for her and give her the refund immediately (on January 15), minus a 10% service charge. She is tempted to take them up on their offer, since she is very short of cash. Also, if they do her taxes, it will take next to no time to get it done.

Susan also has a Visa card, with an annual percentage rate of 18%, and if she wants to, she can borrow $2000 directly from the credit card company by not paying her bill in full. If she files her taxes herself, she can expect her refund by February 15.

Susan values her time at $10 per hour.

The question is: Should Susan file her taxes herself, or should she let them do it for her?

a. What is the present discounted value of letting the company file her taxes for her? Remember to take into account the service fee.

b. What is the present discounted value of the refund of $2000, if she receives it on February 15? Remember that her interest rate is 18% per year.

c. Now, taking into account the value of her time, what is the present discounted value of doing her taxes herself?

d. Should Susan do her own taxes? Why or why not?
4. Consider again the biotechnology firm in question 2. Suppose that it is not sure how long its employees will stay with the firm. On average, they will stay for four years, but some will stay longer and some will leave earlier. The firm expects 10% of people to leave after 1 year, 15% to leave after 2 years, 20% to leave after 3 years, 20% to leave after 4 years, 25% to leave after 5 years, and another 10% to stay for 6 years.

a. Assume that the firm is risk-neutral, and it therefore cares only about its expected profit. It all averages out over the long run. Again, the interest rate is 10% and the gain in productivity per year employed is $1000.
   i. What is the productivity gain from the course if an employee stays for only 1 year?

   ii. What is the productivity gain from the course if an employee stays for 2 years?

   iii. What is the productivity gain from the course if an employee stays for 3 years?

   iv. What is the productivity gain from the course if an employee stays for 4 years?

   v. What is the productivity gain from the course if an employee stays for 5 years?

   vi. What is the productivity gain from the course if an employee stays for 6 years?

   vii. Average these to get the average gain in productivity.

   viii. Is the investment worthwhile? Why or why not?
5. Consider a farmer who is considering entering the futures market in corn at the beginning of the season. If the farmer sells a futures contract, he gets money now and promises to deliver a certain amount of corn at some future date. The current price of a corn future is $950 per ton for September delivery. If the farmer sells a future, he can either deliver his own corn, or he must pay the current price of corn to whoever holds the future at the end of the season.

The price of corn depends on production. If growing weather is good, output will be higher, and the price will be lower. If weather is bad, output will be lower, and the price will be higher. The farmer is better off if the weather is bad, since the price elasticity of demand for corn is extremely high and the price of corn can fall drastically when supply is too high. The farmer might also have problems on his own land that are unrelated to the general quality of the growing season. Assume that the farmer’s costs are not subject to uncertainty. The following are the possible states of the world at the end of the growing season:

- **State 1 (38%)**: Market output is high, and so is the farmer’s own output. The selling price of corn is $800 per ton and the farm produces 13 tons.

- **State 2 (10%)**: Market output is high, but the farmer’s own output is not as high as expected. The selling price of corn is $800 per ton and the farm produces 11 tons.

- **State 3 (42%)**: Market output is low, and so is the farmer’s own output. The selling price of corn is $1100 per ton and the farmer’s output is 11 tons.

- **State 4 (10%)**: Market output is low, but the farmer’s own output is surprisingly good. The selling price of corn is $1100 per ton, and the farm produces 13 tons.

a. Suppose that the farmer does not sell any corn futures. What is his wealth level in each state of the world?

- **State 1**:

- **State 2**:

- **State 3**:

- **State 4**:

- Expected value of wealth:
b. Suppose that the farmer sells 5 tons of corn under a futures contract. What is his wealth level in each state of the world?

- State 1:
- State 2:
- State 3:
- State 4:

- Expected value of wealth:

c. The farmer must decide between two actions:

- Don’t sell any futures.
- Sell 5 tons with a futures contract.

(For simplicity, suppose that other choices are not available.)

Suppose that the farmer is risk-neutral. Which choice will he make and why?

d. What is the farmer’s scaling function if he is risk neutral?

\[ u(v) = \]

e. Now suppose that the farmer is risk-averse. Specifically, suppose that his scaling function is

\[ u(v) = \sqrt{v} \]

This function is very concave. The slope decreases rapidly as \( v \) rises.

i. What is his expected utility if he doesn’t sell any futures?
ii. What is his expected utility if he does sell 5 tons under a futures contract?

iii. Which action will the farmer take?

f. Explain why the farmer’s decision changes.

g. Now assume that the farmer is risk-averse, but he is not as risk averse as before. Suppose now that

\[ u(v) = \sqrt{v} \]

This function is not ”as concave” as the one in part (d).

i. What is his expected utility if he doesn’t sell any futures?

ii. What is his expected utility if he does sell 5 tons under a futures contract?

iii. Which action will the farmer take?

iv. Explain why the farmer’s decision changes.
Mary More is considering buying crop insurance. If she produces a successful crop, she expects it to sell for $100,000. However, there is a 4% chance that, due to circumstances beyond her control, half of her crop will be destroyed (bringing her receipts down to $50,000), and there is a 1% chance of losing the whole crop.

a. What is her expected crop loss in dollars?

b. Suppose that Mary has initial wealth of $100,000. She must spend $50,000 out of pocket on operating expenses. Thus, in a normal year, she will make a profit of $50,000. If she loses the entire crop, her loss is $50,000. What is her total wealth in the following circumstances:
   i. She has a normal crop.
   ii. She loses half the crop.
   iii. She loses the entire crop.

c. Mary has scaling function \( u(x) = \ln x \). What is her expected utility if she does not buy insurance?

d. Norwest Bank is offering crop insurance. It can insure her for all losses for $3,000, paid up-front. What is Mary’s expected utility if she buys the insurance?

e. Should Mary buy insurance? Why or why not?

f. Look again at your result in (c). Calculate the certainty equivalent of her expected utility when she does not buy insurance. (Look at the sample problems for help.)

g. What is the most money that Mary is willing to spend on crop insurance? In other words, how much money, at most, could she spend, and still be no worse off than if she did not buy insurance? Verify that this amount is more than her expected loss.